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On RGWA-locally closed sets in Topological Spaces

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Abstract
In this paper, we introduce three weaker forms of locally closed sets called RGWα-LC sets, RGWα-LC* set and RGWα-LC** sets each of which is weaker than locally closed set and study some of their properties and their relationships with w-lc, θg-lc, lδc, π-lc and rg-lc etc sets.

Keywords: RGWα-closed sets, rgwα-open sets, locally closed sets, rgwα-locally closed sets

Introduction

Kuratowski and Sierpinski [8] introduced the notion of locally closed sets in topological spaces. According to Bourbaki [4], a subset of a topological space (X, τ) is locally closed in (X, τ) if it is the intersection of an open set and a closed set in (X, τ). Stone [12] has used the term FG for locally closed set. Ganster and Reilly [5] have introduced locally closed sets, which are weaker forms of both closed and open sets. After that Balachandran *et al.* [3], Gnanambal [6], Arockiarani *et al.* [1], Pusphalatha [10] and Sheik John [11] have introduced α-locally closed, generalized α-locally closed, semi locally closed, semi generalized locally closed, regular generalized locally closed, strongly locally closed and w-locally closed sets and their continuous maps in topological space respectively. Recently rgwα-closed sets and continuous maps were introduced and studied by Wali *et al.* [13, 15].

2. Preliminaries: A subset A of topological space (X, τ) is called a

1. Locally closed (briefly LC) set [5] if A=U⋂F, where U is open and F is closed in X.
2. regular generalized weakly α-closed set [13] (briefly rgwα-closed set) if rαcl(A)⊆U whenever A⊆U and U is wα-open in (X, τ).
3. regular generalized weakly α-open set if A^c is a [14] rgwα-closed.
4. θg-lc set [2] if A=U⋂F, where U is θg-open and F is θg-closed in X.
5. θg-lc* set [2] if A=U⋂F, where U is θg-open and F closed in X.
6. θg-lc set** [2] if A=U⋂F, where U is open and F θg-closed in X
7. g-lc set [3] if A=U⋂F, where U is g-open and F is g-closed in X.
8. g-lc* set [3] if A=U⋂F, where U is g-open and F closed in X.
9. g-lc set** [3] if A=U⋂F, where U is open and F g-closed in X.
10. w-lc set if A=U⋂F [11] where U is w-open and F is w-closed in X.
11. w-lc* set if A=U⋂F [11] where U is w-open and F closed in X.
12. w-lc** set if A=U⋂F [11] where U is open and F is w-closed in X.
13. rg-lc set if A=U⋂F [1] where U is g-open and F rg-closed in X.
14. rg-lc* set if A=U⋂F [1] where U is g-open and F closed in X.
15. rg-lc** set if A=U⋂F [1] where U is open and F is rg-closed in X.
16. lδg-lc set if A=U⋂F [9] where U is lδg-open and F is lδg-closed in X.
17. lδg-lc* set if A=U⋂F [9] where U is lδg-open and F closed in X.
18. lδg-lc** set if A=U⋂F [9] where U is open and F is lδg-closed in X.
19. πg-lc set if A=U⋂F [12] where U is πg-open and F is πg-closed in X.
20. πg-lc* set if A=U⋂F [12] where U is πg-open and F is closed in X.
21. πg-lc** set if A=U⋂F [12] where U is open and F is πg-closed in X.

Example 2.1: Let X = {a, b, c} and τ ={X, ϕ, {a}, {b}, {a, b}}. Here
Closed sets: {X, ϕ, {c}, {a, c}, {b, c}}
θ-closed sets: {X, ϕ}

θg-closed sts: {X, ϕ,{c},{a, c}, {b, c}}
δ-closed sets: {X, ϕ}
δ g-closed sts: {X, ϕ,{c},{a, c},{b, c}}
π-closed sets: {X, ϕ, {c},{a, c},{b, c}}
πg-closed sts: {X, ϕ, {c},{a, c},{b, c}}
w-closed sets: {X, ϕ, {c},{a, c},{b, c}}
g-closed sets: {X, ϕ, {c}, {a, c}, {b, c}}
rg-closed sets: {X, ϕ, {c}, {a, b}, {a, c}, {b, c}}
rgwα-closed sets: {X, ϕ, {a}, {b}, {c}, {a, c}, {b, c}}

Example 2.2: Let X = {a, b, c} and τ ={X, ϕ, {a}}. Here
Closed sets: {X, ϕ, {b, c}}
θ-closed sets: {X, ϕ}
θg-closed sts: {X, ϕ, {b}, {c}, {a, b}, {a, c}, {b, c}}
δ-closed sets: {X, ϕ}
δ g-closed sts: {X, ϕ, {b}, {c}, {a, b}, {a, c}, {b, c}}
π-closed sets: {X, ϕ}
πg-closed sts: {X, ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}}
w-closed sets: {X, ϕ, {b, c}}
g-closed sets: {X, ϕ, {b}, {c}, {a, b}, {a, c},{b, c}}
rg-closed sets: {X, ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}}
rgwα-closed sets: {X, ϕ, {b, c}}

Lemma 2.3: By seeing above example we can say
(i) Every closed set is rgwα-closed set.
(ii) Every w-closed set is rgwα-closed set.
(iii) Every θ-closed set is rgwα-closed set.
(iv) Every δ-closed set is rgwα-closed set.
(v) Every δ-closed set is rgwα-closed set
(vi) rgwα-closed set and δg-closed, θg-closed, πg-closed g-closed and rg-closed sets are independent of each other.

Lemma 2.4: The space (X, τ) is $\tau_{rgw\alpha}$-space if every rgwα-closed set is closed set.

3. rgwα-Locally Closed Sets in Topological Spaces
Definition 3.1: A Subset A of a Topological space (X, τ) is called regular generalized weakly α-locally closed (briefly rgwα-locally closed) if A=U∩F where U is rgwα-open in (X, τ) and F is rgwα-closed in (X, τ).
The set of all rgwα-locally closed sets of (X, τ) is denoted by RGWα-LC(X, τ).

Example 3.2: Let X = {a, b, c, d, e} and τ ={X, ∅, {a}, {d}, {e}, {a, d}, {a, e}, {d, e}, {a, d, e}}
RGWα C (X, τ)= {X, ∅, {a}, {b}, {c}, {d}, {e}, {a, b}, {a, c}, {a, e}, {b, c}, {b, d}, {b, e}, {c, d}, {c, e}, {a, b, c}, {a, b, d}, {a, b, e}, {a, c, e}, {b, c, d}, {b, c, e}, {b, d, e}, {c, d, e}, {a, b, c, d}, {a, b, d, e}, {a, b, c, e}, {a, c, d, e}, {b, c, d, e}}.

RGWα O (X, τ) = {X, ∅, {a}, {b}, {c}, {d}, {e}, {a, b} {a, c}, {a, d}, {a, e}, {b, d}, {c, d}, {c, e}, {d, e}, {a, b, d}, {a, b, e}, {a, c, d} {a, c, e}, {a, d, e}, {b, c, d}, {b, d, e}, {c, d, e}, {a, b, c, d}, {a, b, d, e}, {a, b, c, e}, {a, c, d, e}, {b, c, d, e}}.

RGWα-LC Set ={X, ∅, {a}, {b}, {c}, {d}, {e}, {a, b}, {a, c}, {a, d}, {a, e}, {b, c}, {b, d}, {b, e}, {c, d}, {c, e}, {d, e}, {a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e}, {a, d, e}, {b, c, d}, {b, c, e}, {b, d, e}, {c, d, e}, {a, b, c, d}, {a, b, d, e}, {a, b, c, e}, {a, c, d, e}, {b, c, d, e}}.

Remark 3.3: The following are well known
(i) A Subset A of (X, τ) is RGWα-LC set iff its complement X-A is the union of a rgwα-open set and a rgwα-closed set.
(ii) Every rgwα-open (resp. rgwα-closed) subset of (X, τ) is a RGWα-LC set.

Theorem 3.4: Every locally closed set is a RGWα-LC set but not conversely.
Proof: The proof follows from definition and fact that every closed (resp. open) set is rgwα-closed (rw-open).

Example 3.5: Let X = {a, b, c, d, e} and τ ={X, ∅, {a}, {d}, {e}, {a, d}, {a, e}, {d, e}, {a, d, e}}.
LC-Set={X, ϕ, {a}, {d}, {e}, {a, d}, {a, e}, {b, c}, {d, e}, {a. b. c}, {a, d, e}, {b, c, d}, {a. c. e}, {a, b, c, d}, {a, b, c, e}, {b, c, d, e}}. RGWα-LC-Set={X, ∅, {a}, {b}, {c}, {d}, {e}, {a, b}, {a, c}, {a, d}, {a, e}, {b, c}, {b, d}, {b, e}, {c, d}, {c, e}, {d, e}, {a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e}, {a, d, e}, {b, c, d}, {b, c, e}, {b, d, e}, {c, d, e}, {a, b, c, d}, {a, b, d, e}, {a, b, c, e}, {a, c, d, e}, {b, c, d, e}}. Here the set {a, b} is RGWα-LC-Set but not LC-set.

Theorem 3.6: Every w-lc set is a RGWα-LC set but not conversely.
Proof: The proof follows from definition and fact that every w-closed (resp. w-open) set is rgwα-closed (rgwα-open).

Example 3.7: Let X={a, b, c, d} and τ ={X, ϕ, {a}, {b}, {a, b}, {a, b, c}} then {a, d} is RGWα-LC set but not a w-locally closed set in (X, τ).

Theorem 3.8: Every θ-lc set is a RGWα-LC set but not conversely.
Proof: The proof follows from definition and fact that every θ-closed (resp. θ-open) set is rgwα-closed (rgwα-open).

Example 3.9: Let X={a, b, c, d} and τ ={X, ϕ, {a}, {b}, {a, b}, {a, b, c}} then {a} is RGWα-LC set but not a θ-locally closed set in (X, τ).

Theorem 3.10: Every lδc set is a RGWα-LC set but not conversely.
Proof: The proof follows from definition and fact that every δ-closed (resp. δ-open) set is rgwα-closed (rgwα-open).

Example 3.11: Let X={a, b, c} and τ ={X, ϕ, {a}, {b}, {a, b}} then {a, b} is RGWα-LC set but not a lδc set in (X, τ).

Theorem 3.12: Every π-lc set is a RGWα-LC set but not conversely.
Proof: The proof follows from definition and fact that every π-closed (resp. π-open) set is rgwα-closed (rgwα-open).

Example 3.13: Let X={a, b, c, d} and τ ={X, ϕ, {a}, {b}, {a, b}, {a, b, c}} then {b, c} is RGWα-LC set but not a π-locally closed set in (X, τ).

Remark 3.14: lgc set, θ-g lc set, lδgc set, πg-lc set, rg-lc sets and RGWα-LC sets are independent of each other as seen from the following example.

Example 3.15: i) Let X ={a, b, c} and τ ={X, ϕ, {a}} then {a, c} is lgc set, θ-g lc set, lδgc set, rg-lc set, but not RGWα-LC set in (X, τ).

ii) Let X = {a, b, c} and τ ={X, ϕ, {a}, {b}, {a, b}} then {b} is RGWα-LC set, but not lgc set, θ-g lc set, lδgc set, πg-lc set rg-lc set in (X,τ).

Definition 3.16: A subset A of (X, τ) is called a RGWα-LC* set if there exists a rgwα-open set G and a closed F of (X, τ) s.t A=G⋂F the collection of all RGWα-LC* sets of (X, τ) will be denoted by RGWα-LC*(X, τ).

Definition 3.17: A subset B of (X, τ) is called a RGWα-LC** set if there exists an open set G and rgwα- closed set F of (X, τ) s.t B=G⋂F the collection of all RGWα-LC** sets of (X, τ) will be denoted by RGWα-LC**(X, τ).

Theorem 3.18:
1. Every locally closed set is a RGWα-LC* set.
2. Every locally closed set is a RGWα-LC** set.
3. Every w-lc* set is RGWα-LC* set.
4. Every w-lc**set is RGWα-LC** set.
5. Every θ-lc* set is a RW-LC* set.
6. Every θ-lc** set is a RW-LC** set.
7. Every lδc* set is a RW-LC* set.
8. Every lδc** set is a RW-LC** set.
9. Every π-lc* set is a RW-LC* set.
10. Every π-lc** set is a RW-LC** set.
11. Every RGWα-LC* set is RGWα-LC set.
12. Every RGWα-LC** set is RGWα-LC set.
13. Every RGWα-LC* set is rg-lc* set.
14. Every RGWα-LC** set is rg-lc** set.
15. Every RGWα-LC* set is rg-lc set.
16. Every RGWα-LC** set is rg-lc set.

Proof: The proofs are obvious from the definitions and the relation between the sets.
However the converses of the above results are not true as seen from the following examples.

Example 3.19: Let X = {a, b, c, d} and τ ={X, ϕ, {a}, {b}, {a, b}, {a, b, c}}
(i) The set {a, d} is RGWα-LC* set but not a locally closed set in (X, τ).
(ii) The set {b, d} is RGWα-LC** set but not a locally closed set in (X, τ).
(iii) The set {a, b, d} is RGWα-LC* set but not a w-lc* set in (X, τ).
(iv) The set {b, d} is RW-LC** set but not a w-lc** set in (X, τ).

Example 3.20: Let X = {a, b, c} and τ ={X, ϕ, {a}, {b}, {a, b}}
(i) The set {a} is RW-LC* set but not a θ-lc* set in (X, τ).

(ii) The set {b} is RW-LC** set but not a θ-lc** set in (X, τ).
(iii) The set {c} is RW-LC* set but not a lδc* set in (X, τ).
(iv) The set {a, b} is RW-LC** set but not a lδc** set in (X, τ).

Example 3.21: Let X = {a, b, c} and τ ={X, ϕ, {a}}
(i) The set {a} is RW-LC* set but not a π-lc* set in (X, τ).
(ii) The set {b, c} is RW-LC** set but not a π-lc** set in (X, τ).

Example 3.22: Let X={a, b, c, d} and τ ={ X, ϕ, {a}, {b}, {a, b}, {a, b, c}}
(i) The set {c} is RGWα-LC set but not a RGWα-LC* set in (X, τ).
(ii)The set {a, b, d} is RGWα-LC set but not a RGWα-LC** set in (X, τ).

Example 3.23: Let X = {a, b, c} and τ ={X, ϕ, {a}}
(ii) The set {a, b} is rg-lc* set but not a RGWα-LC* set in (X, τ).
(iii) The set {a, c} is rg-lc** set but not a RGWα-LC** set in (X, τ).
(iv) The set {b} is rg-lc set but not a RGWα-LC* set in (X, τ).
(v) The set {c} is rg-lc set but not a RGWα-LC** set in (X, τ).

Remark 3.24: lgc* set, θ-glc* set, lδgc* set, πg-lc* sets and RGWα-LC* sets are independent of each other and lgc** set, θ-glc** set, lδgc** set, πg-lc** sets and RGWα-LC** sets are independent of each other as seen from the following examples.

Example 3.25: i) Let X={a, b, c, d} and τ ={ X, ϕ, {a}, {b}, {a, b}, {a, b, c}}. Here
{a, d} is RGWα-LC* but not θ-glc* set and lδgc* set in (X, τ).
{a, b, c} is RGWα-LC* but not πg-lc* set in (X, τ).
{c} is RGWα-LC** but not θ-glc** set and lδgc** set in (X, τ).
{a, b, d} is RGWα-LC** but not πg-lc** set in (X, τ).

ii) Let X = {a, b, c} and τ ={X, ϕ, {a}}. Here
{a, c} is θ-glc* set, lδgc* set, πg-lc* set, but not RGWα-LC* set in (X, τ).
{b} is θ-glc** set, lδgc** set, πg-lc** set, but not RGWα-LC** set in (X, τ).

Example 3.26: i) Let X = {a, b, c} and τ ={X, ϕ, {a}}. Here
{a, b} is lgc* set, but not RGWα-LC* set in (X, τ).
{a, c} is lgc** set, but not RGWα-LC** set in (X, τ).

ii) Let X= {a, b, c, d} and τ ={X, ϕ, {a}, {b}, {a, b}, {a, b, c}}. Here
{b, d} is RGWα-LC* but not lgc* set in (X, τ).
{c} is RGWα-LC** but not lgc** set in (X, τ).

Remark 3.27: RGWα-LC* sets and RGWα-LC** sets are independent of each other as seen from the examples.

Example 3.28: (i) Let X= {a, b, c, d} and τ ={X, ϕ, {a}, {b}, {a, b}, {a, b, c}} then set {a, b, d} is RGWα-LC**
(ii) Let X= {a, b, c, d} and τ = {X, ϕ, {a}, {b}, {a, b}, {a, b, c}} then set {c} is RGWα-LC* set but not a RGWα-LC** set in (X, τ). Set but not a RGWα-LC* set in (X, τ).τ).

Remark 3.29: From the above discussion and known results we have the following implications in the diagram.

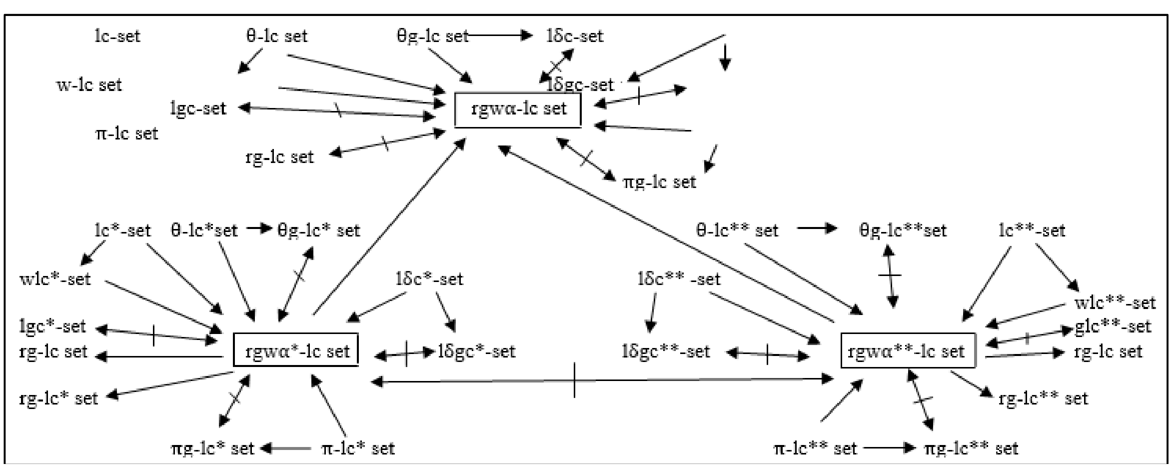

Fig 1

In the above diagram by A →B we mean A implies B but not conversely and
A↮B means A and B are independent of each other.

Theorem 3.30: If RGWαO(X, τ) = τ then
(i) RGWα-LC(X, τ) = LC (X, τ).
(ii) RGWα-LC(X, τ) = W-LC (X, τ).
(iii) RGWα-LC(X, τ) ⊆ GLC (X, τ).

Proof: (i) For any space (X, τ), W.k.t LC(X, τ) ⊆ RGWα-LC(X, τ). Since RGWαO(X, τ) = τ, that is every rgwα-open set is open and every rgwα-closed set is closed in (X, τ), RGWα-LC(X, τ) ⊆LC (X, τ); hence RGWα-LC(X, τ) = LC (X, τ).
(ii) For any space (X, τ), LC(X, τ) ⊆W-LC(X, τ) ⊆RGWα-LC(X, τ). From (i) it follows that RGWα-LC(X, τ) = W-LC(X, τ).
(iii) For any space (X, τ), LC(X, τ) ⊆GLC (X, τ) from (i) RGWα-LC(X, τ) = LC (X, τ) and hence RGWα-LC(X, τ) ⊆GLC (X, τ).

Theorem 3.31: If RGWαO(X, τ) = τ, then RGWα-LC*(X, τ) = RGWα-LC** (X, τ) = RGWα-LC(X, τ).

Proof: For any space (X, τ), LC (X, τ) ⊆ RGWα-LC*(X, τ) ⊆ RGWα-LC(X, τ) and LC (X, τ) ⊆ RGWα-LC**(X, τ) ⊆ RGWα-LC(X, τ) since RGWαO(X, τ) = τ. RGWα-LC(X, τ) =LC(X, τ) by theorem 3.30, it follows that LC(X, τ) = RGWα-LC*(X, τ) = RGWα-LC**(X, τ) = RGWα-LC(X, τ).

Remark 3.32: The converse of the theorem 3.31 need not be true in general as seen from the following example.

Example 3.33: Let X = {a, b, c} with the topology τ = {X, ϕ, {a}, {b}, {a, b}} then RGWα-LC*(X, τ) = RGWα-LC**(X, τ) = RGWα-LC(X, τ) =P(X). However RGWαO(X, τ) ={X, ϕ, {a}, {b}, {a, b}, {b, c}, {a, c}} ≠ τ.

Theorem 3.34: If GO(X, τ) = τ, then GLC(X, τ) ⊆ RGWα-LC(X, τ)

Proof: For any space (X, τ) w.k.t LC(X, τ) ⊆ GLC(X, τ) and LC (X, τ) ⊆ RGWα-LC(X, τ).....(i) GO(X, τ) =τ, that is every g-open set is open and every g-closed set is closed in (X, τ) and so GLC(X, τ) ⊆LC(X, τ) that is GLC(X, τ) =LC(X, τ)......(ii) from (i) and (ii) we have GLC(X, τ) ⊆ RW-LC(X, τ).

Theorem 3.35: If RGWαC(X, τ) ⊆ LC(X, τ) then RGWα-LC(X, τ) = RGWα-LC*(X, τ)

Proof: Let RGWαC(X, τ) ⊆ LC(X, τ), for any space (X, τ), w.k.t RGWα-LC*(X, τ) ⊆ RGWα-LC(X, τ)... (i) Let A∈ RGWα-LC(X, τ), then A= U⋂F, where U is rgwα-open and F is a rgwα-closed in (X, τ). Now, F⊆ RGWα-LC(X, τ) by hypothesis F is locally closed set in (X, τ), then F= G⋂E, where G is an open set and E is a closed set in (X, τ). Now, A= U⋂F=U⋂ (G⋂E)= (U⋂G)⋂ E, where U⋂G is rgwα-open as the intersection of rgwα-open sets is rgwα-open and E is a closed set in (X, τ). It follows that A is RGWα-LC*(X, τ). That is A ∈ RGWα-LC* (X, τ) and so, RGWα-LC(X, τ) ⊆ RGWα-LC*(X, τ)...... (ii).
From (i) and (ii) we have RGWα-LC(X, τ) = RGWα-LC*(X, τ).

Remark 3.36: The converse of the theorem 3.44 need not be true in general as seen from the following example.

Example 3.37: Consider X= {a, b, c, d} and τ = {X, ϕ, {a}, {b}, {a, b}, {a, b, c}}, then RGWα-LC(X, τ) = RGWα-LC*(X, τ) =P(X). But RGWαC(X, τ) ={X, ϕ, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}}and LC(X, τ) = {X, ϕ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {b, c}, {c, d}, {a, b, c}, {a, c, d}, {b, c, d}}. That is RGWαC (X, τ) ⊈ LC(X, τ).

Theorem 3.38: For a subset A of (X, τ) if A∈RGWα-LC(X, τ) then A = U⋂(rgwαcl (A)) for some open set U.

Proof: Let, A∈RGWα-LC(X, τ) then there exist a rgwα-open U and a rgwα-closed set F s.t. A= U⋂F. Since A⊆ F, rgwαcl(A)= rgwαcl(F) =F. Now U⋂(rgwαcl(A))⊆ U⋂F= A, that is U⋂(rgwαcl(A))= A.

Remark 3.39: The converse of the theorem 3.38 need not be true in general as seen from the following example.

Example 3.40: Consider X= {a, b, c, d} with the topology τ= {X, ϕ, {a}, {b}, {a, b}, {a, b, c}} then
Take A= {a, d}, rgwαcl(A)= {a, d} now, A = X⋂(rgwαcl(A)) for some rgwα-open set X but {a, d}∉RGWα-LC (X, τ).

Theorem 3.41: For a subset A of (X, τ), the following are equivalent.
(i) A∈RGWα-LC*(X, τ).
(ii) A= U⋂ (cl(A)) for some rgwα-open set U.
(iii) cl(A)-A is rgwα-closed.
(iv) A⋃ $(cl(A))^c$ is rgwα-open.

Proof: (i) Implies (ii) Let A∈RGWα-LC*(X, τ) then there exists a rgwα-open set U and a closed set F s.t A = U⋂F. Since A ⊆ F, cl(A) ⊆ cl(F)= F. Now U⋂cl(A) ⊆ U⋂F = A that is U⋂cl(A)= A. and also A ⊆ U, and A ⊆ cl(A) implies A ⊆ U⋂cl(A) and therefore A = U⋂cl(A) for some rgwα-open set U. Hence A∈RGWα-LC*(X, τ).

(ii) Implies (i) since U is a rgwα-open set and cl(A) is a closed set, A= U⋂(cl(A)) ∈ RGWα-LC*(X, τ).

(iii) Implies (iv) let F = cl(A)-A, then F is rgwα-closed by the assumption and X-F=X-[cl(A)-A] = X⋂[cl(A)-A]^c = A⋃(X-cl(A)) = A⋃(cl(A))^c. But X- F is rgwα-open. This shows that A⋃ (cl(A))^c is rgwα-open.

(iv) Implies (iii) Let U = A⋃ (cl(A))^c then U is rgwα-open, this implies X-U is rgwα-closed and X-U = X – (A⋃(cl(A)^c)) = cl(A)⋂(X-A) = cl(A)-A is rgwα-closed.

(iv) Implies (ii) Let U = A⋃ (cl(A))^c then U is rgwα-open. Hence we prove that A=U⋂(cl(A)) for some rgwα-open set U.
Now consider U⋂(cl(A))= [A⋃(cl(A))^c]⋂cl(A) = [A⋂(cl(A))] ⋃ [(cl(A))^c⋂ cl(A)] = A⋃ϕ = A.
Therefore A = U⋂(cl(A)) for some rgwα-open set U.

(ii) Implies (iv) Let A = U⋂ (cl(A)) for some rgwα-open set then we p.t A⋃(cl(A))^c is rgwα-open. Now A⋃(cl(A))^c = (U⋂(cl(A)) ⋃ (cl(A))^c = U⋂ (cl(A))⋃(cl(A))^c =U⋂ X = U, which is rgwα-open. Thus A= (cl(A))^c is rgwα-open.

Theorem 3.42: For a subset A of (X, τ) if A∈RGWα-LC**(X, τ), then there exists an open set U s.t A= U ⋂ rgwαcl(A).

Proof: Let A∈RGWα-LC**(X, τ), then there exist an open set U and a rgwα-closed set s.t A = U⋂F Since A⊆U and A⊆rgwαcl(A) we have A ⊆ U⋂ rgwαcl(A) and Since A⊆F and rgwαcl(A)⊆rgwαcl(F) = F, as F is rgwα-closed. Thus U⋂rgwαcl(A)⊆U⋂F = A. That is U⋂rgwαcl(A)⊆A; hence A= U⋂ rgwαcl(A). For some open set U.

Remark 3.43: The converse of the theorem 3.42 need not be true in general as seen from the following example.

Example 3.44: Let X= {a, b, c, d} with the topology τ= {X, ϕ, {a}, {b}, {a, b}, {a, b, c}} take A= {a, b, d}. Then rgwαcl(A) = rgwαcl{a, b, d}={a, b, d}; also A= X⋂rgwαcl(A)= {a, b, c, d}⋂{a, b, d}={a, b, d} for some open set X but {a, b, d}∉RGWα-LC**(X, τ).

Theorem 3.45: For A and B in (X, τ) the following are true.
(i) if A∈RGWα-LC*(X, τ) and B∈RGWα-LC*(X, τ), then A⋂B∈RGWα-LC*(X, τ).
(ii) if A∈RGWα-LC**(X, τ) and B is open, then A⋂B∈RGWα-LC**(X, τ).
(iii) if A∈RGWα-LC(X, τ) and B is rgwα-open, then A⋂B∈RGWα-LC(X, τ).
(iv) if A∈RGWα-LC*(X, τ) and B is rgwα-open, then A⋂B∈RGWα-LC*(X, τ).
(v) if A∈RGWα-LC*(X, τ) and B is closed, then A⋂B∈RGWα-LC*(X, τ).

Proof: (i) Let A, B∈RGWα-LC*(X, τ), it follows from theorem 3. That there exist rgwα-open sets P and Q s.t A = P⋂ cl(A) and B = Q⋂ cl(B). Therefore A⋂B = P⋂cl(A) ⋂ Q⋂cl(B) =P⋂Q⋂[cl(A)⋂cl(B)] where P⋂Q is rgwα-open and cl(A)⋂cl(B) is closed. This shows that A⋂B∈RGWα-LC*(X, τ).
(ii)Let A∈RGWα-LC**(X, τ) and B is open. Then there exist an open set P and rgwα-closed set F s.t A = P⋂F. Now, A⋂B = P⋂F⋂B = (P⋂B) ⋂ F, Where (P⋂B) is open and F is rgwα-closed. This implies A⋂B∈RGWα-LC**(X, τ). (iii) Let A∈RGWα-LC(X, τ) and B is rgwα-open then there exists a rgwα-open set P and rgwα-closed set F s.t A = P⋂F. Now, A⋂B = P⋂F⋂ B = (P⋂B) ⋂ F, Where (P⋂B) is rgwα-open and F is rgwα-closed. This shows that A⋂B∈RGWα-LC(X, τ).
(iv) Let A∈RGWα-LC*(X, τ) and B is rgwα-open then there exists a rgwα-open set p and rgwα-closed set F s.t A = P⋂F. Now, A⋂B = (P⋂F)⋂ B = (P⋂B)⋂F, Where (P⋂B) is rgwα-open and F is closed. This implies that A⋂B∈RGWα-LC*(X, τ).
(v) A∈RGWα-LC*(X, τ) and B is closed. Then there exist an rgwα-open set P and a closed set F s.t A = P⋂F. Now, A⋂B = (P⋂F) ⋂B = P⋂(F⋂B), Where (F⋂B) is closed and P is rgwα-open. This implies A⋂B∈RGWα-LC*(X, τ).

Definition 3.46: A topological space (X, τ) is called RGWα-submaximal if every dense set in it is RGWα-open.

Theorem 3.47: A Topological Space (X, τ) is rgwα-submaximal if and only if P(X) =RGWα-LC*(X, τ)

Proof: Let (X, τ) be rgwα-submaximal, A∈P(X) and V= A⋃(X-cl(A))^c. Then cl(V) = cl(A⋃(X-cl(A))^c)=cl(A)⋃ (X-cl(A)) =X. That is cl(V)= X. It follows that V is dense in (X, τ). By assumption, V is rgwα-open. By Theorem 3.41, A∈ RGWα-LC*(X, τ). Therefore P(X) =RGWα-LC*(X, τ) Conversely, let A be dense in (X, τ) and P(X) =RGWα-LC*(X, τ). Then A=A⋃(X-cl(A)). Since A∈ RGWα-LC*(X, τ), A=A⋃(X-cl(A)) is rgwα-open by Theorem 3.41. Hence (X, τ) is rgwα-submaximal.

Theorem 3.48: If (X, τ) is submaximal space then it is RGWα-submaximal space but converse need not be true in general.

Proof: Let (X, τ) be submaximal space and A be a dense subset of (X, τ). Then A is open. But every open set is rgwα-open and so A is rgwα-open. Therefore (X, τ) is a RGWα-submaximal space.

Example 3.49: Let X = {a, b, c} and τ = {X, ϕ, {a, b}, {c, d}}. Then Topological space (X, τ) is RGWα-submaximal but set A ={b, c} is dense in (X, τ) but not open therefore (X, τ) is not submaximal.

Theorem 3.50: A topological space (X, τ) is RGWα-submaximal if and only if P(X) = RGWα-LC*(X, τ).
Proof

Necessity: Let A∈P(X) and U = A∪(X-cl(A)). Then it follows cl(U)= cl(A∪(X-cl(A))) =cl(A)∪(X-cl(A)) = X. Since (X, τ) is rgwα-sub maximal, U is rgwα-open, so A∈rgwα-LC*(X, τ) from the Theorem 3.41 Hence P(X) = rgwα-LC*(X, τ).

Sufficiency: Let A be dense sub set of (X, τ).Then by assumption and Theorem 3.41 that A∪(X-cl(A)) = A holds, A∈rgwα-LC*(X, τ) and A is rgwα-open. Hence (X, τ) rgwα-sub maximal.

Theorem 3.51: If (X, τ) $\tau_{rgw\alpha}$ -space then RGWα-LC(X, τ) = LC(X, τ).

Proof: Straight Forward.

Theorem 3.52: Let (X, τ) and (Y, σ) be topological spaces.
i) If A∈RGWα-LC(X, τ) and B∈RGWα-LC(Y, σ) then A×B ∈ RGWα-LC (X×Y, τ×σ).
ii) If A∈RGWα-LC*(X, τ) and B∈RGWα-LC*(Y, σ) then A×B ∈ RGWα-LC* (X×Y, τ×σ).
iii) If A∈RGWα-LC**(X, τ) and B∈RGWα-LC**(Y, σ) then A×B ∈ RGWα-LC** (X×Y, τ×σ).

Proof: i) If A∈RGWα-LC(X, τ) and B∈RGWα-LC(Y, σ). Then there exist rgwα-open sets U and V of (X, τ) and (Y, σ) and rgwα-closed sets G and F of X and Y respectively such that A= U ∩ G and B= V ∩ F. Then A×B = (U×V) ∩ (G×F) holds. Hence A×B ∈ RGWα-LC (X ×Y, τ×σ).
ii) and iii) Similarly the follow from the definition.

4. Conclusion
In this paper we have introduced and studied the properties of rgwα-locally closed sets and rgwα*- locally closed sets and rgwα**-locally closed sets. Our future extension is to study rgwα-locally continuous, closed, open maps in Topological Spaces.

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