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Mathematics in Baudhāyana-Śulbasūtras*

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Abstract

My research paper's topic is 'Mathematics in Baudhāyana-Śulbasūtras'. Although references of Indian Mathematics are found in Vedas, Brāhmaṇagranthas, but it is not easy to understand mathematical concepts there. That's why Baudhāyana has made a categorical and systematic arrangement of these mathematical concepts in his sūtras and this is the contribution of Baudhāyana in explaining Vedic Mathematics. Baudhāyana has given rules and operations for the measurements and constructions of the various sacrificial fires and altars which shows Indian Geometry. In this paper, Baudhāyana's theorem of square on the diagonal of square or a rectangle is discussed and the same theorem is known as Pythagoras theorem now. Here is also discussed--methods of construction of square that is sum of unequal squares and also construction of a square that is the difference of two squares. Even then method of transformation of geometrical figures such as rectangle in to square and method of calculating the value of is also discussed in this paper. In this way, this type of research paper will create interest among scholars to find the roots of Indian mathematics. This type of study will also contribute in the explanation of Vedic mathematics.

Keywords: Vedāngas, Kalpasūtras, Śulbasūtras

Introduction

Sanskrit is a rich language by its literature. Sanskrit literature is classified in to two main parts –'Vedic Literature' and 'Laukik Literature'. Vedas are gems of Indian Culture. Vedas are considered as ancient literature of world. Vedas are sources of knowledge because everything is depicted in Vedas like Science, Mathematics, Political Science, Economics, History, Social Sciences etc. There are four Vedas--Rgveda, Yajurveda, Sāmveda and Atharvveda. To study and preservation of the Vedas, Vedāngas play an important role in maintaining the purity and integrity of the Vedic tradition. They are six in number: Phonetics (Śiksā), Ritual (Kalpa), Grammar (Vyākaraṇa), Etymology (Nirukta), Metrics (Chanda) and Astronomy (Jyotiṣa). Śiksā deals with study of sounds and pronunciation with each syllable. Ritual deals with sacrifices with great detail of dimensions and designs of the vedic altars. Vyākaraṇa deals with the study of word and sentence structures. Nirukta deals with the meaning of complex words and phrases. Chanda deals with mastery of rhyme and meter. Jyotiṣa deals with the study of astronomy and astrology.

Among these Vedāngas, Kalpasūtras related to process of performing rituals mainly covers the chief portion of Brāhmaṇa-granthas. Kalpasūtras deal with organization of the rituals which are done by vedic priests to attain wishes. These texts earliest of which are dated prior to 800 BCE, form a part of much larger corpus known as Kalpasūtra. On the behalf of the subject-matter, kalpasūtras are further divided in to four parts---Śrautasūtras, Gṛhyasūtras, Dharmasūtras and Śulbasūtras. Śrautasūtras deal with rituals which are given in Śruti (Vedas) and associated with social welfare and concerned with direction for the laying of the sacrificial fires for the fire-sacrifice (agnihotra), the new and full-moon, the soma and other sacrifices. Gṛhyasūtras deal with rituals related to household e.g. birth, name-giving, marriage sacraments (ceremonies) etc. Dharmasūtras deal with duties and general code of conduct. Śulbasūtras specially concerned with rules for the measurements and constructions of various sacrificial fires and altars.

The word śulba means 'a rope' or 'a string' and its stems from the root 'शूला आन्' [1] (to measure); it means a rope which is used in the act of measurement is known as śulba. This

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class of literature was written in the sūtra or aphoristic style, a composing style characterized by great precision and economy of words. In this way, main theme of the śulbasūtras is to construct sacrificial grounds and fire-altars using rope, that's why śulbasūtras text is considered as oldest text of Indian Geometry. One of the prime occupations of the Vedic people seems to have been performing sacrifices, for which altars of prescribed shapes and sizes were needed; that's why the Vedic priests have composed a class of texts called Śulbasūtras.

The etymological derivation of the word ‘śulba’ can be presented in more than one way, i.e. refers to the act of measuring, the entity/result of measuring and the instrument of measuring.

Although Śulbasūtras are related to every Veda, but Yajurveda related śulbasūtras are only available in recent times and they are explained by different scholars as given below:

1. Baudhāyana-Śulbasūtra (Kṛṣṇa Yajurveda)
2. Āpastamba-Śulbasūtra (Kṛṣṇa Yajurveda)
3. Katyāyana-Śulbasūtra (Śukla Yajurveda)
4. Mānava-Śulbasūtra (Kṛṣṇa Yajurveda)
5. Maitrayāna-Śulbasūtra (Kṛṣṇa Yajurveda)
6. Varāha-Śulbasūtra (Kṛṣṇa Yajurveda)
7. Vādhula-Śulbasūtra (Kṛṣṇa Yajurveda)

Out of these, Baudhāyana-Śulbasūtras texts are considered to be the most ancient one (prior to 800BCE). Besides, it represents the most systematic, logical and associated with detailed explanation of several topics. That's why in this paper, there is given description of units of measurements of altars, methods of construction of squares (that is the sum and difference of two squares) and transformation of rectangle in to square, the theorem of square on the diagonal of a rectangle and the value of $\sqrt{2}$ according to Baudhāyana. Firstly, Baudhāyana has given a list of units of measurements as given below^[2]:

1aṅgula =14 añus = 34 tilas (a unit of measure equal to finger's breadth=8 barley corns)

1 small pada = 10 aṅgulas

1 prādeśa = 12 aṅgulas

1 pada = 15 aṅgulas

1 iṣā = 188 aṅgulas

1 akṣa=104 aṅgulas

1yuga=86 aṅgulas

1jānu=32 aṅgulas

1 śamyā=36 aṅgulas

1bāhu=36 aṅgulas

1 prakrama=2 padas

1 aratni=2 pradeśas=24 aṅgulas

1 puruṣa=5 aratnis=120 aṅgulas

1 vyāma=5 aratnis

1 vyāyāma=4 aratnis

1aṅgula=3/4 inch (approx.) [Inch-a unit of linear measure equal to one twelfth of a foot)

After watching this list of measurement units, it is clear that with the help of these units, mathematical calculation is done and by that calculation, constructions of sacrificial altars are done by Vedic priests. As it is seen that areas of

different chambers and Vedis given by Baudhāyana are given below^[3]:

Name of altar	Geometrical shape	Measurement
Āgnidhriya	square	side=5 aratnis
Cātvāla	square	side=36 aṅgulas
Dhisnas	circle	diameter=2 prādeśas.
Havirdhāna	square	side=10 or 12 prakramas
Mahāvedi	isosceles	face=24 padas, base=30 padas,
	Trapezium	altitude=36 padas; the units maybe also in prakramas.
Mārjāliya	square	side=5 aratnis
Prāgyamśa	rectangle	length=16 prakramas breadth=10 prakramas
		length=27 aratnis
Sadas	rectangle	breadth=10 prakramas or length=18 aratnis breadth=10 prakramas

Baudhāyana introduces the theorem of square on the diagonal of a square or a rectangle (Pythagoras Theorem) by saying that-dīrgacaturaśasyārajjuh pārvamānī tiryāmānī ca yatprthagbhūte kurutastadubhayam karoti^[4].

Its meaning is that the areas (of the squares) produced separately by the length and the breadth of a rectangle together equal the area (of the square) produced by the diagonal, it means that the rope corresponding to the diagonal of a rectangle makes whatever is made by the lateral and vertical sides individually. This theorem of Baudhāyana is known after Pythagoras is Pythagorean Theorem. This theorem states that in a rectangle ABCD (Figure-1), $AC^2=AB^2+BC^2$ ^[5].

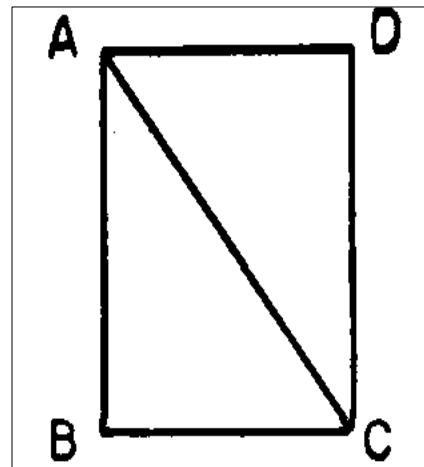


Fig 1. In a rectangle ABCD, $AC^2=AB^2+BC^2$

Baudhāyana says that this theorem can be easily verified from the following relations or Pythagorean triplets and he has given a few examples of these Pythagorean triplets; i.e. $(3^2+4^2=5^2)$, $(5^2+12^2=13^2)$, $(15^2+8^2=17^2)$, $(7^2+24^2=25^2)$, $(12^2+35^2=37^2)$, $(15^2+36^2=39^2)$ i.e. it can be observed in rectangles having sides 3 and 4, 12 and 5, 15 and 8, 7 and 24, 12 and 35, 15 and 36^[6].

³ (i)Baudhāyana- Śulbasūtra, 4.1-4.11

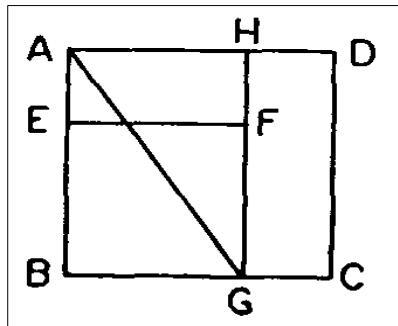
(ii) The Śulbasūtras, P.No177.

⁴ Baudhāyana-Śulbasūtra, 1.12

⁵ The Śulbasūtras, P. No152.

⁶ trikacatuskayordvādaśikapāñcikayoh/pañcadashikayoh/saptikacaturvimśikayordvādaśikapāñcatrimśikayohpañcadashikāṣṭātriṁśikayorityetāśupalabdhiḥ/-Baudhāyana-Śulbasūtra, 1.13

Baudhāyana has given method in his sūtras by which construction of a square that is combination of two squares is possible. He states, if somebody desirous of combining two different squares then a rectangle is formed with a side (karanya) of the smaller (square--kaniyasah); the diagonal of the rectangle (thus formed-vṛdhra) is the side of the sum of two unequal squares [7]. Here is given in figure-2, for the combination of a smaller square EBGF with another square ABCD, when rectangular portion ABGH is cut off by the side of the smaller square whose side is equal to BG. Then AG of this cut-off portion will be the side of the combined square.

Fig 2: $AG^2=AB^2+BG^2$

In this way, it proves: $AG^2=AB^2+BG^2=\text{sum of two squares}$. In spite of it, Baudhāyana has also discussed the method of constructing a square that is the difference of two squares. He states, if somebody desirous of subtracting a square from another square, a (rectangle) part is cut off from the larger (square) with the side (karanya) of the smaller one to be removed. With the (cord corresponding to the longer) side of the cut-off (rectangular) part is placed across so as to touch the opposite side; by this contact (the side) is cut off, with the cut-off (part) the difference (of the two squares) is obtained [8]. Here is given in figure-3, for the construction of a square which is the difference between a smaller square EBGF and other square ABCD, says that the rectangular portion ABGH is cut off by the side BG of the smaller square. Then the side GH of the cut off portion is allowed to fall on AB and P is the point where it falls. Here $GH=GP$, then BP is the side of a square which is equal to the difference of the Squares ABCD and EBGF

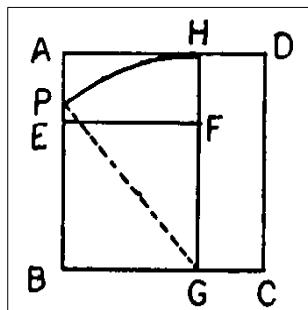


Fig 3: BP is the side of a square which is equal to the difference of the squares ABCD and EBGF

⁷ Nānācaturaśre samasyankamīyasah karanya varṣyaso vṛdhram ullikhet / vṛdrasayākṣṇayārajuḥ samastayoh pārśvamānī bhavati /--Baudhāyana Śulbasūtra, 2.1

⁸ caturāścaturaśram nirjihūṣanyāvannirjihūṣettasya karanyā varṣyaso vṛdhramullikhet/vṛdrasayā pārśvamānīmakṣayetarapārśvamupasamphare / sā yatra nipatettadapacchindiyāt / chinnayā nirastam / - Baudhāyana-Śulbasūtra, 2.2

It is proved by the figure,

$$BP^2=GP^2-BG^2$$

$$=GH^2-BG^2$$

$$=AB^2-BG^2$$

=difference of two squares ABCD and EBGF [9].

It is very interesting to know that how Śulbaśāras construct geometrical figures like square, rectangle, rhombus using only rope, there was no instrument or device to construct them that time. Even then, Vedic Priests are also capable of transforming one figure in to another figure by only using rope, not by instrument or device. That's why Baudhāyana has given methods in his sūtras by which transformation of geometrical figures is possible. He states for transforming a rectangle in to a square, its breadth are taken as the side of a square (and this square on the breadth is cut off from the rectangle.) The remainder (of the rectangle) is divided in to two equal parts and placed on two sides (one part on each). The empty space (in the corner) is filled up with a (square) piece. The removal of it (of the square piece from the square thus formed to get the required square) has been stated [10]. In figure-4, ABCD is the given rectangle, the portion ABFE is cut off such that $AE=AB$ =the breadth of the rectangle. The remaining portion EFCD is cut off in to two equal halves. One half GHCD is placed on the other side and its new position becomes BKLF. A small square FLMH is fitted at the corner.

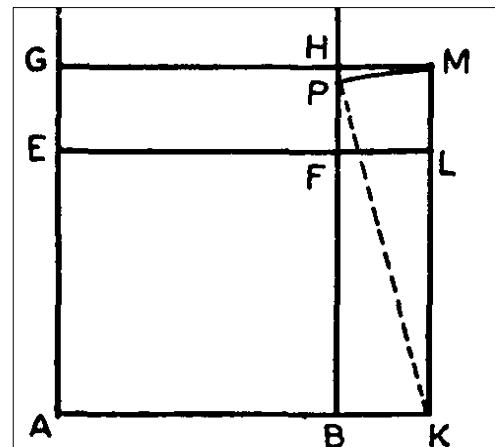


Fig 4: A rectangle in to a square

Now, rectangle ABCD= (sq.) AKMG - (sq.) FLMH, which shows that the rectangle ABCD is expressed as the difference of two squares. Since the method of nīrīhāra has already been taught before by Baudhāyana (2.2), a square equal to the difference of the two squares mentioned above is found by allowing the side KM to fall at P over BH. Then the square on BP will be equal to the difference of two squares, which is equal to the area of the given rectangle.

For, $BP^2=PK^2-BK^2$

$$=MK^2-FL^2$$

$$=\text{sq. ABFE}+\text{rec. EFGH}+\text{rect. FBKL}$$

$$=\text{sq. ABFE}+\text{rect. EFGH}+\text{rect. DGHC}$$

$$=\text{rect. ABCD}$$
 [11].

Like it, method of transformation of other geometrical figures is also given in Baudhāyana-Śulbasūtras.

⁹ The Śulbasūtras, P.No. 155

¹⁰ Baudhāyana-Śulbasūtra, 2.5

¹¹ The Śulbasūtras, P.No. 159

The value of $\sqrt{2}$ is beautifully described by this Baudhāyana-sūtra:

Pramāṇam trītyena vardhayettacca caturthenātmacatustriṁśonena/ saviṣeṣah / [12].

Here is said that the measure is to increased by its third and this (third) again by its own fourth less the thirty-fourth part (of that fourth); this is (the value of) the diagonal of a square (whose side is the measure). In this sūtra, the word ‘saviṣeṣa’ is noteworthy, which has meaning=some specialty i.e. being approximate. It can be written in this way also,

$$\sqrt{2}=1+1/3+1/3\times4 -1/3.4.34 \text{ (approx.)}$$

$$=1+1/3+1/3\times4 (1-1/34)$$

$$=577/408=1.4142156$$

In decimal fraction, the value of $\sqrt{2}=1.41421568627451$ given by Baudhāyana method of calculation, while according to modern calculation, the value of $\sqrt{2}=1.414213562373095$. It shows that the ancient Indians attained a degree of accuracy in calculating an approximate value of $\sqrt{2}$.

Baudhāyana has also discussed in his sūtras to construct different types of sacrificial grounds and fire-altars such as Śyenciti, Kañkacit, Alajacit, Praugacit and Ubhayataḥ-praugacit. In this way, a very systematic and logical arrangement of bricks in the construction of these fire altars given in Baudhāyana-sūtras. Although this Indian geometry or ancient mathematics is discussed in subtle form in Vedās and Brāhmaṇgranthas like Taittirīya-Saṁhitā and Śatpatha-Brāhmaṇa; but these all topics related to mathematics were discussed with details in Baudhāyana-Śulbasūtras and that is his contribution in the explanation of Vedic mathematics.

This type of research paper would inspire scholars to find the roots of Indian Mathematics in Vedic tradition; whether it may be Vedas and Brāhmaṇa-granthas and Vedāṅgas. While there is a great interest among Indian scholars as well as western scholars regarding Vedic Mathematics; this type of study will give a new direction for doing research in this field. By these types of researches, contribution of Vedic scholars in development of mathematics would be come in to light globally.

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¹²Baudhāyana-Śulbasūtra, 2.12