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Generalized b-closed sets in vague topological spaces

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Abstract
The purpose of this paper is to introduce and study the concepts of vague generalized b closed sets in vague topological spaces and their characterizations are investigated. Further the notion of *vague* $_bT_{1/2}$ spaces and *vague* $_{gb}T_{1/2}$ spaces are introduced and discussed.

Keywords: Vague topology, vague generalized b closed sets, *vague* $_bT_{1/2}$ spaces and *vague* $_{gb}T_{1/2}$

1. Introduction

Closed sets are fundamental objects in a topological space. One can define the topology on a set by using either the axioms for the closed sets. In 1970, Levine [7] initiated the study of generalized closed sets. By definition, a subset *S* of a topological space *X* is called *generalized closed* if cl $(A) \subseteq U$ whenever $A \subseteq U$ and *U* is open. This notion has been studied extensively in recent years by many topologists because generalized closed sets are not only natural generalizations of closed sets. More importantly, they also suggest several new properties of topological spaces. The concept of fuzzy sets was introduced by Zadeh [14] in 1965. The idea of fuzzy set is welcome because it handles uncertainty and vagueness which Cantorian set could not address. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. The theory of fuzzy topology was introduced by C.L. Chang [5] in 1967. Several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. In 1986, the concept of intuitionistic fuzzy sets was introduced by Atanassov [2] as a generalization of fuzzy sets. The theory of vague sets was first proposed by Gau and Buehre [6] as an extension of fuzzy set theory and vague sets are regarded as a special case of context-dependent fuzzy sets. The basic concepts of vague set theory and its extensions are defined by [4, 6].We introduce vague generalized b closed sets and their properties are obtained. Also as an application we have defined vague $_bT_{1/2}$ spaces andvague $_{gb}T_{1/2}$spaces.

2. Preliminaries

Definition 2.1: [3] A vague set*A* in the universe of discourse *X* is characterized by two membership functions given by:

1. A true membership function $t_A : X \to [0,1]$and
2. A false membership function $f_A : X \to [0,1]$.

Where t_A(x) is lower bound on the grade of membership of *x* derived from the “evidence for *x*”, f_A(x) is a lower bound on the negation of *x* derived from the “evidence against *x*” and $t_A(x)+ f_A(x) \leq 1$.Thus the grade of membership of *x* in the vague set *A* is bounded by a subinterval$[t_A(x),1- f_A(x)]$ of [0, 1]. This indicates that if the actual grade of membership $\mu(x)$, then $t_A(x) \leq \mu(x) \leq f_A(x)$ The vague set *A* is written as,
A= $\left\{\left\langle x,[t_A(x),1-f_A(x)]\right\rangle / x\in X\right\}$ where the interval $[t_A(x),1- f_A(x)]$ is called the “vague value of *x* in *A* and is denoted by $V_A(x)$.

Definition 2.2: [3]Let *A* and *B* be vague sets of the form A= $\left\{\left\langle x,[t_A(x),1-f_A(x)]\right\rangle / x\in X\right\}$ and B= $\left\{\left\langle x,[t_B(x),1-f_B(x)]\right\rangle / x\in X\right\}$. Then

a) $A \subseteq B$ if and only if $t_A(x) \le t_B(x)$ and $1-f_A(x) \le 1-f_B(x)$.
b) $A=B$ if and only if $A \subseteq B$ and $B \subseteq A$.
c) $A^c = \{\langle x,[f_A(x),1-t_A(x)]\rangle / x \in X\}$
d) $A \cap B = \{\langle x,[(t_A(x) \wedge t_B(x)),((1-f_A(x)) \wedge (1-f_B(x)))]\rangle / x \in X\}$.
e) $A \cup B = \{\langle x,[(t_A(x) \vee t_B(x)),((1-f_A(x)) \vee (1-f_B(x)))]\rangle / x \in X\}$. For the sake of simplicity, we shall use thenotion A= $\{\langle x,[t_A(x),1-f_A(x)]\rangle\}$ instead of A= $\{\langle x,[t_A(x),1-f_A(x)]\rangle / x \in X\}$

Definition 2.3: Let be a (X,τ) topological space. A subset *A* of *X* is called:
i) semi closedset (SCS in short) [8] if int(cl(A) $\subseteq$ A
ii) pre- closed set (PCS in short) [11] if cl(int(A)) $\subseteq$ A
iii) α-closed set (αCS in short) [12] if cl(int(cl(A))) $\subseteq$ A
iv) regular closed set (RCS in short) [14] if A= cl(int(A))
v) b closed set (bCS in short) [13] if cl(int(A)) $\cap$ int(cl(A)) $\subseteq$ A

Definition 2.4: Let (X,τ) be a topological space. A subset *A* of *X* is called:
i) generalized closed (briefly, g-closed) [7] if cl(A)⊆U whenever A ⊆ U and U is open inX.
ii) generalized semi closed (briefly gs-closed) [1] if scl(A)⊆U whenever A ⊆ U and U is open in X.
iii) α-generalized closed (briefly αg-closed) [9] if αcl(A)⊆Uwhenever A ⊆ U and U is open in X .
iv) generalized pre closed (briefly gp-closed) [10] if pcl(A)⊆Uwhenever A ⊆ U and U is open in X .
v) generalized b closed set (briefly gb-closed)[13] if bcl(A) ⊆U whenever A ⊆ U and U is open in X .

Definition 2.5: A vague topology (VT in short) on *X*is a family τ of vague sets (VS in short) in *X* satisfying the following axioms.

- 0, 1 ∈ τ
- $G_1 \cap G_2 \in \tau$
- $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$

In this case the pair (X,τ) is called vague topological space (VTS in short) and vague set in τ is known as vague open set (VOS in short) in *X*.The complement A^C of VOS in VTS is (X,τ) called vague closed set (VCS in short) in *X*.

Definition 2.6: Let (X,τ) be VTS and A= $\{\langle x,[t_A,1-f_A]\rangle\}$be VS in *X*. Then the vague interior and vague closure are defined by

- vint(A) = $\cup$ {G/G is aVOS in X and G $\subseteq$ A}
- vcl(A) = $\cap$ { K / K is aVCS in X and A $\subseteq$ K}

Note that for any vague set A in (X,τ), we A^c =(vint(A))^c and
vint (A^c) = (vcl(A))^c

Definition 2.7: A Vague set A= $\{\langle x,[t_A,1-f_A]\rangle\}$ in a VTS (X,τ) is said to be a

i) vague semi closed set (VSCS in short) if vint(vcl(A)) $\subseteq$ A.
ii) vague semi open set (VSOS in short) if A $\subseteq$ vcl(vint(A)).
iii) vague pre-closed set (VPCS in short) if vcl(vint(A)) $\subseteq$ A.
iv) vague pre-open set (VPOS in short) if A $\subseteq$ vint(vcl(A)).
v) vague α-closed set (VαCS in short) if vcl(vint(vcl(A))) $\subseteq$ A.
vi) vague α-open set (VαOS in short) if A $\subseteq$ vint(vcl(vint(A))).
vii) vague regular open set (VROS in short) if A =vint(vcl(A)).
viii) vague regular closed set (VRCS in short) if A=vcl(vint(A)).

Definition 2.8: Let A be a vague set of VTS (X,τ) Then the vague semi interior of A
(vsint (A) in short)the vague alpha interior of A (vαint (A)in short) and vague alpha closure of A (vαcl (A)in short) are defined by

- vαint(A)= $\cup$ {G/G is a VαOSin X and G $\subseteq$ A}
- vαcl(A)= $\cap$ {K / K is a VαCS in X and A $\subseteq$ K}

Result 2.11: Let A be a vague set (X,τ), then
i) vαcl (A)=A $\cup$ vcl (vint(vcl(A)))
ii) vαint (A)=A $\cap$ vint (vcl(vint(A)))

Definition 2.12: A vague set A of VTS (X,τ) is said to be a vague generalized closed set (VGCS in short) if vcl(A) $\subseteq$U whenever A$\subseteq$U and U is VOS in X.

Definition 2.13: A vague setA of VTS (X,τ) is said to be a vague generalized semi closed set (VGSCS in short) if Vscl (A) $\subseteq$U whenever A$\subseteq$U and U is VOS in X.

Definition 2.14: A vague set A of VTS (X,τ) is said to be vague alpha generalized closed set(VαGCS in short) if Vαcl (A) $\subseteq$U whenever A $\subseteq$U and U is VOS in X.

Definition 2.15: A vague set A of VTS (X,τ) is said to be a vague generalized pre-closed set(VGPCS in short) if vpcl(A) $\subseteq$U whenever A $\subseteq$U and U is VOS in X .

3. Vague generalized b Closed sets

In this section, we introduce vague generalized b closed sets in vague topological space and their properties are deliberated.

Definition 3.1: Let (X, τ) be a VTS and A= $\{\langle x,[t_A,1-f_A]\rangle\}$ be a vague set in X. Then the vague b closure of A (vbcl(A) in short) and vague b interior of A (vbint(A) in short) are defined as vbint(A) = $\cup$ { G / G is an VbOS in X and G $\subseteq$A},
vbcl (A) = $\cap$ { K / K is an VbCS in X and A$\subseteq$ K}

Theorem 3.2: If A is an vague set in X then A $\subseteq$ vbcl(A) $\subseteq$ vcl(A).

Proof: It is obvious.

Proposition 3.3: Let (X, τ) be any VTS. Let A and B be any two vague sets in (X, τ). Then the vague generalized b closure operator satisfies the following properties.(a) vbcl(0) = 0 and vbcl(1) = 1,(b) A$\subseteq$vbcl(A),(c) vbint(A) $\subseteq$A,(d) If A is a VbCS then A = vbcl(vbcl(A)),(e) A $\subseteq$ B $\Rightarrow$ vbcl(A) $\subseteq$vbcl(B),(f) A $\subseteq$ B$\Rightarrow$ vbint(A) $\subseteq$vbint(B).

Definition 3.4: A vague set A in VTS (X, τ) is said to bevague generalized b closed set (VGbCSshort) if vbcl(A) $\subseteq$ U whenever A$\subseteq$U and U is an VOS in (X, τ).The family of all VGbCS of a VTS (X, τ) is denoted by VGbC(X).

Theorem 3.5: Every VCS is VGbCS but not conversely.

Proof: Let A $\subseteq$ U and U isVOS in (X, τ). Since A is VCS and vbcl(A) $\subseteq$vcl(A),vbcl(A) $\subseteq$vcl(A) = A $\subseteq$ U. Therefore A is VGbCS in X.

Theorem 3.6:

i. Every VαCS is VGbCS but not conversely.
ii. Every VPCS is VGbCS but not conversely.
iii. Every VbCS is VGbCS but not conversely.
iv. Every VRCS is VGbCS but not conversely.
v. Every VGCS is VGbCS but not conversely.
vi. Every VαGCS is VGbCS but not conversely.
vii. Every VGPCS is VGbCS but not conversely.
viii. Every VWGCS is VGbCS but not conversely.
ix. Every VSCS is VGbCS but not conversely.

Proof: It is obvious.
Remark 3.7: The converse of the above theorems is need not be true and it is shown by the following examples.

Example 3.8: Let X = {a,b} and let τ = {0,G,1} isVT on X, where
G = $\{\langle x,[0.2,03],[0.6,0.8]\rangle\}$.Then the vague set A= $\{\langle x,[0.5,0.6],[0.3,0.4]\rangle\}$is VGPCS in Xbut not VCS in X.

Example 3.9: Let X = {a,b} and let τ = {0,G,1} isVT on X, where
G = $\{\langle x,[0.5,07],[0.5,0.8]\rangle\}$. Then the vague setA= $\{\langle x,[0.3,0.6],[0.3,0.5]\rangle\}$is VGbCS in X but not VαCS in X.

Example 3.10: Let X = {a,b} and let τ = {0,G_1,G_2,1} is a VT on X, where
G_1 = $\{\langle x,[0.2,03],[0.1,0.5]\rangle\}$$G_2$ = $\{\langle x,[0.2,03],[0.1,0.5]\rangle\}$. Then the vague set
A= $\{\langle x,[0.2,0.7],[0.4,0.8]\rangle\}$is VGbCS in X but not VPCS in X.

Example 3.11: Let X = {a,b} and let τ = {0,G_1,G_2,1} isVT on X, where

$G_1 = \{\langle x,[0.2,03],[0.1,0.5]\rangle\}$ $G_2 = \{\langle x,[0.2,03],[0.1,0.5]\rangle\}$Then the vague set
A= $\{\langle x,[0.2,0.7],[0.4,0.8]\rangle\}$is VGbCS in X but not VbCS in X.

Example 3.12: Let X = {a,b} and let τ = {0,G,1} is VT on X, where
G = $\{\langle x,[0.3,0.5],[0.4,0.6]\rangle\}$.Then the vague set A= $\{\langle x,[0.5,0.8],[0.4,0.6]\rangle\}$is VGbCS in X but not a VRCS in X.

Example 3.13: Let X = {a,b} and let τ = {0,G,1} is VT on X, where
G = $\{\langle x,[0.2,0.4],[0.6,0.8]\rangle\}$.Then the vague set A= $\{\langle x,[0.5,0.6],[0.3,0.4]\rangle\}$is VGbCS in X but not VGCS in X.

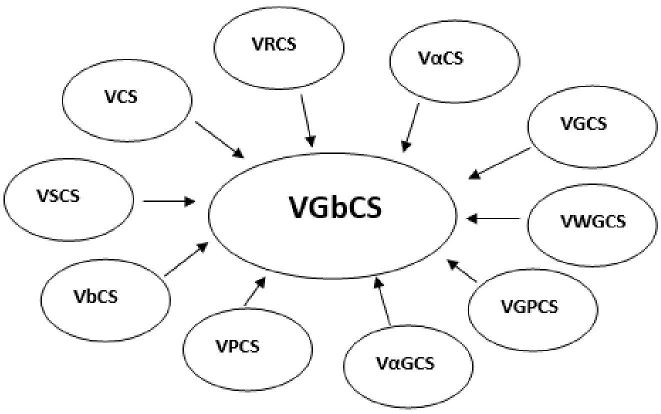
Example 3.14: Let X = {a,b} and let τ = {0,G,1} isVT on X, where
G = $\{\langle x,[0.3,0.8],[0.4,0.5]\rangle\}$.Then the vague set A= $\{\langle x,[0.3,0.5],[0.5,0.7]\rangle\}$is VGbCS in X but not VαGCSinX.

Example 3.15: Let X = {a,b} and let τ = {0,G,1} is a VT on X, where
$G_1 = \{\langle x,[0.2,03],[0.1,0.4]\rangle\}$ and $G_2 = \{\langle x,[0.4,0.5],[0.4,0.7]\rangle\}$.Then the vague set
A= $\{\langle x,[0.4,0.7],[0.5,0.6]\rangle\}$is VGbCS in X but not VGPCS in X.

Example 3.16: Let X = {a,b} and let τ = {0,G,1} isVT on X, where
G = $\{\langle x,[0.3,0.4],[0.6,0.7]\rangle\}$.Then the vague set A= $\{\langle x,[0.3,0.5],[0.1,0.2]\rangle\}$is a VGbCS in X but not VWGCS in X.

Example 3.17: Let X = {a,b} and let τ = {0,G,1} is VT on X, where
G = $\{\langle x,[0.7,0.9],[0.1,0.2]\rangle\}$.Then the vague setA= $\{\langle x,[0.6,0.7],[0.3,0.4]\rangle\}$is VGbCS in X but not VSCS inX.

Remark 3.18: From the above theorems and examples we have the following diagrammatic representation



Remark 3.19: The union of any two VGbCSs need not be VGbCS in general as seen from the following example.

Example 3.20: Let X = {a,b} and let τ = {0,G,1} isVT on X, where
G = $\{\langle x,[0.3,0.7],[0.3,0.8]\rangle\}$ and A = $\{\langle x,[0.2,0.8],[0.6,0.8]\rangle\}$,B= $\{\langle x,[0.5,0.8],[0.5,0.8]\rangle\}$are VGbCSs in X but
$A \cup B$ is not VGbCS in X.

Theorem 3.21: If A isVGbCS in (X, τ) such that A $\subseteq$ B $\subseteq$ vbcl(A) then B is
VGbCSin (X, τ).
Proof: Let B be vague set in VTS (X, τ) such that B $\subseteq$U and U is VOS in X. This implies A $\subseteq$U. Since A is VGbCS, vbcl(A) $\subseteq$U. By hypothesis, we have
vbcl(B) $\subseteq$vbcl(vbcl(A)) = vbcl(A) $\subseteq$U. Hence B is VGbCS in X.

Theorem 3.22: If A is vague b open and vague generalized b closed in VTS(X, τ) then A is vague b closed in (X, τ).

Proof: Since A is vague b open and vague generalized b closed in (X, τ), vbcl(A $\subseteq$ A.
But A $\subseteq$ vbcl(A). Thus vbcl(A) = A and hence A is vague b closed in (X, τ).

4.Vague generalized b open sets
In this section, we introduce vague generalized b open sets in vague topological space and study some of their properties.

Definition 4.1: A vague set A is said to be a vague generalized b open set (VGbOS in short) in (X, τ) if the complement A^cisVGbCS in X.The family of all VGbOSs of VTS (X, τ) is denoted by VGbO(X).

Theorem 4.2: For any VTS (X, τ), we have the following:
i. Every VOS is VGbOS.
ii. Every VbOS is VGbOS.
iii. Every VαOS is VGbOS.
iv. Every VGOS is VGbOS.
v. Every VGPOS is VGbOS.
vi. Every VPOS is VGbOS.
vii. Every VROS is VGbOS.
viii. EveryVαGOS is VGbOS.
ix. Every VWGOS is VGbOS.
x. Every VSOS is VGbOS.

Proof: Straight forward.

Remark 4.3: The converse of the above statements need not be true in general as seen from the following examples.

Example 4.4: Let X = {a,b} and G = $\{\langle x,[0.6,0.7],[0.5,0.7]\rangle\}$.Then τ = {0,G,1} isVT on X. The vague set A= $\{\langle x,[0.7,0.9],[0.6,0.7]\rangle\}$is VGbOS in X but not VOS in X.

Example 4.5: Let X = {a,b} and G = $\{\langle x,[0.6,0.8],[0.2,0.4]\rangle\}$.Then τ = {0,G,1} is VT on X. The vague set A= $\{\langle x,[0.7,0.9],[0.6,0.7]\rangle\}$is VbOS in X but not VGbOS in X.

Example 4.6: Let X = {a,b} and G = $\{\langle x,[0.1,0.2],[0.2,0.3]\rangle\}$.Then τ = {0,G,1} isVT on X. The vague set A= $\{\langle x,[0.1,0.2],[0.2,0.3]\rangle\}$is VGbOS in X but not VαOS in X.

Example 4.7: Let X = {a,b} and G = $\{\langle x,[0.6,0.7],[0.5,0.7]\rangle\}$.Then τ = {0,G,1} isVT on X. The vague set A= $\{\langle x,[0.7,0.9],[0.6,0.7]\rangle\}$ is VGbOS in X but not VGOS in X.

Example 4.8: Let X = {a,b} and G = $\{\langle x,[0.3,0.8],[0.4,0.5]\rangle\}$.Then τ = {0,G,1}is a VT on X. The vague set A= $\{\langle x,[0.5,0.7],[0.3,0.5]\rangle\}$ is VGbOS in X but not VGPOS in X.

Example 4.9: Let X = {a,b} and let τ = {0,G_1,G_2,1} is a VT on X, where
G_1 = $\{\langle x,[0.2,03],[0.1,0.5]\rangle\}$$G_2$ = $\{\langle x,[0.2,03],[0.1,0.5]\rangle\}$. Then the vague set
A= $\{\langle x,[0.3,0.8],[0.2,0.6]\rangle\}$is VGbOS in X but not VPOS in X.

Example 4.10: Let X = {a,b} and let τ = {0,G,1} isVT on X, where
G = $\{\langle x,[0.3,0.5],[0.4,0.6]\rangle\}$.Then the vague set A= $\{\langle x,[0.2,0.5],[0.4,0.6]\rangle\}$ is VGbOS in X but not a VROS in X.

Example 4.11: Let X = {a,b} and let τ = {0,G,1} isVT on X, where
G = $\{\langle x,[0.3,0.8],[0.4,0.5]\rangle\}$.Then the vague set A= $\{\langle x,[0.5,0.7],[0.3,0.5]\rangle\}$is VGbOS in X but not VαGOS in X.

Example 4.12: Let X = {a,b} and let τ = {0,G,1} isVT on X, where
G = $\{\langle x,[0.3,0.4],[0.6,0.7]\rangle\}$.Then the vague set A= $\{\langle x,[0.5,0.7],[0.8,0.9]\rangle\}$is a VGbOS in X but not VWGOS in X.

Example 4.13: Let X = {a,b} and let τ = {0,G,1} is VT on X, where
G = $\{\langle x,[0.7,0.9],[0.1,0.2]\rangle\}$.Then the vague set A= $\{\langle x,[0.3,0.4],[0.6,0.7]\rangle\}$is VGbOS in X but not VSOS in X.

Remark 4.14: The intersection of any two VGbOSs need not be VGbOS in general.

Example 4.15: Let X = {a,b} and let τ = {0,G,1} is VT on X, where
G = $\{\langle x,[0.1,0.2],[0.4,0.5]\rangle\}$ and A = $\{\langle x,[0.6,0.9],[0.5,0.8]\rangle\}$, B= $\{\langle x,[0.6,0.7],[0.6,0.6]\rangle\}$are VGbCSs in X but $A \cap B$ is not VGbCS in X.

Theorem 4.16: A vague set A of VTS (X, τ) is VGbOS if and only if F $\subseteq$ vbint(A) whenever F is VCS and F $\subseteq$ A.

Proof: Necessity: Suppose A is VGbOS in X. Let F be VCS and F $\subseteq$ A. Then F^c isVOS in X such that $A^c \subseteq F^c$. Since A^c isVGbCS,v bcl(A^c) $\subseteq F^c$. Hence $(vbint(A))^c \subseteq F^c$.
This implies F $\subseteq$ vbint(A).

Sufficiency: Let A be any vague set of X and let F $\subseteq$ vbint(A) whenever F is VCS and F $\subseteq$A.Then $A^c \subseteq F^c$ and F^c is VOS. By hypothesis, $(vbint(A))^c \subseteq F^c$.
Hence vbcl(A^c) $\subseteq F^c$.Then A^c is VGbCS in X.Hence A is VGbOS in X.

Theorem 4.17: If A isVGbOS in (X, τ) such that vbint(A) $\subseteq$ B $\subseteq$ A then B is VGbOS in (X, τ).

Proof: By hypothesis, we have vbint(A) $\subseteq$B $\subseteq$ A. This implies $A^c \subseteq B^c \subseteq (vbint(A))^c$. That is, $A^c \subseteq B^c \subseteq$vbcl($A^c$). Since A^c is VGbCS, by theorem 3.21, B^c is VGbCS. Hence B is VGbOS in X.

5. Applications of vague generalized b closed sets
In this section, we introduce vague $_bT_{1/2}$ spaces, vague $_{gb}T_{1/2}$ spaces and vague $_{gb}T_b$ spaces in vague topological space and their properties are discussed.

Definition 5.1: A VTS (X, τ) is called vague $_bT_{1/2}$space ($V_bT_{1/2}$ space in short) if every VbCSin X isVCS in X.

Definition 5.2: A VTS (X, τ) is called vague $_{gb}T_{1/2}$ space ($V_{gb}T_{1/2}$ space in short) if every VGbCS in X is VCS in X.

Definition 5.3: AVTS (X, τ) is called vague $_{gb}T_b$ space ($V_{gb}T_b$ space in short) if every VGbCS in X is VbCS in X.

Theorem 5.4: Every $V_{gb}T_{1/2}$space is $V_{gb}T_b$ space.

Proof: Let (X, τ) be $V_{gb}T_{1/2}$ space and let A be VGbCS in X. By hypothesis, A is VCSin X. Since every VCS is VbCS, A is VbCS in X. Hence (X, τ) is a $V_{gb}T_{1/2}$ space.
The converse of the above theorem need not be true in general as seen from the following example.

Example 5.5: Let X = {a,b} and let τ = {0,G,1} is VT on X, where
G = $\{\langle x,[0.6,0.7],[0.5,0.6]\rangle\}$ and A = $\{\langle x,[0.6,0.7],[0.5,0.6]\rangle\}$.Then (X, τ) is $V_{gb}T_{1/2}$ space. But it is not $_bT_{1/2}$ space since A is VGPCS but not VCS in X.

Theorem 5.6: Let (X, τ) be an VTS and (X, τ) $V_{gb}T_{1/2}$space. Then the following statements hold.
i) Any union of VGbCSisVGbCS. ii) Any intersection of VGbOS isVGbOS.

Proof:(i): Let $\{A_i\}_{i\in J}$ be a collection of VGbCS in a $V_{gb}T_{1/2}$space (X, τ). Therefore every VGbCS isVCS.But the union of VCS isVCS. Hence the union of VGbCS isVGbCS in X.

(ii): It can be proved by taking complement in (i).

Theorem 5.7: A VTS (X, τ) is a $V_{gb}T_b$ space if and only if VGbO(X) = VbO(X).

Proof: Necessity: Let A beVGbOS in X. Then A^c isVGbCS in X. By hypothesis, A^c isVbCS in X. Therefore A isVbOS in X. Hence VGbO(X) = VbO(X).

Sufficiency: Let A beVGbCS in X. Then A^c isVGbOS in X. By hypothesis, A^c isVbOS in X.Therefore A isVbCS in X. Hence (X, τ) is a $V_{gb}T_b$space.

Theorem 5.8: A VTS (X, τ) is$V_{gb}T_{1/2}$ space if and only if VGbO(X) = VO(X).

Necessity: Let A be VGbOS in X. Then A^c is VGbCS in X. By hypothesis, A^c is VCS in X.
Therefore A is VOS in X. Hence VGbO(X) = VO(X).

Sufficiency: Let A be VGbCS in X. Then A^c is VGbOS in X.By hypothesis, A^c is VOS in X. Therefore A is VCS in X. Hence (X, τ) is $V_{gb}T_{1/2}$ space.

6. References

1. Arya S, Nour T. Characterizations of s-normal spaces. Indian J Pure Appl. Math. 1990; 21:717-719.
2. Atanassov KT. Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 1986; 20:87-96.3.
3. Biswas R, Vague groups. Internat J Comput. Cognition. 2006; 4(2):20-23.
4. Bustince H, Burillo P. Vague sets are intuitionistic fuzzy sets, Fuzzy sets and systems. 1996; 79:403-405.
5. Chang CL. Fuzzy topological spaces. J Math Anal Appl. 1968; 24:182-190.
6. Gau WL, Buehrer DJ, Vague sets, IEEE Trans. Systems Man and Cybernet. 1993; 23(2):610-614.
7. Levine N. Generalized closed sets in topological spaces, Rend. Circ. Mat. Palermo. 1970; 19:89-96.
8. Levine N. Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly. 1963; 70:341.
9. Maki H, Balachandran K, Devi R. Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem.Fac. Sci. Kochi Univ. Ser. A Math. 1994; 15:51-63.
10. Maki H, Umehara J, Noiri T. Every topological space is pre- 1/2 T, Mem. Fac. Sci. Kochi. Univ. Ser. A. Mah. 1966; 17:33-42.
11. Mashour AS, Abd ElME, Monsef SN, Deeb El. On pre-continuous mappings and weak pre-continuous mappings, Proc.Math. Phys. Soc. Egypt. 1982; 53:47-53.
12. Njastad O. On some classes of nearly open sets. Pacific J Math. 1965; 15:961-870.
13. Rajarajeswari P, Krishna Moorthy R. On Intuitionistic Fuzzy Generalized b Closed sets.IJCA.63, 14.
14. Stone M. Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc. 1937; 41:374-481.
15. Zadeh LA, Fuzzy Sets. Information and Control. 1965; 8:338-335.