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Modeling of elastic behavior of a functionally graded rotating disc

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Abstract
A mathematical Model has been developed to study elastic behavior of a Functionally Graded (FG) Rotating Disc of constant thickness. The disc is made of FG composite in which SiC_p particles are distributed in a matrix of pure aluminium. The stresses and strain rates are estimated in the FGM disc by varying different profiles of distribution of SiC_p reinforcement in Al Matrix. The study concludes that the strains of a FGM disc with decreasing SiC_p content are significantly lower than uniform composite disc as well as a FGM disc with increasing SiC_p content.

Keywords: functionally graded material, particle gradient, rotating disc

1. Introduction

Functionally Graded Material (FGM) is a kind of composite material in which the volume content of the constituent particles vary along the certain directions [1]. These days the concept of FGMs has led to the development of superior materials while keeping the cost under control. Rotating disc is a very important component in industrial applications. The disc operating at very high speed leads to significant deformations [2].

Zenkour [3] presented a solution procedure to estimate the distribution of elastic stresses in rotating FGM disc of constant thickness. The study indicates that the stresses and deformations in the FGM disc are lower when the material of the disc is tailored in such a way that the elastic modulus and density are more near the inner radius than those noticed towards the outer radius. Afsar [4] carried out finite element analysis of rotating FGM disc subjected to thermal load. It is observed that thermo-elastic characteristics of the FGM disc are significantly influenced by temperature and thickness profile of the disc, apart from angular speed of the disc.

Callioglu [5] obtained closed-form solutions for rotating annular FGM disc subjected to internal as well as external pressure. The elastic modulus and density of the disc were assumed to vary radially according to different power law functions. It is observed that with the increase in property gradient, the stresses and deformations in the disc change significantly. Keeping this in view, it has been decided to analyze elastic behavior of a rotating FGM disc subjected to different kinds of distributions profiles of reinforcement.

2. Distribution of reinforcement and material properties

Consider a FG rotating disc with the inner radius a (=0.04 m) and the outer radius b (=0.1 m) with constant thickness (t) = 5 mm. The FG disc is assumed to be made of Al-SiC_p composite. The SiC_p volume content, $V(r)$ in the Al disc is assumed to vary along the radial distance (r) as given by,

$$V(r) = V_o\left(\frac{r}{b}\right)^n \tag{1}$$

Where V_o is the SiC_p content at the outer radius and n is SiC_p gradation index.

If V_{avg} is average SiC_p content in the FGM disc then on equating total SiC_p content in FGM disc with that in uniform composite disc, we get,

$$\int_a^b 2\pi rt\, V(r)dr = V_{av}\left[\pi(b^2 - a^2)t\right] \quad (2)$$

Substituting eq. (1) into eq. (2), the SiC_p content (V_o) may be estimated as,

$$V_o = \frac{V_{avg}\, b^n\,(2+n)(b^2 - a^2)}{2\,(b^{2+n} - a^{2+n})} \quad (3)$$

The density (ρ) and Young's modulus (E) are assumed to vary radially, according to the following relation,

$$\rho(r) = \rho_0\left(\frac{r}{b}\right)^{n_1} \text{ and } E(r) = E_0\left(\frac{r}{b}\right)^{n_2} \quad (4)$$

where ρ_o and E_o are respectively the values of density and young's modulus at the outer radius of FGM disc. The exponents n_1 and n_2 denotes gradation indices respectively for density and young's modulus.

3. Mathematical formulation

For elastic deformations, stresses and strains are related as [5],

$$\varepsilon_r = \frac{1}{E(r)}(\sigma_r - \nu\sigma_\theta) \quad (5)$$

$$\varepsilon_\theta = \frac{1}{E(r)}(\sigma_\theta - \nu\sigma_r) \quad (6)$$

where σ_r and σ_θ are respectively the radial and tangential stresses in the disc and v (=0.3) is the Poisson's ratio.

The force equilibrium equation for a rotating FGM disc is given as [6],

$$\frac{d}{dr}\left[r\sigma_r\right] - \sigma_\theta + \rho(r)\omega^2 r^2 = 0 \quad (7)$$

where $\rho(r)$ is the density and ω (=1570 rad/s) is angular velocity of the disc.

Equlibirium eq. (7) is solved along with the constitutive eqs. (5-6), to get the functions of radial and tangential stresses as given by,

$$\sigma_r = M_1 r^{\frac{n_2+m-2}{2}} + M_2 r^{\frac{n_2-m-2}{2}} + Ar^{2+n_1} \quad (8)$$

$$\sigma_\theta = \left[\left(\frac{n_2+m}{2}\right)M_1 r^{\frac{n_2+m-2}{2}} + \left(\frac{n_2-m}{2}\right)M_2 r^{\frac{n_2-m-2}{2}} + (3+n_1)Ar^{2+n_1}\right] + \rho(r)r^2\omega^2 \quad (9)$$

Where $A = \dfrac{-\rho_o\,\omega^2\,(3+v+n_1-n_2)}{b^{n_1}\,(8+n_1{}^2+6n_1-n_1 n_2-3n_2+vn_2)}$ and $m = \sqrt{n_2{}^2 - 4vn_2 + 4)}$

The constants M_1 and M_2 are calculated using free-free boundary conditions [7],

$$\sigma_r = 0 \text{ at } r = a \text{ and } \sigma_r = 0 \text{ at } r = b \quad (10)$$

Using the above boundary condition in equation (19), we get,

$$M_1 = \frac{D_2 b^{\frac{2+m-n_2}{2}} - D_1 a^{\frac{2+m-n_2}{2}}}{(b^m - a^m)}$$

and

$$M_2 = \frac{D_1 b^m a^{\frac{2+m-n_2}{2}} - D_2 b^{\frac{2+m-n_2}{2}} a^m}{(b^m - a^m)} \quad (11)$$

where,

$$D_1 = -Aa^{2+n_1} \qquad D_2 = -Ab^{2+n_1} \quad (12)$$

4. Results and Discussion

A computer code is generated on the basis of analysis carried out. The results are estimated for different FGM discs with varying reinforcement gradation index (refer Table 1).

Table 1: Description of FGM discs

Disc Notation	*n*	SiC_p Content (*vol %*)			ρ_o	n_1	E_o	n_2
		$V_i(r=a)$	V_{av}	$V_o(r=b)$				
D1	0.5	14.77	20	23.36	2816.95	0.0172	158.28	0.2529
D2	0	20	20	20	2801.12	0	146	0
D3	-0.5	26.66	20	16.86	2783.67	-0.019	133.56	-0.2678

The radial stress (Fig. 1a) in all the FGM discs is zero at inner and outer radius under the imposed boundary conditions (refer eq. 10). As compared to uniform composite disc D2 (*n*=0), the radial stress in FGM disc D1 (*n*= 0.5) is relatively lower, but in the FGM disc D3 (*n*= -0.5), it is relatively higher. On imposing decreasing SiC_p content in the FGM disc D3 (*n*= -0.5), the tangential stress (Fig. 1b) increases near the inner radius but decreases towards the outer radius as compare the uniform composite disc D2 (*n*=0). The tangential stress in the FGM disc D3 is the highest near the inner radius but the lowest towards the outer radius.

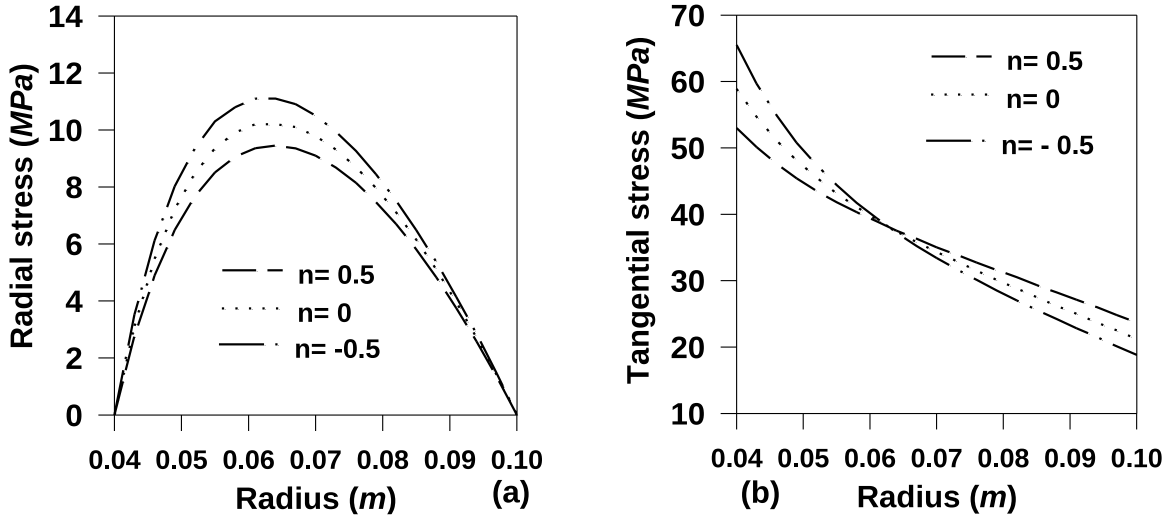

Fig. 1: Variation of: (a) Radial and (b) Tangential stresses in FGM discs.

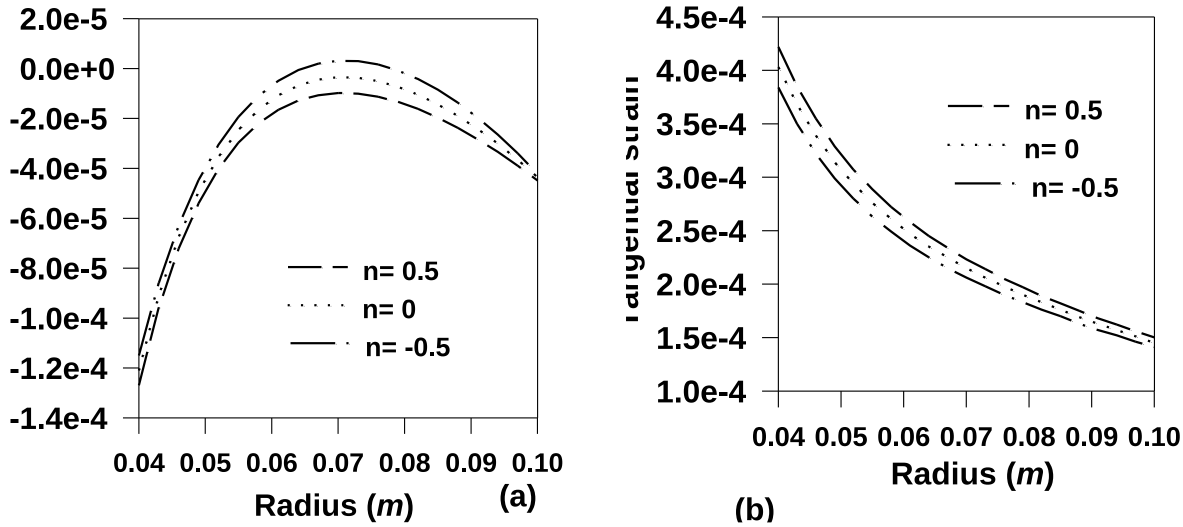

Fig. 2: Variation of: (a) Radial and (b) Tangential strains in FGM discs.

It is clear from Fig. 2a that radial strain in the FGM disc D3 are lower as compare FGM disc D1 and composite disc D2. The effect of imposing SiC_p gradient on the tangential strain rate in the FGM disc (Fig. 2b) is similar to those observed in Fig. 2(a) for radial strain rate.

5. Conclusions

The elastic behavior in rotating FGM disc has been studied by using different distributions of SiC_p content. The radial stress and tangential stress increases slightly for the FGM disc with decreasing SiC_p content distribution as compared to FGM disc with decreasing SiC_p content distribution as well as uniform composite disc. It is observed that by employing decreasing SiC_p content along the radius, the strain rates in the FGM disc can be significantly reduced.

6. References

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