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Supra soft generalized pre-regular closed and open sets in soft supra topological spaces

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Abstract

In this paper, we define supra soft generalized pre-regular closed and open sets in soft supra topological spaces, and we investigate some basic properties of these concepts. Further, we present the notion of supra soft gpr-connectedness and study their characteristics in soft supra topological space.

Keywords: Supra soft topological spaces, supra soft gpr closed, supra soft gpr open, supra soft- gpr-separate and connected.

1. Introduction

The soft set theory is a rapidly processing field of mathematics. Soft set theory was initiated by Molodtsov [11] as a new method for vagueness. Shabir and Naz [12] introduced the soft topological spaces which are defined over an initial universe with a fixed set of parameters and studied some basic notions of soft topological spaces such as soft open and closed sets, soft subspace, soft closure, soft neighbourhood of a point, soft separation axioms. Maji *et al* [10] carried out Molodtsov's idea by introducing several operations in soft set theory. Chen [3] introduced the concept of soft semi-open sets in soft topological spaces. The concept of generalized closed sets was introduced by N. Levine [9]. K. Kannan [7] studied soft generalized closed sets in soft topological spaces along with its properties. After then Yü ksel *et al* [13] defined soft regular generalized closed and open sets in soft topological spaces. Many mathematicians [2, 4, 5] extended the results of generalized closed sets in soft topological spaces.

El-Sheikh and Abd El-Latif [6] introduced the concept of supra soft topological spaces, which is wider and more general than the class of soft topological spaces. They introduced a unification of some types of different kinds of subsets of supra soft topological spaces using the notion of γ -operation and studied the decompositions of some forms of supra soft continuity. After that, Kandil *et al* [8] introduced the concept of soft supra g-closed sets in supra soft topological spaces, which is generalized in [1].

In this paper, we introduce the concept of supra soft generalized pre-regular closed soft sets with respect to a supra soft topological spaces. We investigate many properties of these concepts. We establish several interesting results and present their properties with the help of some examples.

2. Preliminaries

Let U be an initial universe set and E be the set of all possible parameters with respect to U , where parameters are the characteristics or properties of objects in U . Let $P(U)$ denote the power set of U , and let $A \subseteq E$.

Definiton [2.1] [1]

A pair (F, A) is called a soft set over U where $A \subseteq E$ and $F : A \rightarrow P(U)$ is a set valued mapping. In other words, a soft set over X is a parameterized family of subsets of the universe U . For $\forall e \in A$, $F(e)$ may be considered as the set of e -approximate elements of the soft set (F, A) . It is worth noting that $F(e)$ may be arbitrary. Some of them may be empty, and some may have nonempty intersection.

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Definiton [2.2] ^[12]

A soft set (F, A) over U is said to be a null soft set denoted by φ if for all $e \in A, F(e) = \varphi$. A soft set (F, A) over U is said to be an absolute soft set denoted by A if for all $e \in A, F(e) = U$.

Definition [2.3] ^[2]

Let Y be a nonempty subset of X, then \tilde{Y} denotes the soft set (Y, E) over X for which $Y(e) = Y$, for all $e \in E$. In particular, (X, E) will be denoted by \tilde{X} .

Definition [2.4] ^[2]:

For two soft sets (F, A) and (G, B) over U, we say that (F, A) is a soft subset of (G, B) if $A \subseteq B$ and for all $e \in A, F(e)$ and $G(e)$ are identical approximations. We write $(F, A) \tilde{\subseteq} (G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(G, B) \tilde{\subseteq} (F, A)$. Then (F, A) and (G, B) are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition [2.5] ^[12]

For two soft sets (F, A) and (G, B) over a common universe U.

The union of two soft sets (F, A) and (G, B) is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & , \text{if } e \in A - B \\ G(e) & , \text{if } e \in B - A \\ F(e) \cup G(e) & , \text{if } e \in A \cap B \end{cases}$$

We write $(F, A) \cup (G, B) = (H, C)$.

Definition [2.6] ^[2]

The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U denoted $(F, A) \cap (G, B)$ is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)$, for all $e \in C$.

Definition [2.7] ^[5]

For a soft set (F, A) over the universe U, the relative complement of (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \rightarrow P(U)$ is a mapping defined by $F^c(e) = U - F(e)$, for all $e \in A$.

Definition [2.8] ^[2]

Let $\tilde{\tau}$ be the collection of soft sets over a universe U with a fixed set of parameters E, then $\tilde{\tau}$ is said to be a soft topology on U.

- (1) $\varphi, \tilde{U} \in \tilde{\tau}$
- (2) the union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
- (3) the intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triplet (U, $\tilde{\tau}$, E) is called a soft topological space. Every member of $\tilde{\tau}$ is called a soft open set. A soft set (F, E) is called soft closed in U if $(F, E)^c \in \tilde{\tau}$.

Definition [2.9] ^[1]

Let μ be the collection of supra soft sets over a universe U with a fixed set of parameters E, then μ is said to be a supra soft topology on U if

- (1) $\varphi, U \in \mu$
- (2) the union of any number of soft sets in μ belongs to μ .

The triplet (U, μ , E) is called a supra soft topological space.

Every member of μ is called a supra soft open set. A soft set (F, E) is called supra soft closed in U if $(F, E)^c \in \mu$.

Remark [2.10] ^[13]

Every soft topological space is supra soft topological space, but the converse is not true in general as shown in the following example.

3. Supra Soft Generalized Pre-regular Closed Sets

Definition [3.1]

Let (U, μ , E) be a supra soft topological space and (F, E) be a soft set over U.

- (1) The supra soft closure of (F, E) is the supra soft set $(F, E)^{-s} = \cap \{(G, E) : (G, E) \text{ is supra soft closed and } (F, E) \subseteq (G, E)\}$.

(2) The supra soft interior of (F, E) is the supra soft set $(F, E)^{os} = \cup \{(H, E) : (H, E) \text{ is supra soft open and } (H, E) \subseteq (F, E)\}$.

Clearly, $(F, E)^{-s}$ is the smallest supra soft closed set over U which contains (F, E) and

$(F, E)^{os}$ is the largest supra soft open set over U which is contained in (F, E) .

Definition [3.2]

Let (U, μ, E) be a supra soft topological space and (F, E) be a supra soft set over U .

- (i) a supra soft semi open if $(A, E) \subseteq ((A, E)^{os})^{-s}$
- (ii) a supra soft regular open if $(A, E) = ((A, E)^{-s})^{os}$.
- (iii) a supra soft α -open if $(A, E) \subseteq (((A, E)^{os})^{-s})^{os}$
- (iv) a supra soft b-open if $(A, E) \subseteq ((A, E)^{os})^{-s} \cup ((A, E)^{-s})^{os}$
- (v) a supra soft pre-open set if $(A, E) \subseteq ((A, E)^{-s})^{os}$.
- (vi) a supra soft clopen is (A, E) is both supra soft open and supra soft closed.

Definition [3.3]

Let (U, μ, E) be a supra soft topological space. A soft set (F, E) is called supra soft preopen set in U if $(F, E) \subseteq ((F, E)^{-s})^{os}$.

The relative complement of a supra soft preopen set is called a supra soft pre-closed set.

It is obvious that every supra soft closed set is supra soft pre-closed.

Definition [3.4]

Let (U, μ, E) be a supra soft topological space over U and (F, E) be a supra soft set over U .

(1) The supra soft preclosure of (F, E) is the supra soft set $(F, E)^{-sp} = \cap \{(G, E) : (G, E) \text{ is supra soft preclosed and } (F, E) \subseteq (G, E)\}$.

(2) The supra soft preinterior of (F, E) is the supra soft set $(F, E)^{osp} = \cup \{(H, E) : (H, E) \text{ is supra soft preopen and } (H, E) \subseteq (F, E)\}$.

Clearly, $(F, E)^{-sp}$ is the smallest supra soft preclosed set over U which contains (F, E) and $(F, E)^{osp}$ is the largest supra soft preopen set over U which is contained in (F, E) .

Definition [3.5]

Let (U, μ, E) be a supra soft topological space over U . A soft set (F, E) is called a supra soft generalized pre-regular closed set

(supra soft gpr-closed) in U if $(F, E)^{-sp} \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and (G, E) is supra soft regular open in U .

Definition [3.6]

- (1) A supra soft regular generalized closed set (briefly supra soft rg-closed) if $(F, E)^{-s} \subseteq (G, E)$ and (G, E) is supra soft regular open in U .
- (2) A supra soft α -generalized closed set (briefly supra soft α g-closed) if $(F, E)^{-\alpha s} \subseteq (G, E)$ and (G, E) is supra soft open in U .
- (3) A supra soft generalized semi closed set (briefly supra soft gs-closed) if $(F, E)^{-ss} \subseteq (G, E)$ and (G, E) is supra soft open in U .
- (4) A supra soft generalized regular set (briefly supra soft gr-closed) if $(F, E)^{-sr} \subseteq (G, E)$ and (G, E) is supra soft open in U .
- (5) A supra soft generalized closed (supra soft g-closed) in U if $(F, E)^{-s} \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and (U, E) is supra soft open in U .

Theorem [3.7]

1. Every supra soft closed set is supra soft gpr-closed set.
2. Every supra soft rg closed set is supra soft gpr-closed set.
3. Every supra soft α g closed set is supra soft gpr-closed set.
4. Every supra soft gs closed set is supra soft gpr-closed set.
5. Every supra soft gr closed set is supra soft gpr-closed set.
6. Every supra soft π gr closed set is supra soft gpr-closed set.

Proof

1) Suppose $(F, E) \subseteq (G, E)$, where (G, E) is a supra soft regular open in U ,

then $(F, E)^{-s} = (F, E) \subseteq (G, E)$. Thus $(F, E)^{-sp} \subseteq (F, E)^{-s} \subseteq (G, E)$. Hence (F, E) is supra soft gpr-closed.

- 2) Let $(F,E) \subseteq (G,E)$ and (G,E) be supra soft regular open. Then $(F,E)^{-s} \subseteq (G,E)$. since ever supra soft closed set is supra soft pre closed, $(F,E)^{-sp} \subseteq (F,E)^{-s} \subseteq (G,E)$. Hence (F,E) is supra soft gpr closed.
- 3) Let (F,E) be supra soft α g closed set in U and $(F,E) \subseteq (G,E)$ and (G,E) be supra soft regular open. Since every supra soft α closed set is supra soft pre closed set, we have $(F,E)^{-s\alpha} \subseteq (G,E)$. Hence $(F,E)^{-sp} \subseteq (F,E)^{-s\alpha} \subseteq (G,E)$. Then (F,E) is supra soft gpr closed.
- 4) Let $(F,E) \subseteq (G,E)$ and (G,E) be supra soft regular open. By assumption $(F,E)^{-ss} \subseteq (G,E)$. Since every supra soft semi closed set is supra soft pre closed set then $(F,E)^{-sp} \subseteq (F,E)^{-ss} \subseteq (G,E)$. Hence (F,E) is supra soft gpr closed.
- 5) Let (F,E) be supra soft gr closed set in U and $(F,E) \subseteq (G,E)$, where (G,E) be supra soft regular open. since ever supra soft closed set is supra soft pre closed. Hence $(F,E)^{-sp} \subseteq (F,E)^{-sr} \subseteq (G,E)$. Hence (F,E) is supra soft gpr closed.
- 6) Let (F,E) be supra soft π gs-closed set in U and $(F,E) \subseteq (G,E)$, where (G,E) be supra soft regular open. By assumption $(F,E)^{-ss} \subseteq (G,E)$. Also $(F,E)^{-sp} \subseteq (F,E)^{-ss} \subseteq (G,E)$. Hence (F,E) is supra soft gpr closed.

Remark [3.8]

Converse of the above theorem need not be true as seen in the following example.

Example [3.9]

Let $X = \{a,b,c\}$, $E = \{e_1, e_2\}$, $F_1, F_2, F_3, F_4, F_5, F_6$ are functions from E to $P(X)$ and defined as follows;

$$F_1(e_1) = \{X\}, F_1(e_2) = \{a\}, \quad F_2(e_1) = \{b\}, F_2(e_2) = \{a\},$$

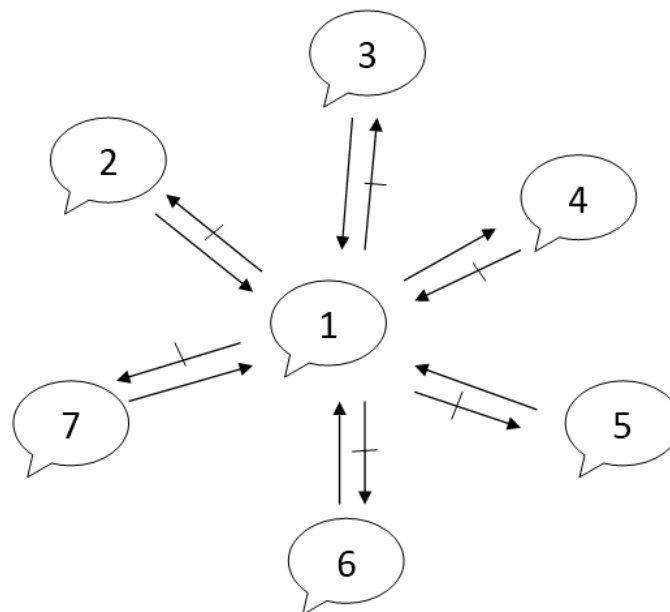
$$F_3(e_1) = \{X\}, F_3(e_2) = \{a,b\}, \quad F_4(e_1) = \{\emptyset\}, F_4(e_2) = \{c\},$$

$$F_5(e_1) = \{X\}, F_5(e_2) = \{a,c\}, \quad F_6(e_1) = \{b\}, F_6(e_2) = \{a,c\}.$$

Then $\mu = \{\emptyset, X, (F_1, E), \dots, (F_6, E)\}$ is supra soft topology and elements of μ is supra soft open -sets.

- (1) The supra soft set $(A,E) = \{\{a\}, \{a,b\}\}$ is gpr-closed, but not supra soft -closed.
- (2) The supra soft set $(A,E) = \{\{c\}, \{X\}\}$ is gpr-closed, but not supra soft rg -closed.
- (3) The supra soft set $(A,E) = \{\{X\}, \{c\}\}$ is gpr-closed, but not supra soft α g-closed.
- (4) The supra soft set $(A,E) = \{\{b\}, \{a,c\}\}$ is gpr-closed, but not supra soft gs -closed.
- (5) The supra soft set $(A,E) = \{\{X\}, \{a,b\}\}$ is gpr-closed, but not supra soft gr-closed.
- (6) The supra soft set $(A,E) = \{\{a,b\}, \{a\}\}$ is gpr-closed, but not supra soft π gs-closed.

Remark [3.10]



- | | |
|---|--|
| 1. supra soft gpr- closed set | 2. supra soft regular generalized closed set |
| 3. supra soft α generalized closed set | 4. supra soft generalized semi soft closed set |
| 5. supra soft gr closed set | 6. supra soft π gs closed set |

Remark [3.11]

The union of two supra soft gpr-closed sets is generally not a supra soft gpr-closed set.

Example [3.12]: Let $X = \{a, b, c\}$, $E = \{e_1, e_2\}$, $F_1, F_2, F_3, F_4, F_5, F_6$ are functions from E to $P(X)$ and are defined as follows;

$$\begin{aligned} F_1(e_1) &= \{X\}, F_1(e_2) = \{a\}, & F_2(e_1) &= \{b\}, F_2(e_2) = \{a\}, \\ F_3(e_1) &= \{X\}, F_3(e_2) = \{a, b\}, & F_4(e_1) &= \{\emptyset\}, F_4(e_2) = \{c\}, \\ F_5(e_1) &= \{X\}, F_5(e_2) = \{a, c\}, & F_6(e_1) &= \{b\}, F_6(e_2) = \{a, c\}. \end{aligned}$$

Proof

Let (A, E) and (B, E) be two soft sets over U such that $A(e_1) = \{\emptyset\}$, $A(e_2) = \{a\}$ and $B(e_1) = \{\emptyset\}$, $B(e_2) = \{c\}$. Clearly, (A, E) and (B, E) are supra soft gpr-closed sets in (X, μ, E) , but $(A, E) \cup (B, E)$ is not a supra soft gpr-closed set.

Remark [3.13]

The intersection of two supra soft gpr-closed sets is generally not a supra soft gpr-closed set.

Example [3.14] : Let $X = \{a, b, c\}$, $E = \{e_1, e_2\}$, $F_1, F_2, F_3, F_4, F_5, F_6$ are functions from E to $P(X)$ and are defined as follows;

$$\begin{aligned} F_1(e_1) &= \{X\}, F_1(e_2) = \{a\}, & F_2(e_1) &= \{b\}, F_2(e_2) = \{a\}, & F_3(e_1) &= \{X\}, F_3(e_2) = \{a, b\}, \\ F_4(e_1) &= \{\emptyset\}, F_4(e_2) = \{c\}, & F_5(e_1) &= \{X\}, F_5(e_2) = \{a, c\}, & F_6(e_1) &= \{b\}, F_6(e_2) = \{a, c\}. \end{aligned}$$

Proof

Let (A, E) and (B, E) be two soft sets over U such that $A(e_1) = \{X\}$, $A(e_2) = \{b\}$ and $B(e_1) = \{a\}$, $B(e_2) = \{X\}$. Clearly, (A, E) and (B, E) are supra soft gpr-closed sets in (X, μ, E) but $(A, E) \cap (B, E)$ is not a supra soft gpr-closed set.

Theorem [3.15]

Let (X, μ, E) be a supra soft topological space and (F, E) be a supra soft set over U . If a supra soft set (F, E) is supra soft gpr-closed, then $(F, E)^{-sp} - (F, E)$ contains only null supra soft regular closed set.

Proof

Suppose that (F, E) is supra soft gpr-closed. Let (H, E) be a supra soft regular closed subset of $(F, E)^{-sp} - (F, E)$. Then $(H, E) \subseteq (F, E)^{-sp} \cap (F, E)^c$ and so $(F, E) \subseteq (H, E)^c$. Since (F, E) is a supra soft gpr-closed set and $(H, E)^c$ is supra soft regular open, we obtain $(F, E)^{-sp} \subseteq (H, E)^c$. Consequently $(H, E) \subseteq ((F, E)^{-sp})^c$. We have already $(H, E) \subseteq (F, E)^{-sp}$. Hence we obtain $(H, E) \subseteq (F, E)^{-sp} \cap ((F, E)^{-sp})^c = \emptyset$. This shows $(H, E) = \emptyset$. Therefore $(F, E)^{-sp} - (F, E)$ contains only null supra soft regular closed set.

Theorem [3.16]

Let (X, μ, E) be a supra soft topological space over U and (F, E) be a supra soft gpr-closed set in U . (F, E) is supra soft gpr-closed if and only if $(F, E)^{-sp} - (F, E)$ is supra soft regular closed.

Proof:

Let (F, E) be a supra soft gpr-closed set. If (F, E) is supra soft gpr-closed, then $(F, E)^{-sp} = (F, E)$. So $(F, E)^{-sp} - (F, E) = \emptyset$. Which is supra soft regular closed. Conversely, suppose that $(F, E)^{-sp} - (F, E)$ is supra soft regular closed. Since (F, E) is supra soft gpr-closed, then $(F, E)^{-sp} - (F, E) = \emptyset$. That is $(F, E)^{-sp} = (F, E)$. Hence (F, E) is supra soft pre-closed.

Theorem [3.17]

Let (U, μ, E) be a supra soft topological space, (F, E) and (G, E) are supra soft sets over U . If (F, E) is supra soft gpr-closed and $(F, E) \subseteq (G, E) \subseteq (F, E)^{-sp}$, then (G, E) is supra soft gpr-closed.

Proof

Let $(G, E) \subseteq (H, E)$ where (H, E) is supra soft regular open. Then $(F, E) \subseteq (G, E)$ implies $(F, E) \subseteq (H, E)$. Since (F, E) is supra soft gpr-closed, $(F, E)^{-sp} \subseteq (H, E)$. $(G, E) \subseteq (F, E)^{-sp}$ implies $(G, E)^{-sp} \subseteq (F, E)^{-sp}$. Thus $(G, E)^{-sp} \subseteq (H, E)$ and this shows that (G, E) is supra soft gpr-closed.

Definition [3.18]

Let (U, μ, E) be a supra soft topological space. A supra soft set (F, E) is called supra generalized preregular open soft (supra soft gpr-open) in U if $(F, E)^c$ is supra soft rg-closed.

Theorem [3.19]

A soft set (F,E) is supra soft gpr-open in a soft topological space (U, μ, E) if and only if $(H,E) \subseteq (F,E)^{os}$ whenever (H,E) is supra soft regular closed in U and $(H,E) \subseteq (F,E)$.

Proof

Suppose that (H,E) is supra soft regular closed and $(H,E) \subseteq (F,E)$ implies

$(H,E) \subseteq (F,E)$. Let $(F,E)^c \subseteq (G,E)$ where (G,E) is supra soft regular open. Then $(G,E)^c \subseteq (F,E)$ where $(G,E)^c$ is supra soft regular closed. By hypothesis $(G,E)^c \subseteq (F,E)^{osp}$. That is $((F,E)^{osp})^c \subseteq (G,E)$. Equivalently $((F,E)^c)^{sp} \subseteq (G,E)$. Thus $(F,E)^c$ is supra soft gpr-closed. Hence we obtain (F,E) is supra soft gpr-open.

Conversely, suppose that (F,E) is supra soft gpr-open, $(H,E) \subseteq (F,E)$ and (H,E) is supra soft regular closed. Then $(H,E)^c$ is supra soft regular open. Then $(F,E)^c \subseteq (H,E)^c$. Since $(F,E)^c$ is supra soft gpr-closed, $((F,E)^c)^{sp} \subseteq (H,E)^c$. Therefore $(H,E) \subseteq (((F,E)^c)^{sp})^c = (F,E)^{osp}$.

Theorem [3.20]

Let (U, μ, E) be a supra soft topological space, (F,E) and (G,E) supra soft sets over U . If (F,E) is supra soft gpr-open in U and $(F,E)^{osp} \subseteq (G,E) \subseteq (F,E)$, then (G,E) is supra soft gpr-open.

Proof

$(F,E)^{osp} \subseteq (G,E) \subseteq (F,E)$ implies $(F,E)^c \subseteq (G,E)^c \subseteq ((F,E)^{sp})^c$.

That is, $(F,E)^c \subseteq (G,E)^c \subseteq ((F,E)^c)^{sp}$. Since $(F,E)^c$ is supra soft gpr-closed, $(G,E)^c$ is supra soft gpr-closed and (G,E) is supra soft gpr-open.

4. Supra Soft gpr Separated**Definition [4.1]**

Two non-null supra soft sets (G,E) and (H,E) of a supra soft topological space (U, μ, E) are said to be supra soft gpr-separated sets if $(G,E) \cap \text{gpr}^s(H,E) = \emptyset$ and $\text{gpr}^s(G,E) \cap (H,E) = \emptyset$.

Definition [4.2]

A soft set (F,E) of a supra soft topological space (U, μ, E) is said to be supra gpr-soft clopen set if it is both supra soft gpr-open set and supra soft gpr-closed set.

Remark [4.3]

(1): Each two supra soft gpr-separated sets are always disjoint.

(2): Each two disjoint supra soft sets, in which both of them either supra soft gpr-open sets or supra soft gpr-closed sets, are supra soft gpr-separated.

Theorem [4.4]

Let (G,E) and (H,E) be non-null supra soft sets of a supra soft topological space (U, μ, E) . Then, the following statements hold:

(1) If (G,E) and (H,E) are supra soft gpr-separated, $(G_1,E) \subseteq (G,E)$ and $(H_1,E) \subseteq (H,E)$ then (G_1,E) and (H_1,E) are supra soft gpr-separated sets.

(2): If (G,E) and (H,E) are supra soft gpr-open soft sets, $(A,E) = (G,E) \cap ((U,E) - (H,E))$ and $(B,E) = (H,E) \cap ((U,E) - (G,E))$, then (A,E) and (B,E) are supra soft gpr-separated sets.

Proof

(1) Since $(G_1,E) \subseteq (G,E)$. Then, $\text{gpr}^s(G_1,E) \subseteq \text{gpr}^s(G,E)$.

Hence, $(H_1,E) \cap \text{gpr}^s(G_1,E) \subseteq (H_1,E) \cap \text{gpr}^s(G,E) = \emptyset$. Similarly,

$(G_1,E) \cap \text{gpr}^s(H_1,E) = \emptyset$. Thus, (G_1,E) and (H_1,E) are supra soft gpr-separated sets.

(2) Let (G,E) and (H,E) be supra soft gpr-open sets. Then, $((U,E) - (G,E))$ and $((U,E) - (H,E))$ are supra soft gpr-closed sets. Assume that, $(A,E) = (G,E) \cap ((U,E) - (H,E))$ and $(B,E) = (H,E) \cap ((U,E) - (G,E))$. Then, $(A,E) \subseteq ((U,E) - (H,E))$ and $(B,E) \subseteq ((U,E) - (G,E))$. Hence, $\text{gpr}^s(A,E) \subseteq ((U,E) - (H,E)) \subseteq ((U,E) - (B,E))$ and $\text{gpr}^s(B,E) \subseteq ((U,E) - (G,E)) \subseteq ((U,E) - (A,E))$. Consequently, $\text{gpr}^s(A,E) \cap (B,E) = \emptyset$ and $\text{gpr}^s(B,E) \cap (A,E) = \emptyset$. Therefore, (A,E) and (B,E) are supra soft gpr-separated sets.

5. Supra Soft gpr-connected**Definition [5.1]**

Let (U, μ, E) be a supra soft topological space. A supra soft gpr-separation of U is a pair of non-null proper supra soft gpr-open sets in μ such that $(F,E) \cap (G,E) = \emptyset$ and $U = (F,E) \cup (G,E)$.

Definition [5.2]

A supra soft topological space (U, μ, E) is said to be supra soft gpr-connected if and only if there is no supra soft gpr-separations of U . If (U, μ, E) has such supra soft gpr-separations, then (U, μ, E) is said to be supra soft gpr-disconnected.

Remark [5.3]

(1): \varnothing is always supra soft gpr-connected.

(2): If $(G, E), (H, E)$ are non-null supra soft gpr-separated sets. Then, the pair $(G, E), (H, E)$ is called the supra soft gpr-disconnection of U .

Theorem [5.4]

Let (U, μ, E) be a supra soft topological space, then the following statements are equivalent:

- (1): U is supra soft gpr-connected.
- (2): U can not be expressed as a soft union of two non-null disjoint supra soft gpr-open sets.
- (3): U can not be expressed as a soft union of two non-null disjoint supra soft gpr-closed sets.
- (4): There is no proper supra soft gpr-clopen set in (U, μ, E) .
- (5): U can not be expressed as a supra soft union of two non-null supra soft gpr-separated sets.

Proof

(1) \Leftrightarrow (2): It is obvious from the above Definition.

(2) \Rightarrow (3): Suppose that $U = (F, E) \cup (G, E)$ for some supra soft gpr-closed sets (F, E) and (G, E) such that $(F, E) \cap (G, E) = \varnothing$. Then, $(F, E) = (G, E)^c$ which is gpr-open supra soft set, $U = (G, E) \cup (G, E)^c$ and $(G, E) \cap (G, E)^c = \varnothing$, which is a contradiction with (2).

(3) \Rightarrow (4): Suppose that there is a proper supra soft gpr-clopen subset (F, E) of U . Then, $(F, E)^c$ is supra soft gpr-clopen set, where $U = (F, E) \cup (F, E)^c$ and $(F, E) \cap (F, E)^c = \varnothing$, which is a contradiction with (3).

(4) \Rightarrow (3): Suppose that $U = (F, E) \cup (G, E)$ for some supra soft gpr-closed sets (F, E) and (G, E) such that $(F, E) \cap (G, E) = \varnothing$. Then, $(F, E) = (G, E)^c$ and $(G, E) = (F, E)^c$. Thus, (F, E) and (G, E) are proper supra soft gpr-clopen sets, which is a contradiction with (4).

(3) \Rightarrow (5): Suppose that $U = (H, E) \cup (G, E)$ for some supra soft gpr-separated sets (H, E) and (G, E) . Then, $(G, E) \cap \text{gpr}^s(H, E) = \varnothing$ and $\text{gpr}^s(G, E) \cap (H, E) = \varnothing$. It follows that,

$(H, E) \cap (G, E) = \varnothing$. Hence, $(H, E) = (G, E)^c$ and $(G, E) = (H, E)^c$. Therefore,

$\text{gpr}^s(H, E) \subseteq (G, E)^c = (H, E)$ and $\text{gpr}^s(G, E) \subseteq (H, E)^c = (G, E)$. But, $(H, E) \subseteq \text{gpr}^s(H, E)$ and $(G, E) \subseteq \text{gpr}^s(G, E)$. Thus, (H, E) and (G, E) are supra soft gpr-closed sets, which is a contradiction with (3).

(5) \Rightarrow (1): Suppose that $U = (F, E) \cup (G, E)$ for some supra soft gpr-open sets (F, E) and (G, E) such that $(F, E) \cap (G, E) = \varnothing$. Then $(F, E) = (G, E)^c$ and $(G, E) = (F, E)^c$. Thus, (F, E) and (G, E) are supra soft gpr-clopen sets. Hence, (F, E) and (G, E) are supra soft gpr-separated sets, which is a contradiction with (5).

6. References

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