International Journal of Applied Research 2017; 3(5): 1117-1122

International Journal of Applied Research

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2017; 3(5): 1117-1122
www.allresearchjournal.com
Received: 07-04-2017
Accepted: 08-05-2017

T Madhumathi
M.Phil Research Scholar,
Department of Mathematics,
Nirmala College for Women,
Coimbatore, Tamil Nadu,
India

F Nirmala Irudayam
Assistant Professor,
Department of Mathematics,
Nirmala College for Women,
Coimbatore, Tamil Nadu,
India

Correspondence
T. Madhumathi
M.Phil Research Scholar,
Department of Mathematics,
Nirmala College for Women,
Coimbatore, Tamil Nadu,
India

On certain forms of contra continuity in simple extension topological spaces

T Madhumathi and F Nirmala Irudayam

Abstract
This paper is brings out the idea of a new classes of functions named contra- Ω_{gb}^{+}-continuous and almost contra- Ω_{gb}^{+}-continuous functions in the light of simple extended topological spaces. We also initiate to investigate some of its basic properties and relations concerning the new functions.

Keywords: Contra continuity, simple extension, topological spaces

1. Introduction

The notation of generalized open sets in a topological space called b-open sets was introduced Andrijevic [1]. In 1996, Dontchev [4] initiated the notion of contra continuous functions and an year later Dontchev, Ganster and Reilly [6] studied a new class of functions called regular set connected functions. Dontchev and Noiri [5], Jafari and Noiri [8, 9] investigated the concepts of contra semi-continuous functions, contra pre-continuous functions and contra α- continuous functions between topological spaces respectively. Nasef [16] defined the so called contra b-continuous functions in topological spaces. A.A. Omari and M.S.M. Noorani [18] discussed the further properties of contra b-continuous functions and established the idea of almost contra b-continuous functions. Caldas, Jafari, Noiri and Simoes [3] proposed a new class of functions called generalized contra continuous (contra g-continuous) functions. Metin Akdag and Alkan Ozkan [14] introduced some of the fundamental properties of contra generalized b-continuous (contra gb-continuous) via the concept of gb-open sets.

In 1963 Levine [10] introduced the concept of a simple extension of a topology τ as $\tau(B) = \{(B \cap O) \cup O' / O, O' \in \tau \text{ and } B \notin \tau\}$. Sr. I. Arockiarani and F. Nirmala Irudayam [15] introduced the concept of b^{+}-open sets in simple extended topological spaces. S. Reena and F. Nirmala Irudayam [19] devised a new form of continuity and T. Noiri, Sr. I. Arockiarani and F. Nirmala Irudayam [16] coined the idea of Ω_{gb}^{+*}, $\mho_{gb}^{+*}$ sets in simple extended topological spaces. T. Madhumathi and F. Nirmala Irudayam [12] proposed the idea of $\Omega_{gb}^{+}(S)$ and $\mho_{gb}^{+}(S)$ sets in simple extension ideal topological spaces. T. Madhumathi and F. Nirmala Irudayam [13] formulated the concept of Ω_{gb}^{+}-closed sets in extended topological spaces. The purpose of this paper is to introduce the notion of contra Ω_{gb}^{+}-continuous, almost contra Ω_{gb}^{+}-continuous and study some of their properties.

2. Preliminaries

All through the paper the space X is a simple extended topological spaces in which no separation axioms are assumed unless and otherwise stated.

Definition 2.1: A subset A of a topological space (X,τ) is said to be,

i) Regular open set [23], if A = int(cl(A)) and a b-open set [1], if A ⊆ cl(int(A)) ∪ int(cl(A)).
ii) A generalized closed (briefly g-closed) [11] if cl(A)⊆U whenever A⊆U and U is open.
iii) A generalized b-closed (briefly gb-closed) [11] if bcl(A)⊆U whenever A⊆U and U is open.
iv) πgb-closed [21] if bcl(A)⊂A whenever A⊂U and U is π-open in (X,τ). By πGBC(X,τ) we mean the family of all πgb-closed subsets of the space (X,τ)

Definition 2.2: A subset A of a topological space (X,τ^+) is said to be,
1. Regular$^+$ open set[15] if $A = int(cl^+(A))$ and b^+-open set, if $A \subseteq cl^+(int(A)) \cup int(cl^+(A))$.
2. A generalized$^+$ closed[15] (briefly g^+-closed) if $cl^+(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
3. A generalized b^+-closed[15](briefly gb^+-closed) if $bcl^+(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
4. πgb^+-closed[19] if $bcl^+(A) \subset A$ whenever $A \subset U$ and U is π^+-open in (X,τ^+). By $\pi GB^+C(X,\tau^+)$ we mean the family of all πgb^+-closed subsets of the space (X,τ^+).

Definition 2.3 [12]: Let S be a subset of a topological space (X, τ^+) we define the sets $\Omega_{gb}^+(S)$and$\mho_{gb}^+(S)$asfollows,
$\Omega_{gb}^+(S) = \cap\{G \mid G \in \pi GB^+O(X,\tau^+) \text{ and } S \subseteq G\}$, $\mho_{gb}^+(S) = \cup\{F \mid F \in \pi GB^+C(X,\tau^+) \text{ and } S \supseteq F\}$.

Definition 2.4 [12]: A space (X, τ^+) is called a Ω_{gb}^+ -closed set if $A=S\cap C$ where S is Ω_{gb}^+-set and C is closed set.

Definition 2.5 [19]: A function f: $(X,\tau^+) \to (Y,\sigma^+)$ is called
1. π^+-irresolute if $f^{-1}(V)$ is π^+-closed in (X,τ^+) for every π^+-closed set V of (Y,σ^+).
2. b^+-irresolute if for each b^+-open set V in (Y,σ^+), $f^{-1}(V)$ is b^+-open in (X,τ^+).
3. b^+-continuous if for each open set V in (Y,σ^+), $f^{-1}(V)$ is b^+-open in (X,τ^+).
4. Ω_{gb}^+-continuous [13] if every $f^{-1}(V)$ is Ω_{gb}^+-closed in (X,τ^+) for every closed set V of (Y,σ^+).
5. Ω_{gb}^+-irresolute [13] if $f^{-1}(V)$ is Ω_{gb}^+-closed in (X,τ^+) for every Ω_{gb}^+-closed set V in (Y,σ^+).

Definition 2.6 [13]: A topological space X is a Ω_{gb}^+ space if every Ω_{gb}^+- closed set is closed.

Definition 2.7: A function $f:(X,\tau^+) \to (Y,\sigma^+)$ is called
1. contra –continuous [4] if $f^{-1}(V)$ is closed in X for each open set V of Y.
2. contra pre-continuous [9] if $f^{-1}(V)$ is pre-closed in X for each open set V of Y.
3. contra semi-continuous [5] if $f^{-1}(V)$ is semi-closed in X for each open set V of Y.
4. contra α-continuous [8] if $f^{-1}(V)$ is α-closed in X for every open set V of Y.
5. contra-b-continuous [17] if $f^{-1}(V)$ is b-closed in X for each open set V of Y.
6. contra-πg-continuous [23] if $f^{-1}(V)$ is πg-closed in X for each open set V of Y. (vii)contra-πgα-continuous [2] if $f^{-1}(V)$ is πgα-closed in X for each open set V of Y.
7. almost continuous [20] (almost contra$^+$-continuous) if $f^{-1}(V)$ is open (closed) in X for each regular open set V of Y.
8. regular set connected [24], if $f^{-1}(V)$ is clopen in X for every regular open set V of Y.

3. Contra Ω_{GB}^+ -Continuous Function

In this section we promote the new idea of contra Ω_{gb}^+-continuous functions and almost contra Ω_{gb}^+-continuous functions in simple extended topological spaces.

Definition 3.1: A function $f:(X,\tau^+) \to (Y,\sigma^+)$ is called
1. contra$^+$ –continuous if $f^{-1}(V)$ is closed in X for each open set V of Y.
2. contra pre$^+$-continuous if $f^{-1}(V)$ is pre$^+$-closed in X for each open set V of Y.
3. contra semi$^+$-continuous if $f^{-1}(V)$ is semi$^+$-closed in X for each open set V of Y.
4. contra α^+-continuous if $f^{-1}(V)$ is α^+-closed in X for every open set V of Y.
5. contra-b^+-continuous if $f^{-1}(V)$ is b^+-closed in X for each open set V of Y.
6. contra-πg^+-continuous if $f^{-1}(V)$ is πg^+-closed in X for each open set V of Y.
7. contra-$\pi g\alpha^+$-continuous if $f^{-1}(V)$ is $\pi g\alpha^+$-closed in X for each open set V of Y.

Definition 3.2: A function $f:(X, \tau^+)\to(Y,\sigma^+)$ is called contra- Ω_{gb}^+-continuous if $f^{-1}(V)$ is Ω_{gb}^+-closed in (X,τ^+) for each open set V of (Y, σ^+).

Example 3.3: Let X = Y = {a, b, c} with topologies $\tau = \{\Phi,X,\{a\}\}$, B={b}, $\tau^+ = \{\Phi,X,\{a\},\{b\},\{a,b\}\}and\sigma=\{\Phi,Y,\{b\},\{b,c\}\}$, B={c}, $\sigma^+=\{\Phi,Y,\{b\},\{c\},\{b,c\}\}$. Define $f:(X, \tau^+) \to (Y,\sigma^+)$ by f(a)=c, f(b)=a, f(c)=b . Then the function f is contra Ω_{gb}^+-continuous.

Theorem 3.4: Every contra$^+$-continuous function is contra Ω_{gb}^+-continuous but not conversely.

Proof: Let $f:(X, \tau^+)\to(Y,\sigma^+)$ be contra$^+$ -continuous. Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is closed in X. Since every closed set is Ω_{gb}^+-closed, $f^{-1}(V)$ is Ω_{gb}^+-closed in X. Therefore f is contra Ω_{gb}^+-continuous.

Example 3.5: Let X = Y = {a, b, c} be two topology spaces with topologies $\tau = \{\Phi,X,\{a\}\}$, B={b},$\tau^+ = \{\Phi,X,\{a\},\{b\},\{a,b\}\}$and $\sigma=\{\Phi,Y,\{b\},\{b,c\}\}$, B={c}, $\sigma^+=\{\Phi,Y,\{b\},\{c\},\{b,c\}\}$. Define $f:(X, \tau^+) \to (Y,\sigma^+)$ by f(a)=a, f(b)=c, f(c)=b. Then the function f is contra Ω_{gb}^+-continuous but not contra$^+$-continuous.

Theorem 3.6: Every contra pre$^+$-continuous function is contra Ω_{gb}^+-continuous but not conversely.

Proof: Let $f:(X, \tau^+)\to(Y,\sigma^+)$ be contra pre$^+$ -continuous. Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is pre$^+$ closed in X. Since every pre$^+$ closed set is Ω_{gb}^+-closed, $f^{-1}(V)$ is Ω_{gb}^+-closed in X. Therefore f is contra Ω_{gb}^+-continuous.

Example 3.7: Let X = Y = {a, b, c} be two topology spaces with topologies $\tau = \{\Phi,X,\{a,b\}\}$, B={a}, $\tau^+ = \{\Phi,X,\{a\},\{a,b\}\}$ and $\sigma=\{\Phi,Y,\{b\}\}$, B={a,c}, $\sigma^+=\{\Phi,Y,\{b\}\{a,c\}\}$. Define $f:(X, \tau^+) \rightarrow (Y,\sigma^+)$ by f(a)=a, f(b)=c, f(c)=b. Then the function f is contra Ω_{gb}^+-continuous but not contra pre^+-continuous.

Theorem 3.8: Every contra $semi^+$-continuous function is contra Ω_{gb}^+-continuous but not conversely.
Proof: Let $f:(X, \tau^+)\rightarrow(Y,\sigma^+)$ be contra $semi^+$-continuous. Let V be any open set in Y. By the property of contra $semi^+$-continuity we have the inverse image $f^{-1}(V)$ to be $semi^+$-closed in X. But we know that every $semi^+$-closed set is Ω_{gb}^+-closed. Hence $f^{-1}(V)$ is Ω_{gb}^+-closed in X. Therefore f is contra Ω_{gb}^+-continuous.

Example 3.9: Let X = Y = {a, b, c} be two topology spaces with topologies $\tau = \{\Phi,X,\{a,b\}\}$, B={b}, $\tau^+ = \{\Phi,X,\{b\},\{a,b\}\}$ and $\sigma = \{\Phi,Y,\{b\}\}$, B = {a,b}, $\sigma^+ = \{\Phi,Y,\{b\}\{a,b\}\}$. Define $f:(X, \tau^+) \rightarrow (Y,\sigma^+)$ by f(a)=b, f(b)=c, f(c)=a. Then the function f is contra Ω_{gb}^+-continuous but not contra $semi^+$-continuous.

Theorem 3.10: Every contra α^+-continuous function is contra Ω_{gb}^+-continuous but not conversely.
Proof: Let $f:(X, \tau^+)\rightarrow(Y,\sigma^+)$ be contra α^+ -continuous. Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is α^+ closed in X. Since every α^+ closed set is Ω_{gb}^+-closed, $f^{-1}(V)$ is Ω_{gb}^+-closed in X. Therefore f is contra Ω_{gb}^+-continuous.

Example 3.11: Let X = Y = {a, b, c} be two topology spaces with topologies $\tau = \{\Phi,X,\{a,c\}\}$, B={c}, $\tau^+ = \{\Phi,X,\{c\},\{a,c\}\}$ and $\sigma = \{\Phi,Y,\{a\}\}$, B = {a,b}, $\sigma^+ = \{\Phi,Y,\{a\}\{a,b\}\}$. Define $f:(X, \tau^+) \rightarrow (Y,\sigma^+)$ by f(a)=b, f(b)=a, f(c)=c. Then the function f is contra Ω_{gb}^+-continuous but not contra α^+-continuous.

Theorem 3.12: Every contra b^+-continuous function is contra Ω_{gb}^+-continuous but not conversely.
Proof: Let $f:(X, \tau^+)\rightarrow(Y,\sigma^+)$ be contra b^+-continuous. Let V be any open set in Y. By the property of contra b^+-continuity we have the inverse image $f^{-1}(V)$ to be b^+-closed in X. But we know that every b^+-closed set is Ω_{gb}^+-closed. Hence $f^{-1}(V)$ is Ω_{gb}^+-closed in X. Therefore f is contra Ω_{gb}^+-continuous.

Example 3.13: Let X = Y = {a, b, c, d} be two topology spaces with topologies $\tau=\{\Phi,X,\{a\},\{a,d\}\}$, B={d}, $\tau^+=\{\Phi,X,\{a\},\{d\},\{a,d\}\}$ and $\sigma =\{\Phi,Y,\{a\}\}$, B={b}, $\sigma^+=\{\Phi,Y,\{a\},\{b\},\{a,b\}\}$. Define $f:(X, \tau^+) \rightarrow (Y,\sigma^+)$ by f(a)=b, f(b)=c, f(c)=d,f(d)=a. Then the function f is contra Ω_{gb}^+-continuous but not contra b^+-continuous.

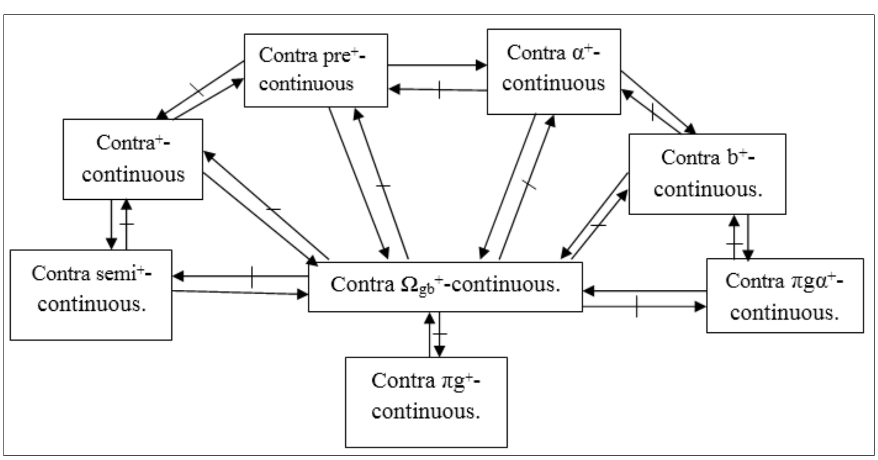
Theorem 3.14: Every contra-πg^+-continuous function is contra- Ω_{gb}^+-continuous but not conversely.
Proof: Let $f:(X, \tau^+)\rightarrow(Y,\sigma^+)$ be contra πg^+-continuous. Let V be any open set in Y. By the property of contra πg^+-continuity we have the inverse image $f^{-1}(V)$ to be b^+-closed in X. But we know that every πg^+-closed set is Ω_{gb}^+-closed. Hence $f^{-1}(V)$ is Ω_{gb}^+-closed in X. Therefore f is contra Ω_{gb}^+-continuous.

Example 3.15: Let $X=\{a,b,c,d\}$, $\tau=\{\Phi,X,\{a\},\{a,d\}\}$, B={d}, $\tau^+=\{\Phi,X,\{a\},\{d\},\{a,d\}\}$, $\sigma =\{\Phi,X,\{a\}\}$, B={b}, $\sigma^+=\{\Phi,X,\{a\},\{b\},\{a,b\}\}$.Then the identity function f: $(X,\tau^+) \rightarrow(X,\sigma^+)$ is contra- Ω_{gb}^+ - continuous but not contra πg^+ continuous.

Theorem 3.16: Every contra-$\pi g\alpha^+$-continuous function is contra- Ω_{gb}^+-continuous but not conversely.
Proof: Let $f:(X, \tau^+)\rightarrow(Y,\sigma^+)$ be contra $\pi g\alpha^+$-continuous. Let V be any open set in Y. By the property of contra $\pi g\alpha^+$-continuity we have the inverse image $f^{-1}(V)$ to be b^+-closed in X. But we know that every $\pi g\alpha^+$-closed set is Ω_{gb}^+-closed. Hence $f^{-1}(V)$ is Ω_{gb}^+-closed in X. Therefore f is contra Ω_{gb}^+-continuous.

Example3.17: Let X = Y = {a,b,c,d} be two topology spaces with topologies $\tau=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,c,d\}\}$,B={b},$\tau^+=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,c,d\}\}$,$\sigma=\{\Phi,X,\{a\},\{a,c\}\}$,B={c}, $\sigma^+=\{\Phi,X,\{a\},\{c\},\{a,c\}\}$.Then the identity function f: $(X,\tau^+) \rightarrow (X,\sigma^+)$ is contra- Ω_{gb}^+ continuous but not contra $\pi g\alpha^+$- continuous.

Remark 3.18: The following diagrammatic representation is the capsulation of the above proved theorems .



Remark 3.19: The following two examples will show that the concept of Ω_{gb}^+-continuous and contra Ω_{gb}^+-continuous are independent from each other.

Example3.20: Let X = Y = {a,b,c,d} be two topology spaces with topologies τ={Φ,X,{a},{c,d},{a,c,d}},B={d},τ^+={Φ,X,{a},{d},{a,d},{c,d},{a,c,d}},σ={Φ,X,{a},{a,d},{a,c,d}},B={d}, σ^+={ Φ,X,{a},{d},{a,c},{a,c,d}}. Then the identity function f: $(X,\tau^+) \to (X,\sigma^+)$ is Ω_{gb}^+-continuous but not contra Ω_{gb}^+- continuous.

Example3.21: Let X = Y = {a,b,c,d} be two topology spaces with topologies τ={Φ,X,{b},{c,d},{b,c,d}},B={c},τ^+={Φ,X,{b},{c},{b,c},{c,d},{b,c,d}},σ={Φ,X,{a},{a,d},{a,c,d},{a,b,d}},B={d}, σ^+={ Φ,X,{a},{d},{a,d},{a,c,d},{a,b,d}}.Then the identity function f: $(X,\tau^+) \to (X,\sigma^+)$ is contra Ω_{gb}^+-continuous but not Ω_{gb}^+-continuous.

Theorem 3.22: If f:$(X, \tau^+) \to (Y,\sigma^+)$ is contra Ω_{gb}^+-continuous map and g:$(Y, \tau^+) \to (Z,\sigma^+)$ is continuous map, then their composition g ∘ f:$(X, \tau^+) \to (Z,\sigma^+)$ is contra Ω_{gb}^+-continuous.
Proof: Let U be any open set in Z. Since g:$(Y, \tau^+) \to (Z,\sigma^+)$ is continuous, $g^{-1}(U)$ is open in Y. Since f:$(X, \tau^+) \to (Y,\sigma^+)$ is contra Ω_{gb}^+-continuous, $f^{-1}(g^{-1}(U))$ is Ω_{gb}^+-closed in X. Hence $(g \circ f)^{-1}(U)$ is Ω_{gb}^+-closed in X. Thus g ∘ f is contra Ω_{gb}^+-continuous.

Theorem 3.23: If f:$(X, \tau^+) \to (Y,\sigma^+)$ is Ω_{gb}^+-irresolute map and g:$(Y, \tau^+) \to (Z,\sigma^+)$ is Ω_{gb}^+-continuous map, then their composition gof:$(X, \tau^+) \to (Z,\sigma^+)$ is contra Ω_{gb}^+-continuous.
Proof: Let U be any open set in Z. Then $g^{-1}(U)$ is Ω_{gb}^+-closed in Y, because g:$(Y, \tau^+) \to (Z,\sigma^+)$ is contr a Ω_{gb}^+-continuous. Since f:$(X, \tau^+) \to (Y,\sigma^+)$ is Ω_{gb}^+-irresolute, $f^{-1}(g^{-1}(U)) =(g \circ f)^{-1}(U)$ is Ω_{gb}^+-closed in X. Thus g ∘ f is contra Ω_{gb}^+-continuous.

Remark 3.24: The composition of two contra Ω_{gb}^+-continuous maps cannot be a contra Ω_{gb}^+-continuous map as seen from the following example.

Example 3.25: Let X={a,b,c,d},τ ={X, ϕ,{a},{a,d},{c,d}},{a,c,d}}, and B={d}, τ^+ ={X,Φ,{a},{d},{a,d},{c,d},{a,c,d}},σ ={ X, ϕ, X,{b},{b,d}} and B={d}, σ^+ ={X,Φ,{b},{d},{b,d}}. η ={ X, ϕ,{c},{a,c},{a,b.d}} and B={d}, η^+ ={X,Φ,{c},{d},{a,c},{c,d},{a,c,d},{a,b,d. Define f:$(X,\tau^+)\to(X,\sigma^+)$ by f(a)=a, f(b)=b, f(c)=c. Define g:$(X,\sigma^+)\to(X,\eta^+)$ by g(a)=a, g(b)=b, g(c)=c. Then f and g are contra Ω_{gb}^+-continuous but g ∘ f is not a contra Ω_{gb}^+-continuous.

Definition 3.26: A space (X,τ^+) is called a Ω_{gb}^+-locally indiscrete if every Ω_{gb}^+-open set in it is closed.

Example 3.27: Let X = {a, b} with the topology τ ={X, ϕ,{a}} and B={b}, τ^+ ={X,Φ,{a},{b}}.Then (X,τ^+) is Ω_{gb}^+-locally indiscrete space.

Theorem 3.28: If a function f:$(X,\tau^+)\to(Y,\sigma^+)$ is Ω_{gb}^+-continuous and (X,τ^+) is Ω_{gb}^+-locally indiscrete then f is contra$^+$-continuous.
Proof: Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is Ω_{gb}^+-open in X as f is Ω_{gb}^+-continuous. Since (X,τ^+) is Ω_{gb}^+-locally indiscrete $f^{-1}(V)$ is closed in X. Hence f is contra$^+$-continuous.

Theorem 3.29: If a function f:$(X,\tau^+)\to(Y,\sigma^+)$ is contra Ω_{gb}^+-continuous and X is the space where "every Ω_{gb}^+-closed set is closed", then f is contra$^+$-continuous.
Proof: Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is Ω_{gb}^+-closed in X as f is contra Ω_{gb}^+-continuous. By hypothesis, $f^{-1}(V)$ is closed in X. Hence f is contra+- continuous.

Theorem 3.30: If a function f:$(X,\tau^+)\to(Y,\sigma^+)$ is Ω_{gb}^+-irresolute map with Y as Ω_{gb}^+-locally indiscrete space and g:$(Y,\sigma^+)\to(Z,\eta^+)$ is contra Ω_{gb}^+-continuous map, then their composition g ∘ f: $(X,\tau^+)\to(Z,\eta^+)$ is contra Ω_{gb}^+-continuous.
Proof: Let U b e any closed set in Z. Since g:$(Y,\sigma^+)\to(Z,\eta^+)$ is contra Ω_{gb}^+-continuous, $g^{-1}(U)$ is Ω_{gb}^+- open in Y. Since Y is Ω_{gb}^+-locally indiscrete, $g^{-1}(U)$ is closed in Y. Hence $g^{-1}(U)$ is Ω_{gb}^+-closed set in Y. Since f:$(X,\tau^+)\to(Y,\sigma^+)$ is Ω_{gb}^+-irresolute,$f^{-1}(g^{-1}(U))= (g \circ f)^{-1}(U)$ is Ω_{gb}^+-closed in X. Thus g ∘ f is contra Ω_{gb}^+-continuous.

4. Almost Contra Ω_{gb}^+-Continuous Function

Definition 4.1: A function f:$(X,\tau^+) \to (Y,\sigma^+)$ is called

1. almost$^+$ continuous (almost contra$^+$-continuous) if $f^{-1}(V)$ is open (closed) in X for each regular$^+$ open set V of Y.
2. almost contra pre$^+$-continuous if $f^{-1}(V)$ is pre$^+$- closed in X for each regular$^+$ open set V of Y.
3. almost contra semi$^+$-continuous if $f^{-1}(V)$ is semi$^+$- closed in X for each regular$^+$ open set V of Y.
4. almost contra α^+-continuous if $f^{-1}(V)$ is α^+-closed in X for each regular$^+$ open set V of Y.
5. almost contra b^+-continuous if $f^{-1}(V)$ is b^+-closed in X for each regular$^+$ open set V of Y.

Definition 4.2: A function f:$(X,\tau^+)\to(Y,\sigma^+)$ is called almost contra Ω_{gb}^+-continuous if $f^{-1}(V)$ is Ω_{gb}^+-closed in X for every regular$^+$ open set V of Y.

Example 4.3: Let X = Y = {a, b, c} be two topology spaces with topologies τ = {Φ,X,{a,c}}, B={c}, τ^+ = {Φ,X,{c},{a,c}} and σ = {Φ,Y,{a}}, B = {a,b}, σ^+ = {Φ,Y,{a} {a,b}}. Define f: (X, τ^+) → (Y,σ^+) by f(a)=a, f(b)=c, f(c)=b. Then the function f is almost contra $\Omega_{gb}{}^+$-continuous.

Theorem4.4

1. Every almost contra$^+$-continuous function is almost contra $\Omega_{gb}{}^+$-continuous function.
2. Every almost contra pre$^+$-continuous function is almost contra $\Omega_{gb}{}^+$-continuous function.
3. Every almost contra semi$^+$-continuous function is almost contra $\Omega_{gb}{}^+$-continuous function.
4. Every almost contra α^+-continuous function is almost contra $\Omega_{gb}{}^+$-continuous function. (v) Every almost contra b$^+$-continuous function is almost contra $\Omega_{gb}{}^+$-continuous function. **Proof:** The proof is obvious.

Remark 4.5: Converse of the above statements is not true as shown in the following example.

Example4.6: (i) Let X = Y = {a, b, c} be the two topology spaces with topologies τ = {Φ,X,{a,b}}, B={b}, τ^+ = {Φ,X,{b},{a,b}} and σ = {Φ,Y,{a},{a,b}}, B = {b}, σ^+ = {Φ,Y,{a},{b},{a,b}}. Define f: (X, τ^+) → (Y,σ^+) by f(a)=b, f(b)=a, f(c)=c. Then the function f is almost contra $\Omega_{gb}{}^+$-continuous but not almost contra$^+$-continuous.
(ii) Let X = Y = {a, b, c} be the two topology spaces with topologies τ = {Φ,X,{a},{a,b}}, B={b},τ^+ = {Φ,X,{a},{b},{a,b}} and σ = {Φ,Y,{a,c}}, B = {b}, σ^+ = {Φ,Y,{b},{a,c}}. Define f: (X, τ^+) → (Y,σ^+) by f(a)=c, f(b)=a, f(c)=b. Then the function f is almost contra $\Omega_{gb}{}^+$-continuous but not almost contra pre$^+$-continuous.
(iii) Let X = Y = {a, b, c} be the two topology spaces with topologies τ = {Φ,X,{a,b}}, B={b}, τ^+ = {Φ,X,{b},{a,b}} and σ = {Φ,Y,{a},{a,b}}, B = {b}, σ^+ = {Φ,Y,{a},{b},{a,b}}. Let f: (X, τ^+) → (Y,σ^+) be the identity function. Then the function f is almost contra $\Omega_{gb}{}^+$-continuous but not almost contra semi$^+$-continuous.
(iv) Let X = Y = {a, b, c, d} be the two topology spaces with topologies τ = {Φ,X,{a},{c,d},{a,c,d}}, B={d},τ^+ = {Φ,X,{a},{d},{a,d},{c,d},{a,c,d}} and σ = {Φ,Y,{d},{a,d}}, B = {a}, σ^+ = {Φ,Y,{a},{d},{a,d},{c,d}}. Define f: (X, τ^+) → (Y,σ^+) by f(a)=a, f(b)=c, f(c)=d,f(d)=b.Then the function f is almost contra $\Omega_{gb}{}^+$-continuous but not almost contra α^+-continuous.
(v) Let X = Y = {a, b, c} be the two topology spaces with topologies τ = {Φ,X,{a,c}}, B={c},τ^+ = {Φ,X,{c},{a,c}} and σ = {Φ,Y,{a},{a,b}}, B = {b}, σ^+ = {Φ,Y,{a},{b},{a,b}}. Define f: (X, τ^+) → (Y,σ^+) by f(a)=a, f(b)=c, f(c)=b.Then the function f is almost contra $\Omega_{gb}{}^+$-continuous but not almost contra b$^+$-continuous.

Theorem 4.7: If a map f: (X, τ^+) → (Y,σ^+) from a topological space X into a topological space Y, then the following statements are equivalent:

1. f is almost contra $\Omega_{gb}{}^+$-continuous.
2. for every regular$^+$ closed set F of Y f^{-1}(F) is $\Omega_{gb}{}^+$- open in X.

Proof: (a) ⇒ (b) : Let F be a regular$^+$ closed set in Y, then Y – F is a regular$^+$ open set in Y. By (a), f^{-1}(Y – F) = X – f^{-1}(F) is $\Omega_{gb}{}^+$-closed set in X. This implies f^{-1}(F) is $\Omega_{gb}{}^+$-open set in X. Therefore (b) holds.
(b) ⇒ (a) : Let G be a regular$^+$ open set of Y. Then Y – G is a regular$^+$ closed set in Y. By (b), f^{-1}(Y – G) is $\Omega_{gb}{}^+$-open set in X. This implies X – f^{-1}(G) is $\Omega_{gb}{}^+$-open set in X, which implies f^{-1}(G) is $\Omega_{gb}{}^+$-closed set in X. Therefore (a) holds.

Definition 4.8: A function f: (X, τ^+) → (Y,σ^+) is said to be regular$^+$ set connected if f^{-1}(V) is clopen in X for every regular$^+$ open set V of Y.

Theorem 4.9: If a function f: (X, τ^+) → (Y,σ^+) is almost contra $\Omega_{gb}{}^+$-continuous and almost$^+$ continuous together with X is the space where "every $\Omega_{gb}{}^+$-closed set is closed", then f is regular$^+$ set connected.
Proof: Let V be a regular$^+$ open set in Y. Since *f* is almost contra $\Omega_{gb}{}^+$-continuous and almost$^+$ continuous, f^{-1}(V) is $\Omega_{gb}{}^+$-closed and open in X. By hypothesis, f^{-1}(V) is clopen in X. Therefore f is regular$^+$ set connected.

References

1. Andrijevic D. On b-open sets, Mat. Vesink. 1996, 48, 59- 64.
2. Arokiarani I, Balachandran K, Janaki C. On contra-πgα-continuous functions, Kochi. J. Math. 2008; 3:201-209.
3. Caldas M, Jafari S, Noiri T, Simoes M, A new generalization of contra-continuity via Levine's g-closed sets," Chaos, Solitons and Fractals. 2007; 32:1597-1603.
4. Dontchev J, Contra-continuous functions and strongly S-closed spaces, Int. J. Math. Sci. 1996; 19(2):303-310.
5. Dontchev J and Noiri T., "On contra semi-continuous functions," Mathematica pannonica. 1999; 10(2):159-168.
6. Dontchev J, Ganster M, Reilly I. More on almost s-continuity, Indian J. Math. 1999; 41:39-146.
7. Ekici E. On contra-πg-continuous, Chaos, Solitons and Fractals. 2008; 35(1):71-81.
8. Jafari S, Noiri T. On contra α-continuous functions between topological spaces, Iran. Int. J. Sci, 2001; 2(2):153-167.
9. Jafari S, Noiri T. On contra pre-continuous functions, Bull. Malays. Math. Sci. Soc. 2002; 25(2):115-128.
10. Levine N. Simple extension of topology, Amer. Math. Monthly. 1964; 71:22-105.
11. Levine N, Generalized closed sets in topology, Rend. Circ. Mat. Palermo. 1970; 19(2)89-96.
12. Levine N, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly. 1963; 70:36- 41.
13. Madhumathi T, Nirmala Irudayam F, On $\Omega_{gb}{}^+$ and $\upsilon_{gb}{}^+$ sets in simple extension Ideal topological spaces"Mathematical Sciences International Research Journal. 2016; 5(2):187-190.

14. Madhumathi T, Nirmala Irudayam F. On Ω_{gb}^{+}-closed sets in simple extension topological spaces,"Elixir Appl. Math. 2017; 105:46305-46307.
15. Metin Akdag, Alkan Ozkan. Some properties of contra gb-continuous functions, Journal of New Results in Science. 2012; 1:40-49.
16. Nirmala Irudayam Fand Arockiarani Sr I. A note on the weaker form of bI set and its generalization in SEITS, International Journal of Computer Application. 2012; 4(2):42-54.
17. Noiri T, Arockiarani Sr I, Nirmala Irudayam F, On Some Topologies Induced By BI^{+} Open Sets in Simple Extension Ideal Topological Spaces", International Journal of Mathematical. 2013; 4(1):30-135.
18. Omari AA, Noorani MSM. On generalized b-closed sets," Bull. Malays. Math. Sci. Soc. 2009; 32(1):19-30.
19. Omari AA, Noorani MSM. Some properties of contra b-continuous and almost contra b-continuous functions," European Journal of Pure and Applied Mathematics. 2009; 2(2):213-230.
20. Reena S and Nirmala Irudayam F., "A new weaker form of πgb-continuity," International Journal of Innovative Research in Science, Engineering and Technology. 2016; 5(5):8676-8682.
21. Singal MK, Singal AR. Almost continuous mappings," Yokohama Math. 1968; 3:63-73.
22. Sreeja D, Janaki C. On πgb- Closed Sets in Topological Spaces, International Journal of Mathematical. 2011; 2(8):1314-1320.
23. Sreeja D, Janaki C. On Contra -πgb -Continuous Functions and Approximately –πgb-Continuous Functions inTopological Spaces", International Journal of Statistika and Mathematika. 2011; 1(2):46-51.
24. Stone MH. Applications of the theory of Boolean rings to general topology, Trans. Am. Math. Soc. 1937; 41:375-81.
25. Thirumalaiswamy M, Saranya S. On some new functions of g*b-continuity in topological spaces, Int. J. of Math. 20123(11):4081-4084.