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sgβ-closed sets in Biclosure spaces

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Abstract
The aim of this paper is to introduce the concept of semi generalized β-closed Sets and semi generalized β-open Sets in BiČech closure spaces and investigate their characterizations.

Keywords: BiČech closure operator, BiČech closure spaces, BiČech sgβ-closed sets, BiČech sgβ-open sets

1. Introduction
Čech spaces were introduced by Eduard Čech [2] (i.e., sets endowed with a grounded, Extensive and additive closure operators) and studied by many others [5, 10]. BiČech closure spaces were introduced by K. Chandrasekhara Rao, R Gowri and V. Swaminathan [3].
N. Levine [11] introduced g-closed sets. D. Andrijevic [1] initiated the study of β-open sets and β-closed sets. In this paper we introduce the concept of BiČech sgβ- closed sets and BiČech sgβ- open sets and discuss some of its properties.

2. Preliminaries
Definition 2.1
Two maps k_1 and k_2 from power set X to itself are called BiČech closure operator on X and the pair (X, k_1,k_2) is called a BiČech closure spaces if the following axioms are satisfied
1. $k_1(\varphi) = \varphi$ & $k_2(\varphi) = \varphi$
2. $A \subseteq k_1(A)$ & $A \subseteq k_2(A)$ for every $A \subseteq X$
3. $k_1(A \cup B) = k_1(A) \cup k_1(B)$ and $k_2(A \cup B) = k_2(A) \cup k_2(B)$ for all $A,B \subseteq X$.

Definition 2.2 [3] A subset A in a BiČech closure space (X, k_1, k_2) is said to be
1. k_i-regular open if $A = int_{k_i}(k_i(A))$, i = 1, 2
2. k_i -regular closed if $A = k_i(int_{k_i}(A))$, i = 1, 2
3. k_i-semi open if $A \subseteq k_i(int_{k_i}(A))$, i = 1, 2
4. k_i-semi closed if $int_{k_i}(k_i(A)) \subseteq A$, i = 1, 2
5. k_i-pre open if $A \subseteq int_{k_i}(k_i(A))$, i = 1, 2
6. k_i -pre closed if $k_i(int_{k_i}(A)) \subseteq A$, i = 1, 2
7. k_i -α open if $A \subseteq int_{k_i}(k_i(int_{k_i}(A)))$, i = 1, 2
8. k_i -α closed if $k_i(int_{k_i}(k_i(A))) \subseteq A$, i = 1, 2
9. k_i -β open if $A \subseteq k_i(int_{k_i}(k_i(A)))$, i = 1, 2
10. k_i -β closed if $int_{k_i}(k_i(int_{k_i}(A))) \subseteq A$, i = 1, 2.

Definition 2.3: A subset A of a BiČech closure space (X, k_1, k_2) is called biclosed if $k_1A = A = k_2A$ and called biopen if its complement is biclosed.

Definition 2.4: A subset A of a BiČech closure space (X, k_1, k_2) is said to be (k_1, k_2)-g biclosed if $k_2(A) \subseteq G$ whenever $A \subseteq G$ and G is k_1 open set in X.

Definition 2.5: A subset A of a BiČech closure space (X, k_1, k_2) is said to be (k_1, k_2)-πgα biclosed if $k_{2\alpha}(A) \subseteq G$ whenever $A \subseteq G$ and G is k_1 π -open set in X.

Definition 2.6: Let (X, k_1, k_2) be a BiČech closure space. A subset $A\subseteq X$ is said to be (k_1, k_2)-w -biclosed set if $k_2(A) \subseteq G$ whenever $A \subseteq G$ and G is k_1 semi-open set in X.

3. (k_1, k_2)-sgβ closed sets

Definition 3.1: Let (X, k_1, k_2) be a BiČech closure space. A subset $A\subseteq X$ is called (k_1, k_2)-sgβ closed set if $k_{2\beta}(A)\subseteq G$ whenever $A\subseteq G$ and G is semi-open subset of (X,k_1) where $k_{2\beta}(A)$ is the smallest β-closed set containing A.

Theorem 3.2: Every biclosed set is (k_1, k_2)-sgβ closed set.

Proof: Let G be a semi open subset of (X, k_1) such that $A \subseteq G$. Since A is biclosed $k_1(A) = k_2(A) = A$. Therefore $k_{2\beta}(A) \subseteq k_{1\text{-}}(A) = k_2(A)=A \subseteq G$, i.e) $k_{2\beta}(A) \subseteq G$ where G is semi open in (X, k_1) which implies A is (k_1,k_2)- sgβ closed set.

Remark 1: The Converse is not true as can be seen from the following example.

Example 3.3: Let X = {a,b,c} and let k_1 and k_2 be defined as:
$k_1\{\varphi\} =\{\varphi\}$, $k_1\{a\}= k_1\{a,c\} = \{a,c\}$, $k_1\{b\}= k_1\{b,c\}=\{b,c\}$, $k_1\{c\}= \{c\}$, $k_1\{a,b\}= k_1\{X\} =X$, $k_2\{\varphi\} =\{\varphi\}$, $k_2\{b\} = \{b\}$, $k_2\{a\} = k_2\{a,b\} = \{a,b\}$, $k_2\{c\}= k_2\{b,c\} = \{b,c\}$, $k_2\{a,c\} = k_2\{X\} =X$. Now (X, k_1,k_2) is biČech closure space.
(k_1, k_2)-sgβ closed set={X, φ, {a}, {b}, {c}, {a,b}, {b,c}}
Biclosed set={X, φ, {b,c}}
Here A={a, b} is (k_1,k_2)- sgβ closed set but not in Biclosed set.

Theorem 3.4: Every g-biclosed set is (k_1, k_2)- sgβ closed set.

Proof: Let G be a semi open subset of (X, k_1) such that $A\subseteq G$. Since A is g-biclosed set $k_2(A) \subseteq G$. Therefore $k_{2\beta}(A) \subseteq k_2(A) \subseteq G$, i.e., $k_{2\beta}(A) \subseteq G$, where G is semi open subset of (X,k_1). Therefore A is (k_1, k_2)- sgβ closed set.

Remark 2: Converse of the above theorem need not be true which can be seen from the following example

Example 3.5: Let X={a, b, c} and let k_1 and k_2 be defined as:
$k_1\{\varphi\} =\{\varphi\}$, $k_1\{a\}= \{a\}$, $k_1\{b\} = k_1\{a, b\}=\{a, b\}$, $k_1\{c\}= k_1\{a, c\}= \{a, c\}$, $k_1\{b, c\}= k_1\{X\} = X$, $k_2\{\varphi\} =\{\varphi\}$, $k_2\{a\}= \{a\}$, $k_2\{b\}=k_2\{c\} = k_2\{b, c\} = \{b, c\}$, $k_2\{a, c\} = k_2\{a, b\}= k_2\{X\} =X$. (X, k_1, k_2) is bi Čech closure space.
(k_1, k_2)- sgβ closed set={X, φ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
g-biclosed set={X, φ, {a}, {a, b}, {b, c} {a, c}}
Here {c} is (k_1,k_2)- sgβ closed set but not in g-biclosed.

Theorem 3.6: Every πgα-biclosed set is (k_1, k_2)- sgβ closed set.

Proof: Let G be a semi open subset of (X, k_1) such that $A\subseteq G$. Since A is πgα--biclosed set $k_{2\alpha}(A) \subseteq G$. Therefore $k_{2\beta}(A) \subseteq k_{2\alpha}(A) \subseteq G$, i.e., $k_{2\beta}(A) \subseteq G$, where G is semi open set in (X,k_1). Therefore A is (k_1, k_2)- sgβ closed set.

Remark 3: Converse of the above theorem need not be true which can be seen from the following example

Example 3.7: Let X={a, b, c} and let k_1 and k_2 be defined as:
$k_1\{\varphi\} =\{\varphi\}$, $k_1\{a\}= \{a\}$, $k_1\{b\} = k_1\{a, b\}=\{a, b\}$, $k_1\{c\}= k_1\{a, c\}= \{a, c\}$, $k_1\{b, c\}= k_1\{X\}= X$, $k_2\{\varphi\}=\{\varphi\}$, $k_2\{a\} = \{a\}$, $k_2\{b\}=k_2\{c\} = k_2\{b, c\} = \{b, c\}$, $k_2\{a, c\} = k_2\{a, b\}= k_2\{X\} =X$. Now (X, k_1, k_2) is bi Čech closure space.
(k_1, k_2)- sgβ closed set={X, φ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}.
πgα-biclosed set={ X, φ, {a}, {a, b}, {b, c}, {a, c}}
Here {c} is (k_1, k_2)- sgβ closed set but not in πgα-biclosed set.

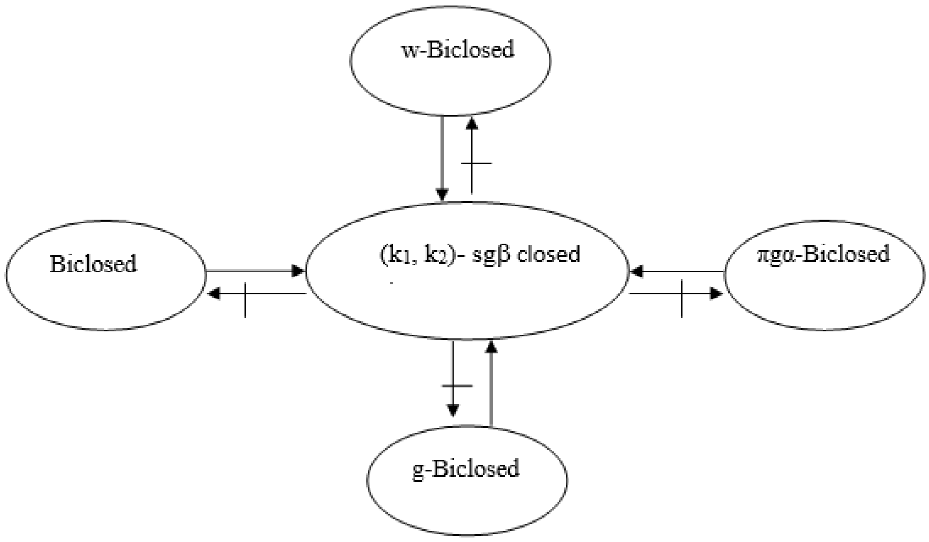
Theorem 3.8: Every w-biclosed set is (k_1, k_2)- sgβ closed set.

Proof: Let G be a semi open subset of (X, k_1) such that $A\subseteq G$. Since A is w-biclosed set $k_2(A) \subseteq G$. Therefore $k_{2\beta}(A) \subseteq k_2(A) \subseteq G$, i.e., $k_{2\beta}(A) \subseteq G$, where G is semi open subset of (X,k_1). Therefore A is (k_1,k_2)- sgβ closed set.
Remark 4: Converse of the above theorem need not be true which can be seen from the following example

Example 3.9: Let X={a, b, c} and let k_1 and k_2 be defined as:

$k_1\{\varphi\} = \{\varphi\}$, $k_1\{a\}=\{a\}$, $k_1\{b\}= k_1\{a, b\}=\{a, b\}$, $k_1\{c\}= k_1\{a, c\}= \{a, c\}$, $k_1\{b, c\}= k_1\{X\} = X$, $k_2\{\varphi\} =\{\varphi\}$, $k_2\{b\}=\{b, c\}$, $k_2\{c\} = k_2\{a, c\}=\{a, c\}$, $k_2\{a, b\} = k_2\{b,c\}= k_2\{X\} =X$.
(X, k_1,k_2) is biČech closure space.
(k_1,k_2)- sgβ closed set=$\{X, \varphi, \{a\},\{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$
w-biclosed set=$\{X, \varphi, \{b, c\}, \{a, c\}\}$
Here $\{a\}$ is (k_1, k_2)- sgβ closed set but not in w-biclosed.

From the above results we have the following implications.



Proposition 3.10: Let (X, k_1,k_2) be a bi Čech closure space. If A and B are (k_1,k_2)- sgβ closed subsets of (X, k_1,k_2), then $A \cup B$ is (k_1,k_2)- sgβ closed.

Proof: Let G be an semi open subset of (X,k_1) such that $A \cup B \subseteq G$. Then $A \subseteq G$ and $B \subseteq G$. Since A and B are (k_1, k_2)- sgβ closed, $k_{2\beta}(A) \subseteq G$ and $k_{2\beta}(B) \subseteq G$, Consequently $k_{2\beta}(A \cup B) = k_{2\beta}(A) \cup k_{2\beta}(B) \subseteq G$. Hence $(A \cup B)$ is (k_1, k_2)- sgβ closed.

Proposition 3.11: Let (X, k_1,k_2) be a biČech closure space and if A is (k_1,k_2)- sgβ closed subset of (X, k_1,k_2) then $k_{2\beta}(A) – A$ contains no nonempty k_1 semi closed set.

Proof: Let F be a nonempty k_1 semi closed set such that $F \subseteq k_{2\beta}(A) –A$. Now $F\subseteq k_{2\beta}(A)$ and $F \subseteq A^c$, $A \subseteq F^c$. Since A is (k_1,k_2)- sgβ closed, F^c is k_1 semi open, $k_{2\beta}(A) \subseteq F^c$. Consequently, $F \subseteq [k_{2\beta}(A)]^c$ and $F \subseteq k_{2\beta}(A) \cap [k_{2\beta}(A)]^c = \varphi$. Hence $F = \varphi$.

Proposition 3.12: Let A be (k_1, k_2)-sgβ closed set and if A is k_1 semi open then $A = k_{2\beta}(A)$.

Proof: Let A be (k_1,k_2)- sgβ closed subset of a biČech closure space (X, k_1,k_2) and let A be k_1 semi open. Then $k_{2\beta}(A) \subseteq G$ whenever $A \subseteq G$ and G is k_1 semi open in X. Since A is k_1 semi open and $A \subseteq A$, we have $k_{2\beta}(A) \subseteq A$. But $A \subseteq k_{2\beta}(A)$.Thus $A = k_{2\beta}(A)$.

Proposition 3.13: Let (X, k_1, k_2) be a biČech closure space. If A is (k_1,k_2)- sgβ closed and F is semi biclosed in (X, k_1,k_2), then $A \cap F$ is (k_1,k_2)- sgβ closed.

Proof: Let G be an semi open set of (X, k_1) such that $A \cap F \subseteq G$.Then $A \subseteq G \cup (X – F)$. Since A is (k_1, k_2)- sgβ closed and $G \cup (X – F)$ is an semi open subset of (X, k_1), $k_{2\beta}(A) \subseteq G \cup (X – F)$. Since F is semi biclosed subset of (X, k_2), $k_{2\beta}(A \cap F) \subseteq G$. Consequently $k_{2\beta}(A \cap F) \subseteq G$. Thus $A \cap F$ is (k_1, k_2)- sgβ closed.

Proposition 3.14: Let (X, k_1, k_2) be a biČech closure space. If A is (k_1, k_2)- sgβ closed set and B is any set $A\subseteq B\subseteq k_{2\beta}(A)$, then B is (k_1,k_2)- sgβ closed set.

Proof: Let G be an semi open subset of (X, k_1) such that $B \subseteq G$. Then $A \subseteq B$ which implies $A \subseteq G$. Since A is (k_1, k_2)- sgβ closed and $A \subseteq G$ then $k_{2\beta}(A) \subseteq G$. Hence B is (k_1, k_2)- sgβ closed set.

Proposition 3.15: Let (X, k_1,k_2) be a biČech closure space and if A is (k_1,k_2)- sgβ closed subset of (X, k_1,k_2), then $k_{1\beta}(x) \cap A \neq \varphi$ holds for each $x \in k_{2\beta}(A)$.

Proof: Let A be (k_1, k_2)-sgβ closed set. Suppose $k_{1\beta}(x) \cap A = \varphi$ for some $x \in k_{2\beta}(A)$. we have $A \subseteq [k_{1\beta}(x)]^c$. Now $k_{1\beta}(x)$ is k_1 semi closed. Therefore $[k_{1\beta}(x)]^c$ is k_1 semi open. Since A is (k_1, k_2)- sgβ closed set, $k_{2\beta}(A) \subseteq [k_{1\beta}(x)]^c$ which implies $k_{2\beta}(A) \cap k_{1\beta}(x) = \varphi$. Thus x does not belong to $k_{2\beta}(A)$ which is a contradiction. Hence $k_{1\beta}(x) \cap A \neq \varphi$ holds for each $x \in k_{2\beta}(A)$.

Proposition 3.16: Let (X, k_1,k_2) be a biČech closure space, for each x in X, $\{x\}$ is k_1 semi closed or $\{x\}^c$ is (k_1,k_2)- sgβ closed.

Proof: Let (X, k_1,k_2) be a biČech closure space. Suppose that $\{x\}$ is not k_1 semi closed set, $\{x\}^c$ is not k_1 semi open. Since the only k_1 semi open set containing $\{x\}^c$ is X. Thus $\{x\}^c \subset X$. Now $k_{2\beta}\{x\}^c \subseteq k_{2\beta}(X) = X$. Hence $\{x\}^c$ is (k_1,k_2)- sgβ closed.

Proposition 3.17: Let $A \subseteq Y \subseteq X$ and suppose that A is (k_1, k_2)- sgβ closed (X, k_1,k_2).Then A is (k_1,k_2)- sgβ closed relative to Y.

Proof: Let S be any k_1 semi open in Y such that $A \subseteq S$. Then $S = G \cap Y$ for some G which is k_1 semi open in X. Therefore $A \subset G \cap Y$ implies $A \subseteq G$. Since A is (k_1, k_2)- sgβ closed set in X, We have $k_{2\beta}(A) \subseteq G$. Hence $Y \cap k_{2\beta}(A) \subseteq Y \cap G = S$. Thus A is (k_1,k_2)- sgβ closed relative to Y.

4. (k_1, k_2)-sgβ open sets

Definition 4.1: A Subset A in biČech closure space (X, k_1,k_2) is called (k_1,k_2)- sgβ open set if A^c is (k_1,k_2)- sgβ closed set in (X, k_1,k_2).

Proposition 4.2: Let (X, k_1, k_2) be a biČech closure space. Then A is (k_1,k_2)- sgβ open subset of (X, k_1,k_2) if and only if $F \subseteq X - k_{2\beta}(X - A)$ for every F is semi closed subset of (X, k_1) with $F \subseteq A$.

Proof: Assume that A is (k_1,k_2)- sgβ open subset of (X, k_1,k_2) and let F be a semi closed subset of (X, k_1) such that $F \subseteq A$. Then $X - A \subseteq X - F$. Since $X - A$ is (k_1, k_2)- sgβ closed and $X - F$ is semi open subset of (X, k_1). It follows that $k_{2\beta}(X - A) \subseteq X - F$. Therefore, $F \subseteq X - k_{2\beta}(X - A)$. Conversely, Let G be an semi open subset of (X, k_1) such that $X - A \subseteq G$.Then $X - G \subseteq A$. Since $X - G$ is a semi closed subset of (X, k_1), $X - G \subseteq X - k_{2\beta}(X - A)$. Consequently $k_{2\beta}(X - A) \subseteq G$. Hence $X - A$ is (k_1, k_2)- sgβ closed and so A is (k_1,k_2)- sgβ open.

Proposition 4.3: Let (X, k_1, k_2) be a biČech closure space. If A is a(k_1,k_2)- sgβ open subset of (X,k_1,k_2), then $G = X$ whenever G is an semi open subset of (X, k_1) and $(X - k_{2\beta}(X-A)) \cup (X - A) \subseteq G$.

Proof: Suppose that A is (k_1, k_2)- sgβ open subset of (X, k_1,k_2). Let G be an semi open subset of (X, k_1) and $(X - k_{2\beta}(X-A)) \cup (X - A) \subseteq G$. Then, $X - G \subseteq X -(X - k_{2\beta}(X-A)) \cup (X- A)$ implies $X - G \subseteq k_{2\beta}(X-A) \cap A$ or $X - G \subseteq k_{2\beta}(X-A) \cap (X-A)$. But $X - G$ is semi closed subset of (X, k_1) and $X - A$ is (k_1, k_2)- sgβ closed. By proposition 3.11, $X - G = \varphi$. Hence $X = G$.

Proposition 4.4: A subset A of (X, k_1, k_2) is called (k_1,k_2)- sgβ open set if and only if $F \subset (\text{int}_{k_{2\beta}}(A))$whenever F is k_1 – semi closed and $F \subseteq A$.

Proof: Suppose A is (k_1, k_2)- sgβ open in (X, k_1,k_2).Let F be k_1 semi closed and $F \subseteq A$. Then F^C is k_1 semi open and $A^C \subseteq F^C$. Since A^C is (k_1,k_2)- sgβ closed set, we have $k_{2\beta}(A^C) \subseteq F^C \Rightarrow F \subseteq k_{2\beta}[(A^C)]^c = (\text{int}_{k_{2\beta}}(A))$. That is $F \subseteq (\text{int}_{k_{2\beta}}(A))$ whenever F is k_1 semi closed and $F \subseteq A$. Conversely, let V be any k_1 semi open set in X such that $A^C \subseteq V$. Thus $V^C \subseteq A$ and V^C is k_1 semi closed set. Since $V^C \subseteq (\text{int}_{k_{2\beta}}(A))$. Then $(\text{int}_{k_{2\beta}}(A))^C \subseteq V \Rightarrow k_{2\beta}(A^C) \subseteq V$ gives that A^C is (k_1,k_2)- sgβ closed set and A is (k_1,k_2)- sgβ open set.

Proposition 4.5: A subset A of (X,k_1,k_2) is (k_1,k_2)- sgβ closed set, then $k_{2\beta}(A) - A$ is (k_1,k_2) - sgβ open set.

Proof: Let F be a k_1 semi closed set such that $F \subseteq k_{2\beta}(A) - A$. But $F = \varphi$ by proposition 3.11. Therefore, $F \subseteq \text{int}_{k_{2\beta}}\{k_{2\beta}(A)- A\}$. Hence $\{k_{2\beta}(A)- A\}$ is (k_1,k_2)- sgβ open set.

Proposition 4.6: If A and B be (k_1, k_2)- sgβ open sets, then so is $A \cap B$.

Proof: Let $(A^C \cup B^C) \subseteq G$, where G is k_1 semi open set.$A^C \subseteq G$ and $B^C \subseteq G$ gives $k_{2\beta}(A^C) \subseteq G$ and $k_{2\beta}(B^C) \subseteq G$.Thus $k_{2\beta}(A^C) \cup k_{2\beta}(B^C) \subseteq G$. Thus $k_{2\beta}(A^C \cup B^C) \subseteq G$, $(A \cap B)^c$ is (k_1,k_2)- sgβ closed. Therefore, $A \cap B$ is (k_1,k_2)- sgβ open set.

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