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## A study on strongly g-closed sets and strongly $g^{**}$ - closed sets in topological spaces

**P Ponmalar**

### Abstract

In this dissertation, we study the notions of g-closed sets and  $g^*$ -closed sets and strongly  $g^*$ -closed sets. Also, we study strongly g-closed sets and  $sg^{**}$ -closed sets in topological spaces and also some examples. And every closed set is strongly  $g^*$ -closed set. The above theorem is need not be true we explain some examples.

**Keywords:** g-closed sets,  $g^*$ -closed sets, strongly g-closed sets, strongly  $g^{**}$ - closed sets

### Introduction

The word topology derived from two area words, topo's meaning discovers or study topology thus literally means the study of surface. modern topology depends strongly on the ideas of theory, developed by GEORG CANTOR in the later part of the 19<sup>th</sup> century during the period up to 1960's, researchs in the field of general topology flourished and settled many important. Since the 1960's researches in general topology has moved into several new areas that invoice intricate mathematical tools, including set theoretic methods. In the late 1960's research worked to generalized some of the topological properties of infinite dimensional Hilbert space.

Maurice freched (1878-1973) was the first to expand topological consider beyond Euclidean space. He introduced metric space in 1960 in a context the permitted one to consider abstract objects and not just real numbers on n tuples of real numbers topology emerged as a concerned discipline in 1914 when Felix hausdroff (1868-1942) published his classical treatise grunzunge

Hausdroaff defined topological space in term of neighborhood of members of a set. These concepts where introduce immediately after georg cantor(1845-1918) had developed a general theory of sets in the general theory of sets in the general theory of sets in the 1880's but even before cantor, Bernard Riemann (1826-1866) had fore seen study of abstract space.

## 2. Preliminaries

### Basic Definitions 2.1

#### Definition 2.1.1

A topology on a set X is collection  $\tau$  of a subsets of X having the following properties

1.  $\emptyset$  and X are in  $\tau$ .
2. The union of elements of any collection of  $\tau$  is in  $\tau$ .
3. The intersection of the elements of any finite sub collection of  $\tau$  is in  $\tau$

The elements of  $\tau$  are known as open set and the elements of  $\tau^c$  are known as closed set.

#### Definition 2.1.2

A set X to gether with a topology  $\tau$  defined on it is called a topological space. And it is denoted by  $(X, \tau)$

#### Definition 2.1.3

Let X be any set. The collection of all subsets of X is a topology on X it is called a Discrete Topology.

**Definition 2.1.4**

The collection consisting of  $\emptyset$  and  $X$ . only is also a topology on  $X$  it is called a Indiscrete Topology.

**Definition 2.1.5**

A subset of  $A$  of a topological space  $X$  is said to be open if the set  $X-A$  is closed.

**Definition 2.1.6**

Let  $X$  be a topological space and let  $A \subset X$ . then, the interior of  $A$  [denoted by  $A^0$  (or)  $\text{int}(A)$ ] is defined as the union of all open sets contained in  $A$ .

**Definition 2.1.7**

Let  $X$  be a topological space and let  $A \subset X$ .then, the clusur of  $A$  [denoted by  $A(\text{OR})\text{cl}(A)$ ] is defined as the intersection of all closed set containing  $A$ .

**Definition 2.1.8**

A subset  $A$  of  $X$  is said to be semi open if  $A \subseteq \text{cl}(\text{int}(A))$ .

**Definition 2.1.9**

A subset of  $A$  of  $X$  is said to be semi closed if  $\text{int}(\text{cl}(A)) \subseteq A$

**3. g-closed and  $g^*$  - closed sets in topological spaces.**

In this chapter, we study the concept of  $g$ -closed and  $g^*$  - closed sets in topological spaces

**Generalized Closed Sets 3.1**

**Definition 3.1.1**

A subset of a topological space  $(X, \tau)$  is called generalized closed set (brifly  $g$ -closed) if  $\text{cl}(A) \subseteq U$  Whenever  $A \subseteq U$  and  $U$  is open in  $X$ .

**Definition 3.1.2**

A subset  $A$  of a topological space  $(X, \tau)$  is called

1. A semi- Generalized closed set (briefly  $sg$ -closed) if  $\text{scl}(A) \subseteq U$ , Whenever  $A \subseteq U$  and  $U$  is semi open in  $(X, \tau)$ .
2. A Generalized semi-closed set (briefly  $gs$ -closed) if  $\text{scl}(A) \subseteq U$ , Whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
3. A Generalized  $\alpha$ -closed set (briefly  $g\alpha$ -closed) if  $\alpha\text{cl}(A) \subseteq U$ , Whenever  $A \subseteq U$  and  $U$  is  $\alpha$  - open in  $(X, \tau)$ .
4. A  $\alpha$  - Generalized closed set (briefly  $ag$ -closed) if  $\alpha\text{cl}(A) \subseteq U$ , Whenever  $A \subseteq U$  and  $U$  is  $\alpha$  - open in  $(X, \tau)$ .
5. A  $\alpha^{**}$  - Generalized closed set (briefly  $\alpha^{**}g$ -closed) if  $\alpha\text{cl}(A) \subseteq \text{int}(\text{cl}(U))$ , Whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
6. A  $g\alpha^*$  - closed set if  $\alpha\text{cl}(A) \subseteq \text{int}(U)$ , Whenever  $A \subseteq U$  and  $U$  is  $\alpha$  - open in  $(X, \tau)$ .
7. A Generalized semi-pre closed set (briefly  $gsp$ -closed) if  $\text{sacl}(A) \subseteq U$ , Whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
8. A regular generalized closed set (briefly  $r$ - $g$ -closed) if  $\text{cl}(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is regular open in  $(X, \tau)$
9. A Generalized pre closed set (briefly  $gp$ -closed) if  $\text{pcl}(A) \subseteq U$ , Whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
10. A Generalized pre *regular* closed set (briefly  $gpr$ -closed) if  $\text{pcl}(A) \subseteq U$ , Whenever  $A \subseteq U$  and  $U$  is Regular- open in  $(X, \tau)$ .

**4. Basic Propertices of  $G^*$ -Closed Sets**

**Definition 4.1**

A subset  $A$  of  $(X, \tau)$  is called  $g^*$ -closed set if  $\text{cl}(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $(X, \tau)$

**Theorem 4.1.1: Every closed set is a  $g^*$ -closed set.**

The following examples supports that  $\alpha$   $g^*$ -closed set need not be closed in general.

**Example 4.1.2**

Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$ . here let  $A = \{a, b\}$   $A$  is  $\alpha$   $g^*$ -closed set but not a closed set of  $(X, \tau)$ .

**Theorem 4.1.3**

If  $A$  is  $\alpha$   $g^*$ -closed set of  $(X, \tau)$  such that  $A \subseteq B \subseteq \text{cl}(A)$ , then  $B$  is also  $\alpha$   $g^*$ -closed set in  $(X, \tau)$ .

**Proof**

Let  $U$  be  $\alpha$   $g$ -open set of  $(X, \tau)$  such that  $B \subseteq U$ . then  $A \subseteq U$ . since  $A$  is  $g^*$ -closed, then  $\text{cl}(A) \subseteq U$ . now

$\text{Cl}(B) \subseteq \text{cl}(\text{cl}(A)) = \text{cl}(A) \subseteq U$ . therefore  $B$  is also  $\alpha$   $g^*$ -closed set of  $(X, \tau)$

**Strongly  $G^*$ -Closed Sets in Topological Spaces 4.2**

In this chapter, we study the concept of strongly  $g^*$ -closed sets in topological spaces.

**Definition 4.2.1**

Let  $(X, \tau)$  be a topological space and  $A$  be its subset, then  $A$  is strongly  $g^*$ -closed set if  $\text{cl}(\text{int}(A)) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $g$ -open.

**Theorem 4.2.2**

Every closed set is strongly  $g^*$ -closed set.

**Proof**

The proof is immediate from the definition of closed set.

**Remark 4.2.3**

The converse of the above theorem is need not be true.

**Example 4.2.4**

Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$ . and  $A = \{a, b\}$ . Then clearly  $A$  is strongly  $g^*$ -closed set of  $(X, \tau)$

**Strongly  $G$ -Closed Sets and Strongly  $G^{**}$ -Closed Sets 4.3:**

In this chapter, we study the concept of strongly  $g$ -closed and strongly  $g^{**}$ -closed sets.

**Definition 4.3.1:**

A subset  $A$  of a space  $(X, \tau)$  is called a regular generalized closed (brifly  $rg$ -closed) set if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $(X, \tau)$

**Example 4.3.2:**

Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$ . then regular generalized closed sets  $\{\emptyset, \{a\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$ .

**Definition 4.3.3:**

A subset  $A$  of a space  $(X, \tau)$  is called a generalized  $g$  star closed (brifly  $g^*$ -closed) set if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$  open in  $(X, \tau)$

**Example 4.3.4**

Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$ . then generalized g star closed sets  $\{\emptyset, \{b\}, \{a, b\}, \{b, c\}, X\}$ .

**Definition 4.3.5**

A subset A of a space  $(X, \tau)$  is called a generalized g star star closed (brifly  $g^{**}$ -closed) set if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  U is  $g^*$  open in  $(X, \tau)$

**Example 4.3.6**

Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$ . Then generalized g star star closed sets  $\{\emptyset, \{b\}, \{a, b\}, \{b, c\}, X\}$ .

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