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A study on non-newtonian mathematical modeling of two phase human pulmonary blood flow in venules during lung cancer

Dheerendra Kumar, V Upadhyay, Surya Kant Chaturvedi and PN Pandey

Abstract

The main aim of present study is to examine a Non-Newtonian mathematical model of two phased blood flow in human pulmonary Venules, keeping in view the nature of pulmonary blood circulation. Herschel Bulkley Non-Newtonian model in Bio- fluid mechanical setup is applied with respect to the help of clinical data in case of Lung Cancer for hemoglobin versus blood pressure. In present study overall presentation is in tensorial form and the solution solution technique adopted is analytical as well as numerical. The role of hematocrit is explicit in the determination of blood pressure drop in particular circulation during pulmonary disease Lung Cancer.

Keywords: Pulmonary Blood Flow, Herschel Bulkley, Non-Newtonian model etc.

Introduction

A. Structure and Function of the Lungs-

The human lungs are paired organs in the chest and divided in two lobes. Where the right lung has three lobes, the left one has two lobes. It is smaller to accommodate to the heart, which takes up the space of the third lobe in the chest cavity. Air enters the lungs via the bronchial tree, a series of smaller branches off the windpipe, the left bronchus branches into the right lung. These bronchi branch into bronchioles, which terminate at the alveolar sacs. The alveolar sacs contain the s thin- walled air pouches known as alveoli. The alveoli are the smallest units of the lung tissue. The lungs behave just like purification station for blood [Trivedi *et al.*, 2013] [15]. The main function of the lungs is (rapid) gas exchange. This is accomplished by a well-coordinated interaction of the lungs with the central nervous system, the diaphragm and chest wall muscul use, and the circulatory system. Their principal function is to transport oxygen from the atmosphere into the blood stream, and to release carbon-die-oxide from the blood stream into the atmosphere. This exchange of gases is accomplished in the mosaic of specialized cells that form millions of tiny, exceptionally thin-walled air sacs called alveoli [Tripathi *et al.*, 2016] [13].

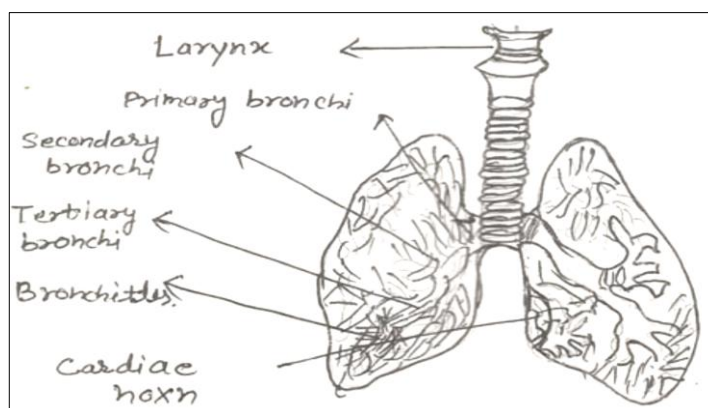


Fig 1

B. Structure and function of the pulmonary Venules-

The function of the arteries is to transport blood under high pressure to the tissues. For this reason, the arteries have strong vascular walls, and blood flows at a high velocity in the arteries. The arterioles are the last small branches of the arterial system; they act as control conduits through which blood is released into the capillaries. The arteriole has a strong muscular wall that can close the arteriole completely or can, by relaxing, dilate it several fold, thus having the capability of vastly altering blood flow in each tissue bed in response to the need of the tissue. The function of the capillaries is to exchange fluid, nutrients, electrolytes, hormones, and other substances between the blood and the interstitial fluid. To serve this purpose, the capillary walls are very thin and have numerous minute capillary pores permeable to water and other small molecular substances. The Venules collect blood from the capillaries, and they gradually coalesce into progressively larger veins. The veins function as conduits for transport of blood from the venules back to the heart; equally important, they serve as a major reservoir of extra blood. Because the pressure in the venous system is very low, the venous walls are thin. Even so, they are muscular enough to contract or expand and there by act as a controllable reservoir for the extra blood, either a small or a large amount, depending on the needs of the circulation [Guyton and Hall, 2006] ^[5].

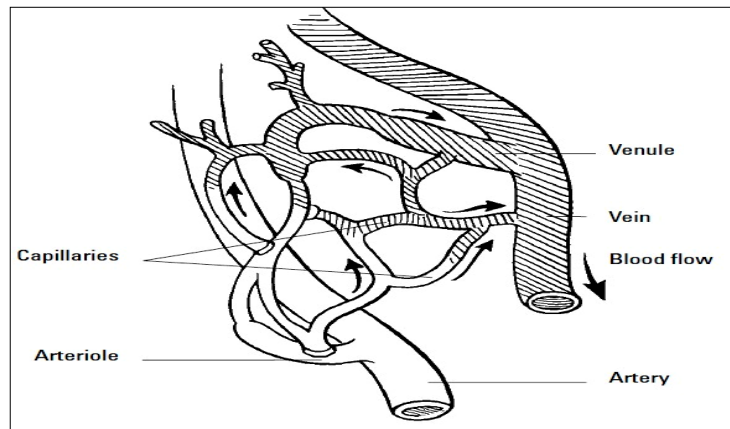


Fig 2

C. Constitution of blood

Blood is the "the river of life" that surges within us. It transports everything that must be carried from one place to another within the body; nutrients, wastes, and body heat, through blood vessels [Dheerendra *et al.*, 2016] ^[2]. Normal erythrocytes are biconcave discs with a mean diameter of 6 to 8 μm and a maximal thickness of 1.9 μm . The average volume of an erythrocyte is 90 μm^3 . Their number per cubic millimeter of blood is approximately 5 to 6 $\times 10^6$ and they represent approximately 40 to 45% by volume of the normal human blood and more than 99% of all blood cells. The first percentage is called hematocrit. Thrombocytes are a vital component of the blood clotting mechanism. The total volume concentration of leukocytes and thrombocytes is only about 1% [Bessonov *et al.*, 2016] ^[9].

D. Description of lung cancer

According to Tina M. St. John, MD, Cancer is a condition caused by the uncontrolled growth of cells. Lung cancer develops when normal lung cells sustain genetic damage that eventually leads to uncontrolled cell proliferation. Lung cancer is sometimes referred to as bronchiogenic cancer or bronchiogenic carcinoma. Most lung cancers begin in the cells lining the bronchi of the lungs. [Tina M. St. John, MD, 2003] ^[12].

Lung cancer is the leading cause of cancer deaths not only in the United States, but worldwide. The World Health Organization (WHO) reports that over 1.1 million people die of lung cancer each year. This number increases every year. As a result, WHO has identified lung cancer as one of the major problems facing the world in this new century [Tina M. St. John, MD, 2003] ^[12].

Lung cancer is a leading cause of cancer-related mortality throughout the world. Although the majority of lung cancer is attributed to tobacco smoke, approximately 25% of lung cancers worldwide occur in lifelong never smokers [Young Joo Lee *et al.*, 2011] ^[18].

Lung cancer develops when the cells that line the lungs sustain genetic damage. Scientists have identified several different chemicals and environmental factors that are capable of causing the kind of genetic damage that can lead to lung cancer. Substances capable of producing cancerous changes in cells are called carcinogens [Tina M. St. John, MD, 2003] ^[12].

The majority of lung cancers occur in people who are either current or former smokers. While the relationship between smoking and lung cancer is well-established, other factors also come into light. We know this because only about one out of every ten smokers develops lung cancer. Further, approximately one out of every six people who develops lung cancer never smoked [Tina M. St. John, MD, 2003] ^[12].

Lung cancer arises from abnormal epithelial cells in the airways of the lungs. Epithelial cells form the covering over free surfaces in the body such as the airways. Lung cancer is divided into two main types based on how it looks under the microscope: small cell lung cancer (SCLC) and non-small cell lung cancer (NSCLC). In the United States, approximately 80% of lung cancers are NSCLC and 20% are SCLC. SCLC and NSCLC have different patterns of growth and spread [Tina M. St. John, MD, 2003] ^[12].

Real Model

A. Choice of frame of reference

A frame of reference was selected for mathematical modeling of two phase blood flow of the state of a moving blood. It was observed in view the difficulty and generality of the problem of blood flow and selected generalized three-dimensional orthogonal curvilinear co-ordinate system, briefly prescribed as E3, called as 3-dim Euclidean space. It was interpreted the quantities related to blood flow in tensorial form which was comparatively more realistic, the biophysical laws thus expressed fully hold good in any co-ordinate system, which was compulsion for the truthfulness of the law. Now, let the co-ordinate axes be OX^i , O denotes origin and superscript $i=1, 2, 3$ let X^i be the co-ordinates of any point P in space. The mathematical description of the state if a moving blood was affected by means of functions which give the distribution of the blood velocity $v^k = v^k(X^i, t)$, $k=1, 2, 3$ and of any two thermodynamic quantities pertaining to the blood, for instance the pressure $p = p(X^i, t)$ and the density $\rho = \rho(X^i, t)$. As was well known, all the thermodynamic quantities are determined by the values of any two of them, together with the equation of state. Hence, if we are given five quantities, namely the three components of velocity v^k , the pressure p and the density ρ , the state of moving blood was completely determined. All these quantities are functions of the co-ordinates X^i , $i=1, 2, 3$ and of the time t . It emphasized that $v^k(X^i, t)$ was the velocity of the blood at a given point X^i in space and at a given t , *i.e.* it refers to fixed points in space and not to fixed particles of the blood; in the course of time, the latter move about in space. The same remarks apply to p and ρ Blood was a mixed fluid. [Upadhyay, 2000] ^[17].

B. Choice of parameters and there uses symbols-

We have considered two phase blood flow in human body. In this present study we have applied Herschel - Bulkley Non Newtonian model transformed in to biofluid mechanical set up, there are five parameters used but three parameter components of velocity are frequently used namely velocity, pressure P and density ρ and uses many symbols:

X – Position of blood cells in unit volume

η_c – Viscosity coefficient of blood cells

η_p – Viscosity coefficient of blood plasma

η_m – Viscosity coefficient of mixture of two phases

ρ_p – Density of blood plasma

ρ_c – Density of blood

ρ_m – Density of mixture of two phases

ρ_m – Density of mixture of two phases

C. Choice of constitutive equation-

Generally blood is non-homogeneous mixture of plasma and blood cells. Thought for practically purpose it may be considered to be homogeneous two phase mixture of plasma and blood cells. The constitutive equations proposed for whole blood mixture. Blood is the Non-Newtonian fluids, then using this constitutive equation for fluids.

$$\tau = \eta e^n$$

(Where, $n \neq 1$)

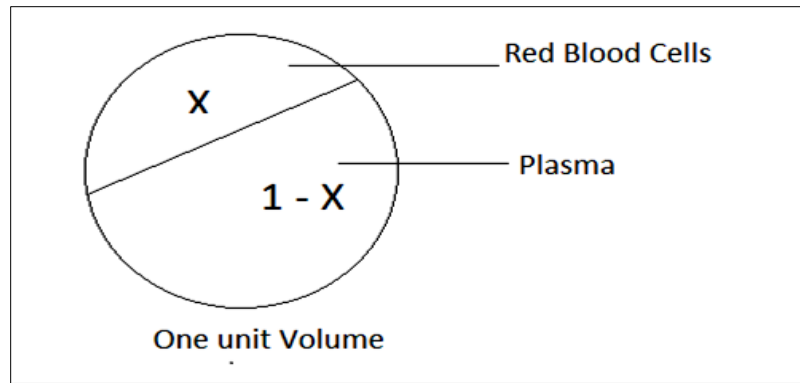
If $n = 1$ then the nature of fluid is Newtonian and if $n \neq 1$ then the nature of fluid is Non-Newtonian fluids. Where, τ is denoted by stress, e is denoted by strain rate and n is denoted by the parameter, these equation uses equation of motion. Through veinules and whose constitutive equation is as follows-

$$T' = \eta_m e^n + T_p (T' \geq T_p) \text{ where, } T_p \text{ is the yield stress.}$$

When strain rate $e = 0$ ($T' < T_p$) a core region is formed which flows just like a plug [Upadhyay, 2000] ^[17]. Which is known as Herschel Bulkley Law.

D. Constitution of two phase blood volume-

The first and foremost reason is that the blood is not an ideal fluid but it is a mixture of the two phases one is of plasma and other one is of blood cells. These blood cells, semi permeable packages of liquid of a density greater than that of plasma, are capable of changing their shape and size while flowing through different blood vessels [Sherman, & Sherman, 1989] ^[11]. Plasma is a liquid containing semi permeable packages of RBCs. The behavior of blood is almost Newtonian at high shear rate, while at low shear rate the blood exhibits yield stress and non-Newtonian behavior [Shah, 2011] ^[10].



Fig

The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells. Let the volume portion covered by blood cells in unit volume be X , X is replaced by $H/100$, where H is the hematocrit the volume percentage of blood cells. The hematocrit is normally about three times the hemoglobin concentration (reported as grams per deciliter) [Berkow, Robert, 1997]^[11]. Then the volume portion covered by the plasma will be $(1-X)$.

If mass ratio of cells to plasma is r then clearly: $r = \frac{X\rho_c}{(1-X)\rho_p}$.

Where ρ_c and ρ_p are densities of blood cells and plasma respectively. Usually this mass ratio is not a constant. Even then this may be supposed to constant in present context [Upadhyay, 2000]^[17].

We have envisage Non-Newtonian mathematical modeling with the help of two phase Herschel Bulkley Non Newtonian model, we explain the blood flow in various Venules in the human heart. But present work will focus on pulmonary circulatory system; pulmonary circulatory system is a sub system of whole circulatory system. On the basis of previous work in present study be achieve new findings which are discussed separately below.

Mathematical Model

Upadhyay V. and Pandey P. N. [Upadhyay *et al.*, 2012]^[16] have already considered the blood flow as two phased. One of which is that of red blood cells and other is plasma. The second important principle of fluid dynamics is that of conservation of momentum. The equation of motion is based on this principle. According to this principle, the total momentum of any fluid system is conserved in absence of external force.

$$\frac{dp}{dt} + P - F_{v(\text{viscosity})} = 0 \text{ (External Force)}$$

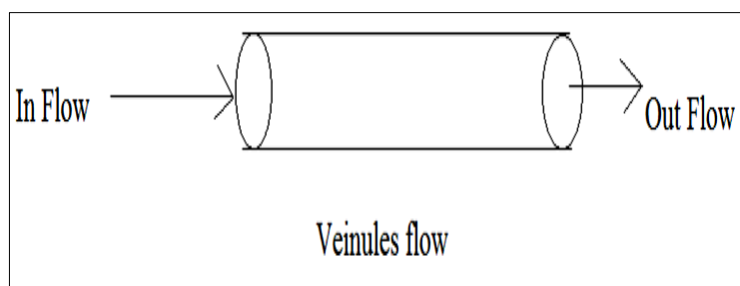
The blood can be considered as homogeneous mixtures of two phases. We derive the fundamental equation of continuity, which is a mathematical expression of principal of conservation of matter.

Equation of Continuity

If mass ratio of cells to plasma is r then clearly.

$$r = \frac{X\rho_c}{(1-X)\rho_p} \tag{2.1}$$

Where ρ_c and ρ_p are densities of blood cells and plasma respectively. Usually this mass ratio is not a constant. Even then this may be supposed to constant in present context. The both phase of blood, i.e. blood cells and plasma move with the common velocity.



Fig

According to Campbell and Pitcher [Campbell and Pitcher, 1957]^[6], V. Upadhyay and P. N. Pandey have already discussed about two phase model. It has been transformed in to biofluid mechanical set up. For this purpose, blood has been assumed to be constituted by plasma and blood cells which is realistic so for. [Upadhyay *et al.*, 2012]^[16]. The principles of conservation of mass in pulmonary circulatory system, equation of continuity for two phases are following as.

$$\frac{\partial(X\rho_c)}{\partial t} + (X\rho_c V^i)_{,i} = 0 \tag{2.2}$$

$$\frac{\partial(1-X)\rho_p}{\partial t} + (1-X)\rho_p V^i_{,j} = 0 \tag{2.3}$$

Where V is the common velocity of two phase blood cells and plasma. If we define the uniform density of blood ρ_m as

$$\text{follows: } \frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \tag{2.4}$$

Then equation (3.2) and (3.3) can be combined together as:

$$\frac{\partial\rho_m}{\partial t} + (\rho_m V^i)_{,j} = \tag{2.5}$$

Equation of motion for blood-flow

The hydro dynamical pressure P between the two phases of blood can be supposed to be uniform because the both phases i.e. blood cells and plasma is always in equilibrium state in blood. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum in pulmonary circulatory system, we get the equation of motion for the phase of blood cells as follows:

$$X\rho_c \frac{\partial V^i}{\partial t} + (X\rho_c V^j) V^i_{,j} = -X_{p,j} g^{ij} + X\eta_c (g^{jk} V^i_{,k})_{,j} \tag{2.6}$$

Similarly, taking the viscosity coefficients of plasma to be equation of motion for plasma will be as follows-

$$(1-X)\rho_c \frac{\partial V^i}{\partial t} + \{(1-X)\rho_c V^j\} V^i_{,j} = -(1-X)_{p,j} g^{ij} + (1-X)\eta_c (g^{jk} V^i_{,k})_{,j} \tag{2.7}$$

Now adding equation (3.6) and (3.7) and using relation (3.4), the equation of motion for blood flow with the both phases will be as follows-

$$\rho_m \frac{\partial V^i}{\partial t} + (\rho_m V^j) V^i_{,j} = -P_{,j} + \eta_m (g^{jk} V^i_{,k})_{,j} \tag{2.8}$$

Where $\eta_m = X\eta_c + (1-X)\eta_p$ are the viscosity coefficients of blood as a mixture of two phase. Hence a yield stress is produced. Though this yield stress is very small, even then the viscosity of blood is increased nearly ten times.

The Herschel- Bulkley law holds good on the two phase blood flow through Venules and whose constitutive equation is as follows- $T' = \eta_m e^n + T_p (T' \geq T_p)$. where, T_p is the yield stress.

When strain rate $e = 0 (T' < T_p)$ a core region is formed which flows just like a plug. Let the radius of the plug be r_p . The stress action on the surface of plug will be T_p . equating the forces acting on the plug, we get, and whose generalizes from will be as follows-

$$T^{ij} = -P g^{ij} + T_e^{ij} \tag{2.9}$$

Where $T^{ij} = \eta_m (e^{ij})^n$ while $e^{ij} = (g^{jk} V^i_{,k} + g^{ik} V^j_{,k})$

Where, symbols have their usual meanings.

Now we have considered the basic equation for Herschel-Bulkley flow as follows.

Equation of Continuity

$$\frac{1}{\sqrt{g} \sqrt{(gV^i)_{,i}}} = 0 \tag{2.10}$$

Equation of Motion

$$\rho_m \frac{\partial V^i}{\partial t} + \rho_m V^j V^i_{,j} = -T^{ij} e_{,j} \tag{2.11}$$

Where, all the symbols have their usual meaning.

Solution

Since, we have supposed the blood vessels are cylindrical; the above governing equations have to be transformed into cylindrical co-ordinates. As we know earlier: $X^1 = r, X^2 = \theta, X^3 = z$.

Matrix of metric tensor in cylindrical co-ordinates is as follows- $[g^{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

While matrix of conjugate metric tensor is as follows- $[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Where the Christoffel's symbols of 2^{nd} kind as follows-

$$\left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} = -r, \left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} = \left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} = \frac{1}{r} \text{ Remaining others is zero.}$$

Relation between contra variant and physical components of velocity of blood flow will be as follows-

$$\sqrt{g^{11}v^1} = v_r \Rightarrow v_r = v^1, \sqrt{g^{22}v^2} = v_\theta \Rightarrow v_\theta = rv^2, \sqrt{g^{33}v^3} = v_z \Rightarrow v_z = v^3, \text{ again the physical components of } p_{,j}g^{ij} \text{ are } \sqrt{g_{ij}p_{,j}g^{ij}}$$

Now, equation (3.9) and (3.10) are transformed into cylindrical form so as to solve as power law to get

Equation of continuity: $\frac{\partial V}{\partial z} = 0$

The equation of motion- r-component: $-\frac{\partial p}{\partial z} = 0, \theta - \text{component: } 0 = 0, z\text{-component: } 0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} \left[r \left(\frac{\partial V_z}{\partial r} \right)^n \right]$

Here, this fact has been taken in view that the blood flow the axially symmetric in arteries concerned, i.e. $V_\theta = 0$ and V_r, V_z and $p = p(z)$ and

$$0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} \left[r \left(\frac{\partial V_z}{\partial r} \right)^n \right] \tag{3.1}$$

Since, pressure gradient $-\frac{dp}{dz} = P$

$$r \left(\frac{dv}{dz} \right)^n = -\frac{Pr^2}{2\eta_m} + A, \text{ we apply boundary condition at } r = 0. V = V_0 \text{ than } A = 0.$$

$$\Rightarrow -\frac{dv}{dr} = \left(\frac{Pr}{2\eta_m} \right)^{\frac{1}{n}} \text{ Replace from } r - r_p$$

$$-\frac{dv}{dr} = \left(\frac{\frac{1}{2}Pr - \frac{1}{2}Pr}{\eta_m} \right)^{\frac{1}{n}} \Rightarrow \frac{dv}{dr} = -\left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} (r - r_p)^{\frac{1}{n}} \tag{3.2}$$

Integrating above equation (3.2) under the no slip boundary condition $V = 0$ at $r = R$ so we get:

$$v = \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} \frac{n}{n+1} \left[(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1} \right] \tag{3.3}$$

This is the formula for velocity of blood flow in Venules. Putting $r = r_p$ to get the velocity v_p of plug flow as follows:

$$v_p = \frac{n}{n+1} \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} \tag{3.4}$$

Where, the value of r_p is taken form (2.7).

Result and Discussion

Examination: Hematocrit v/s blood pressure in during Lung Cancer patient.

Patient name: ...X..... (Male) **Age:** 42 years old

Diagnosis: Lung cancer (Pulmonary disease)

Table

Date	HB(Hemoglobin) (gram/dl)	Hematocrit ($3 \times HB$) (kg/liter ³)	Blood Pressure (BP) (mmhg)	Venules Pressure D in Pascal-second $\frac{2}{3} \left(\frac{(S+D)}{2} + D \right) - \frac{(S+D)}{3} + D$
01/03/2014	8.9	0.025191	150/90	-3106.4135
08/11/2014	8.0	0.02265	140/90	-3032-4512
03/04/2015	7.4	0.02095	120/70	-2440.7536
10/06/2015	7.0	0.01982	110/70	-2366.9712
10/06/2016	7.2	0.02038	100/60	-2070.9423

The flow two phased blood flow in Venules is-

$$Q = \int_0^{r_p} 2\pi r v_p + \int_{r_p}^R 2\pi r v dr$$

$$= \int_0^{r_p} 2\pi r \frac{n}{n+1} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} dr + \int_0^{r_p} 2\pi r \frac{n}{n+1} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \left[(R - r_p)^{\frac{1}{n}+1} - (R - r_p)^{\frac{1}{n}+1} \right] dr$$

Using (3.2) and (3.4), we get

$$Q = \frac{2\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} \left[\frac{r^2}{2} \Big|_0^{r_p} + \frac{2\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \left[\frac{r^2}{2} (R - r_p)^{\frac{1}{n}+1} - \frac{r(r-r_p)^{\frac{1}{n}}}{\frac{1}{n}+1} + \frac{(r-r_p)^{\frac{1}{n}+3}}{(\frac{1}{n}+1)(\frac{1}{n}+3)} \right] \Big|_{r_p}^R \right]$$

$$Q = \frac{2\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} r_p^2 (R - r_p)^{\frac{1}{n}+1} + R^2 (R - r_p)^{\frac{1}{n}+1} - \frac{2R(R-r_p)^{\frac{1}{n}+2}}{(\frac{1}{n}+2)} + \frac{2R(R-r_p)^{\frac{1}{n}+3}}{(\frac{1}{n}+2)(\frac{1}{n}+3)} - r_p^2 (R - r_p)^{\frac{1}{n}+1}$$

$$Q = \frac{\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} R^{\frac{1}{n}+3} \left[\frac{r_p^2}{r^2} \left(1 - \frac{r_p^2}{R}\right)^{\frac{1}{n}+1} + \left(1 + \frac{r_p}{R}\right)^{\frac{1}{n}+1} \left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2} - \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2}}{(\frac{1}{n}+2)} + \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+3}}{(\frac{1}{n}+2)(\frac{1}{n}+3)} \right] \tag{4.2}$$

P = Pressure gradient, ν = Viscosity of mixture (Blood), n = Parameter

Now, we have, $Q = 425 \frac{ml}{min}$, $Q = 0.00708333 m^3/second$ and $R = 1$, $r_p = \frac{1}{3}$,

According to Gustafson, Daniel R. (1980), $\eta_p = 0.0013$ pascal second

According to Glenn Elert (2010), $\eta_m = 0.0271$ pascal second and $H = 0.020378$

Pressure drop ($P_f - P_i$) = 2070.9423 Pascal second

Pulmonary venules average length = 0.15 cm or $0.15 \times 10^{-9} m^3$ (According to J.T. Ottesen, M.S. Olufsen, and J.K. Larsen, 2006)

By using relation $\eta_m = \eta_c X + \eta_p (1 - X)$ Where, $X = \frac{H}{100}$, and we get η_c

$$\eta_m = \eta_c X + \eta_p (1 - X), \Rightarrow 0.0271 = \eta_c(0.0002038) + 0.0013(0.9997962)$$

$$\Rightarrow \eta_c = 126.5961 \text{ Pascal second}$$

Again using this relation and change in to the hematocrit-

$$\eta_m = \eta_c X + \eta_p (1 - X) \Rightarrow \eta_m = 1.265961H + 0.0012998$$

Now substituting the values of r_p and R in Equation (4.1)-

$$Q = \frac{\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} R^{\frac{1}{n}+3} \left[\frac{r_p^2}{r^2} \left(1 - \frac{r_p^2}{R}\right)^{\frac{1}{n}+1} + \left(1 + \frac{r_p}{R}\right)^{\frac{1}{n}+1} \left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2} - \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2}}{(\frac{1}{n}+2)} + \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+3}}{(\frac{1}{n}+2)(\frac{1}{n}+3)} \right]$$

And we get equation-

$$Q = \pi \left(\frac{2P}{6\eta_m}\right)^{\frac{1}{n}} \left(\frac{2}{27}\right) \left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1} \right] \text{ Or, } \frac{27 \times Q}{2\pi} = \left(\frac{P}{3\eta_m}\right)^{\frac{1}{n}} \left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1} \right]$$

$$\text{Let } A = \left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1} \right] \Rightarrow \frac{P}{3\eta_m} = \left(\frac{27 \times Q}{2\pi A}\right)^n \Rightarrow P = \left(\frac{27 \times Q}{2\pi A}\right)^n \cdot 3\eta_m \Rightarrow P = -\frac{dp}{dz} \Rightarrow -dp = Pd z$$

And limit from the pressure from Z_f to Z_i then-

$$\int_{P_f}^{P_i} dP = - \int_{Z_f}^{Z_i} \left(\frac{27 \times Q}{2\pi A}\right)^n \cdot 3\eta_m dz$$

Where $P_f - P_i$ = pressure drop and $Z_f - Z_i$ = pulmonary venules length.

$$P_f - P_i = \left(\frac{27 \times Q}{2\pi A}\right)^n \cdot 3\eta_m \cdot (Z_f - Z_i) \text{ Or, } \frac{27 \times Q}{2\pi A} = \left(\frac{P_f - P_i}{(Z_f - Z_i) 3\eta_m}\right)^{1/n}$$

$$\frac{27 \times Q}{2\pi} = \left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1} \right] \left(\frac{P_f - P_i}{(Z_f - Z_i) 3\eta_m}\right)^{1/n}$$

Substituting the value of Q , η_m , $(P_f - P_i)$ and $(Z_f - Z_i)$ and solve by numerical methods

$$\frac{27 \times 0.0070833}{6.28} = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right] \left(\frac{2070.9423}{0.15 \times 10^{-9} \times 0.0813} \right)^{1/n}$$

$$0.03045669 = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right] (169819000000000)^{1/n}$$

And we get $n = -6.7$

Now again using equation $P_f - P_i = \left(\frac{27 \times Q}{2\pi A} \right)^n \cdot 3\eta_m \cdot (Z_f - Z_i)$

$$\Delta P = \left(\frac{27 \times Q}{2\pi A} \right)^n \cdot 3\eta_m \cdot (Z_f - Z_i), \Delta P = \left(\frac{27 \times 0.007084}{6.28 \times 4.00408} \right)^{-6.7} \times 3\eta_m \times (0.15 \times 10^{-9})$$

$$\Delta P = (1.570705 \times 10^{14}) \times (0.15 \times 10^{-9}) \times 3\eta_m$$

$$\Delta P = 23560.57 (0.000772H + 0.0008581)$$

$$\Delta P = 89480.289H + 91.873$$

Table: Table for Hematocrit v/s blood Pressure drop in Clinical data-

Date	1/3/2014	8/11/2014	3/4/2015	10/6/2015	10/6/2016
H (Hematocrit) In kg/m ³	0.025191	0.02265	0.02095	0.01982	0.02038
BPD(Blood Pressure drop) In Pascal-second	2254.09797	2118.60155	1966.48506	1865.37233	1915.30233

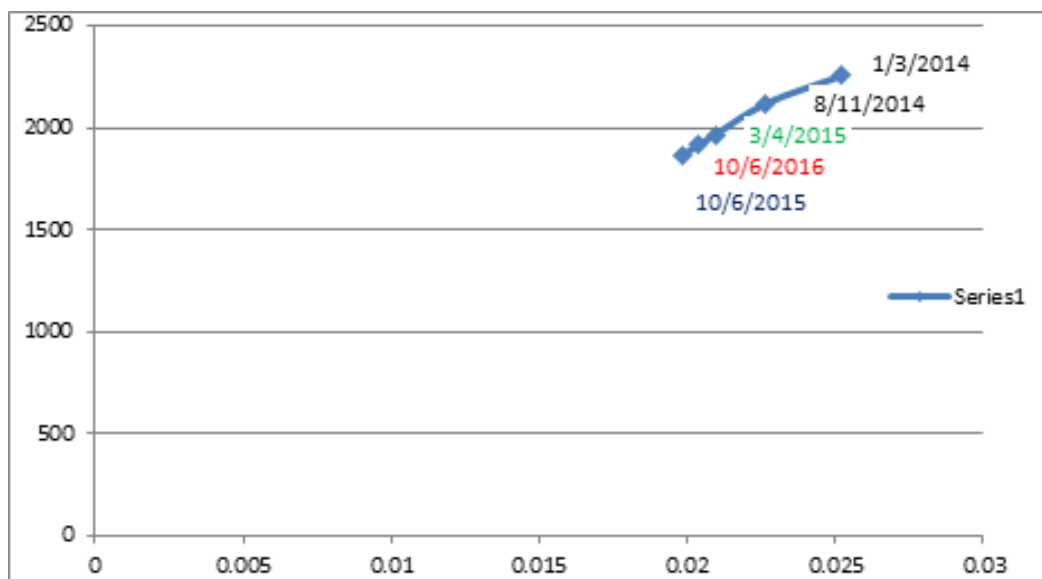


Fig VI: Graphical presentation of clinical data between hematocrit and pressure drop

Observation

A mathematical investigation of the graph from 1/3/2014 to 3/4/2015 convex nature in decreasing since via 8/11/2014, And straight line in decreasing since between 3/4/2015 to 10/6/2016 but increasing with straight line since from 10/6/2015 to from 10/6/2016.

Conclusion

When blood pressure drop is increased then we cannot suggest for operation but when blood pressure drop is decreased we suggest for successful operation. Between 3/4/2015 to 10/6/2016 successful operation is suggested otherwise not.

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