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## On Soft $\pi g * b *$ - closed sets in soft topological spaces

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### Abstract

The aim of this paper is to introduce a new class of sets called soft  $\pi$  generalized  $b^*$ -closed set (briefly soft  $\pi g^* b^*$ -closed) in soft topological space which is defined over the universe of the given set with a fixed set of parameters. Further, we investigate its properties and its relationship with other soft closed.

**Keywords:** soft sets, soft topological space, soft  $\pi g^* b^*$ -closed sets, soft  $\pi g^* b^*$ -continuous function, soft  $\pi g^* b^* - T_{1/2}$ -space

### 1. Introduction

Molodtsov <sup>[1]</sup> initiated the concept of soft set theory as a new mathematical tool and presented the fundamental results of the soft sets. soft set theory has a wider application and its progress is very rapid in different fields. Levine <sup>[2]</sup> introduced  $g$ -closed sets in general topology. Kannan <sup>[3]</sup> introduced soft  $g$ -closed sets in soft topological spaces. Muhammad Shabir and Munazza Naz <sup>[4]</sup> introduced soft topological spaces and the notions of soft open sets, soft closed sets, soft closure, soft interior points. The idea of soft  $\pi g b$ -closed sets were introduced by D. Sreeja and C. Janaki <sup>[5]</sup>.

The concept of  $\pi g^* b^*$ -closed sets in topological spaces was introduced by Vaiyomathi. K <sup>[6]</sup>. In this paper, we introduce and study soft  $\pi g^* b^*$ -closed sets in soft topological spaces and obtain its relationship with other soft closed sets. Further, we obtain the basic results and properties.

### 2. Preliminaries

Let  $U$  be an initial universe set and  $E$  be a collection of all possible parameters with respect to  $U$ , where parameters are the characteristics or properties of objects in  $U$ . Let  $P(U)$  denote the power set of  $U$ , and let  $A \subseteq E$ .

**Definition 2.1.** <sup>[7]</sup>: Let  $X$ , be an initial universal set and  $E$  be the set of parameters.

Let  $P(X)$  denote the power set of  $X$ , and  $A \subseteq E$ . The pair  $(F, A)$  is called a soft set over  $X$ , where  $F$  is a mapping given by  $F : A \rightarrow P(X)$ .

**Definition 2.2.** <sup>[3]</sup>: A soft set  $(F, E)$  over is said to be

- i) A null soft set, denoted by  $\phi$ , if  $\forall e \in E, F(e) = \phi$ .
- ii) An absolute soft set, denoted by  $X$ , if  $e \in E, F(e) = X$ .

**Definition 2.3.** <sup>[3]</sup>: Let  $\tau$  be the collection of soft sets over  $X$ , then  $\tau$  is said to be a soft Topology on  $X$  if

- i)  $\Phi, X$  are belongs to  $\tau$ .
- ii) The union of any number of soft sets in  $\tau$  belongs to  $\tau$ .
- iii) The intersection of any two soft sets in  $\tau$  belongs to  $\tau$ .

The triplet  $(X, \tau, E)$  is called a soft topological space over  $X$  and any member of  $\tau$  is known as soft open set in  $X$ . The complement of a soft open set is called soft closed set over  $X$ .

**Definition 2.4.** [3]: Let  $(X, \tau, E)$  be a soft topological space over  $X$  and  $(F, E)$  be a soft set over  $X$  Then

- i) soft interior of a soft set  $(F, E)$  is defined as the union of all soft open sets contained in  $(F, E)$ . Thus  $\text{int}(F, E)$  is the largest soft open set contained in  $(F, E)$ .
- ii) soft closure of a soft set  $(F, E)$  is the intersection of all soft closed super sets  $(F, E)$ . Thus  $\text{cl}(F, E)$  is the smallest soft closed set over  $X$ , which contains  $(F, E)$ .

**Definition 2.5** [7]: A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F : A \rightarrow P(U)$ . In other words, a soft set over  $U$  is a parameterized family of subsets of the universe  $U$ . For a particular  $e \in A$ .  $F(e)$  may be considered the set of e-approximate elements of the soft set  $(F, A)$ .

**Definition 2.6**

A soft subset  $(A, E)$  of  $X$  is called

- i) a soft pre-open[9] set if  $(A, E) \tilde{\subset} \text{Int}(\text{Cl}(A, E))$ .
- ii) a soft  $\alpha$ -open[9] if  $(A, E) \tilde{\subset} \text{Int}(\text{Cl}(\text{Int}(A, E)))$
- iii) a soft b-open[9] if  $(A, E) \tilde{\subset} \text{Cl}(\text{Int}(A, E)) \text{Int}(\text{Cl}(A, E))$
- iv) a soft regular open[9] if  $(A, E) = \text{Int}(\text{Cl}(A, E))$ .

The complement of the soft pre-open, soft  $\alpha$ -open, soft b-open, soft regular open sets are their respective soft pre -closed, soft  $\alpha$ -closed, soft b-closed and soft regular closed sets.

**Definition 2.7** [11]: Let A subset  $(A, E)$  of a soft topological space  $(X, \tau, E)$  is called a soft  $g$  - closed set if  $\text{cl}(A, E) \tilde{\subset} (U, E)$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft open in  $(X, \tau, E)$ .

**Definition 2.8** [12]: Let A subset  $(A, E)$  of a soft topological space  $(X, \tau, E)$  is called a soft  $\pi g$  - closed set if  $\text{cl}(A, E) \tilde{\subset} (U, E)$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft  $\pi$ -open in  $(X, \tau, E)$ .

**Definition 2.9** [10]: Let A subset  $(A, E)$  of a soft topological space  $(X, \tau, E)$  is called a soft  $\pi gb$  - closed set if  $(\text{bcl}(A, E)) \tilde{\subset} (U, E)$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft  $\pi$ -open in  $(X, \tau, E)$

**Definition 2.10** [9]: Let A subset  $(A, E)$  of a soft topological space  $(X, \tau, E)$  is called a soft  $\pi gr$  - closed set if  $(\text{rcl}(A, E)) \tilde{\subset} (U, E)$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft  $\pi$ -open in  $(X, \tau, E)$

**Definition 2.11** [8]: Let A subset  $(A, E)$  of a soft topological space  $(X, \tau, E)$  is called a soft  $\pi gp$  - closed set if  $(\text{pcl}(A, E)) \tilde{\subset} (U, E)$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft  $\pi$ -open in  $(X, \tau, E)$

**3. Soft  $\pi g^*b^*$ -closed sets and Soft  $\pi g^*b^*$ -open sets**

**Definition 3.1:** Let A subset  $(A, E)$  of a soft topological space  $(X, \tau, E)$  is called a soft  $\pi g^*b^*$ - closed set if  $\text{int}(\text{bcl}(A, E)) \tilde{\subset} (U, E)$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft  $\pi g$ -open in  $(X, \tau, E)$

**Definition 3.2:** Let A subset  $(A, E)$  of a soft topological space  $(X, \tau, E)$  is called a soft  $\pi^{\wedge}g$ - closed set if  $\text{gcl}(A, E) \tilde{\subset} (U, E)$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft  $\pi$ -open in  $(X, \tau, E)$ .

**Definition 3.3:** Let A subset  $(A, E)$  of a soft topological space  $(X, \tau, E)$  is called a soft  $\pi^*g$ -closed set if  $\text{cl}(\text{int}(A, E)) \tilde{\subset} (U, E)$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft  $\pi$ -open in  $(X, \tau, E)$

**Definition 3.4:** Let A subset  $(A, E)$  of a soft topological space  $(X, \tau, E)$  is called a soft  $\pi gb^*$ - closed set if  $\text{int}(\text{bcl}(A, E)) \tilde{\subset} (U, E)$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft  $\pi$ -open in  $(X, \tau, E)$

**Theorem 3.5:** Every soft closed set in a soft topological space is soft  $\pi g^*b^*$ -closed set.

**Proof:** Let  $(A,E)$  be soft closed set of  $(X, \tau, E)$ , such that  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi g$ -open in  $(X, \tau, E)$ . since  $bcl(A,E) \tilde{\subset} cl(A,E) = (A,E)$ . Since every soft  $\pi g$ -closed set is soft  $\pi gb$ -closed. We have,  $int(bcl(A,E) \tilde{\subset} int(A,E) \tilde{\subset} int(U,E) = (U,E)$ . Hence  $(A,E)$  is soft  $\pi g^*b^*$ -closed set.

**Theorem 3.6:** Every soft  $\alpha$ -closed set in a soft topological space is soft  $\pi g^*b^*$ -closed set.

**Proof:** Let  $(A,E)$  be soft  $\alpha$ -closed set of  $(X, \tau, E)$ , such that  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi g$ -open in  $(X, \tau, E)$ . since  $bcl(A,E) \tilde{\subset} \alpha cl(A,E) = (A,E)$ . Since every soft  $\alpha$ -closed set is soft b-closed. We have,  $int(bcl(A,E) \tilde{\subset} int(A,E) \tilde{\subset} int(U,E) = (U,E)$ . Hence  $(A,E)$  is soft  $\pi g^*b^*$ -closed set.

**Theorem 3.7:** Every soft  $g$ -closed set in a soft topological space is soft  $\pi g^*b^*$ -closed set.

**Proof:** Let  $(A,E)$  be soft  $g$ -closed set of  $(X, \tau, E)$ , such that  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi g$ -open in  $(X, \tau, E)$ . Also  $cl(A,E) \tilde{\subset} (U,E)$ . since  $bcl(A,E) \tilde{\subset} cl(A,E) \tilde{\subset} (U,E)$ . Every soft  $\pi g$ -open set is open. We have,  $int(bcl(A,E) \tilde{\subset} int(A,E) \tilde{\subset} int(U,E) = (U,E)$ . Hence  $(A,E)$  is soft  $\pi g^*b^*$ -closed set.

**Theorem 3.8:** Every soft  $\pi g$ -closed set in a soft topological space is soft  $\pi g^*b^*$ -closed set.

**Proof:** Let  $(A,E)$  be soft  $\pi g$ -closed set of  $(X, \tau, E)$ , such that  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi g$ -open in  $(X, \tau, E)$ . Also  $cl(A,E) \tilde{\subset} (U,E)$ . since  $bcl(A,E) \tilde{\subset} rcl(A,E) = (A,E)$ . Every soft  $\pi g$ -closed set is soft  $\pi gb$ -closed. We have,  $int(bcl(A,E) \tilde{\subset} int(A,E) \tilde{\subset} int(U,E) = (U,E)$ . Hence  $(A,E)$  is soft  $\pi g^*b^*$ -closed set.

**Theorem 3.9:** Every soft  $\pi^{\wedge}g$ -closed set in a soft topological space is soft  $\pi g^*b^*$ -closed set.

**Proof:** Let  $(A,E)$  be soft  $\pi^{\wedge}g$ -closed set of  $(X, \tau, E)$ , such that  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi g$ -open in  $(X, \tau, E)$ . Also  $gcl(A,E) \tilde{\subset} (U,E)$ . since  $bcl(A,E) \tilde{\subset} gcl(A,E) = (A,E)$ . Every soft  $\pi g$ -closed set is soft  $\pi gb$ -closed. We have,  $int(bcl(A,E) \tilde{\subset} int(A,E) \tilde{\subset} int(U,E) = (U,E)$ . Hence  $(A,E)$  is soft  $\pi g^*b^*$ -closed set.

**Theorem 3.10:** Every soft  $\pi^*g$ -closed set in a soft topological space is soft  $\pi g^*b^*$ -closed set.

**Proof:** Let  $(A,E)$  be soft  $\pi^*g$ -closed set of  $(X, \tau, E)$ , such that  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi g$ -open in  $(X, \tau, E)$ . Also  $cl(int(A,E)) \tilde{\subset} (U,E)$ . since  $bcl(A,E) \tilde{\subset} cl(int(A,E)) = (A,E)$ . We have,  $int(bcl(A,E) \tilde{\subset} int(U,E) = (U,E)$ . Hence  $(A,E)$  is soft  $\pi g^*b^*$ -closed set.

**Theorem 3.11:** Every soft  $\pi g^b$ -closed set in a soft topological space is soft  $\pi g^*b^*$ -closed set.

**Proof:** Let  $(A,E)$  be soft  $\pi g^b$ -closed set of  $(X, \tau, E)$ , such that  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi g$ -open in  $(X, \tau, E)$ . Also  $bcl(A,E) \tilde{\subset} (U,E)$ . since  $bcl(A,E) = (A,E)$ . since every soft  $\pi$ -closed set is soft  $\pi^g$ -closed. We have,  $int(bcl(A,E) \tilde{\subset} int(A,E) \tilde{\subset} int(U,E) = (U,E)$ . Hence  $(A,E)$  is soft  $\pi g^*b^*$ -closed set.

**Theorem 3.12:** Every soft  $\pi gb^*$ -closed set in a soft topological space is soft  $\pi g^*b^*$ -closed set.

**Proof:** Let  $(A,E)$  be soft  $\pi gb^*$ -closed set of  $(X, \tau, E)$ , such that  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi g$ -open in  $(X, \tau, E)$ . Also  $int(bcl(A,E) \tilde{\subset} (U,E)$ . since every soft  $\pi$ -closed set is soft  $\pi g$ -closed. Hence  $(A,E)$  is soft  $\pi g^*b^*$ -closed set.

**Theorem 3.13:** Every soft  $\pi gp$ -closed set in a soft topological space is soft  $\pi g^*b^*$ -closed set.

**Proof:** Let  $(A,E)$  be soft  $\pi gp$ -closed set of  $(X, \tau, E)$ , such that  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi g$ -open in  $(X, \tau, E)$ . Also  $pcl(A,E) \tilde{\subset} (U,E)$ . since  $bcl(A,E) \tilde{\subset} pcl(A,E) = (A,E)$ . since every soft  $\pi gp$ -closed set is  $\pi gb$ -closed. We have,  $int(bcl(A,E) \tilde{\subset} int(A,E) \tilde{\subset} int(U,E) = (U,E)$ . Hence  $(A,E)$  is soft  $\pi g^*b^*$ -closed set.

**Theorem 3.14:** Every soft  $\pi g$ -closed set in a soft topological space is soft  $\pi g^*b^*$ - closed set.

**Proof:** Let  $(A, E)$  be soft  $\pi g$ - closed set of  $(X, \tau, E)$ , such that  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft  $\pi g$ -open in  $(X, \tau, E)$ . Also  $\text{rcl}(A, E) \tilde{\subset} (U, E)$ . since  $\text{bcl}(A, E) \tilde{\subset} \text{rcl}(A, E) = (A, E)$ . since every soft  $\pi g$ - closed set is soft  $\pi g b$ - closed. We have,  $\text{int}(\text{bcl}(A, E)) \tilde{\subset} \text{int}(A, E) \tilde{\subset} \text{int}(U, E) = (U, E)$ . Hence  $(A, E)$  is soft  $\pi g^*b^*$ - closed set.

**Remark 3.15**

Converse of the above theorems need not be true as seen in the following example.

**Example 3.16**

Let  $X = \{a, b, c\}$ ,  $E = \{e_1, e_2\}$ . Let  $F_1, F_2, \dots, F_7$  are functions from  $E$  to  $P(X)$  and are defined as follows :

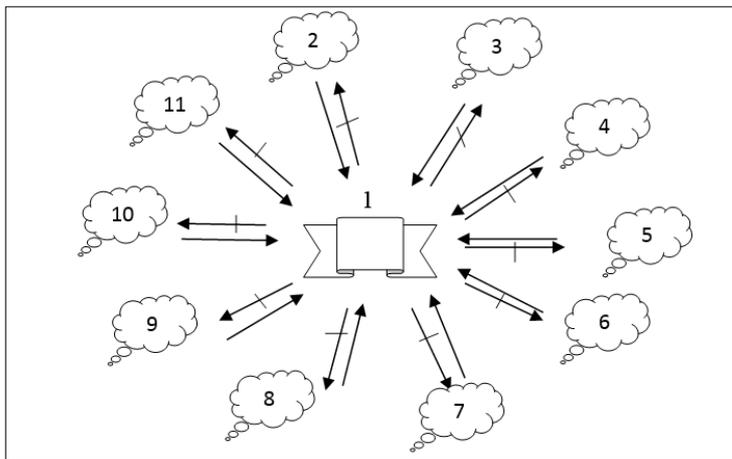
$F_1(e_1) = \{a\}, F_1(e_2) = \{\phi\}, F_2(e_1) = \{b\}, F_2(e_2) = \{c\}, F_3(e_1) = \{c\}, F_3(e_2) = \{a\},$   
 $F_4(e_1) = \{a, b\}, F_4(e_2) = \{c\}, F_5(e_1) = \{a, c\}, F_5(e_2) = \{a\}, F_6(e_1) = \{b, c\}, F_6(e_2) = \{a, c\},$   
 $F_7(e_1) = \{X\}, F_7(e_2) = \{a, c\}.$

Then  $\tau = \{\phi, X, (F_1, E), (F_2, E), \dots, (F_7, E)\}$  is a soft topology and elements in  $\tau$  are soft open sets

1. The soft set  $(A, E) = \{\{a\}, \{b, c\}\}, \{\{b, c\}, \{b\}\}$  are soft  $\pi g^*b^*$ - closed set but not soft closed.
2. (ii) The soft set  $(B, E) = \{\{a, b\}, \{a, b\}\}, \{\{b, c\}, \{b, c\}\}$  are soft  $\pi g^*b^*$ - closed set but not soft  $\alpha$  - closed.
3. The soft set  $(C, E) = \{\{b\}, \{a, c\}\}, \{\{a, b\}, \{a\}\}$  are soft  $\pi g^*b^*$ - closed set but not soft  $g$  - closed.
4. The soft set  $(D, E) = \{\{a\}, \{a\}\}, \{\{b\}, \{c\}\}$  are soft  $\pi g^*b^*$ - closed set but not soft  $\pi g$  - closed.
5. The soft set  $(E, E) = \{\{a, c\}, \{a, c\}\}, \{\{c\}, \{a\}\}$  are soft  $\pi g^*b^*$ - closed set but not soft  $\pi^\wedge g$ - closed.
6. The soft set  $(F, E) = \{\{a\}, \{c\}\}, \{\{c\}, \{a, c\}\}$  are soft  $\pi g^*b^*$ - closed set but not soft  $\pi^*g$ - closed.
7. The soft set  $(G, E) = \{\{a, b\}, \{a, c\}\}, \{\{a, c\}, \{c\}\}$  are soft  $\pi g^*b^*$ - closed set but not soft  $\pi g b$  - closed.
8. The soft set  $(H, E) = \{\{X\}, \{a, c\}\}$  are soft  $\pi g^*b^*$ - closed set but not soft  $\pi g b^*$  - closed.
9. The soft set  $(I, E) = \{\{a\}, \{a, c\}\}, \{\{b, c\}, \{a, c\}\}$  are soft  $\pi g^*b^*$ - closed set but not soft  $\pi g p$  - closed.
10. The soft set  $(J, E) = \{\{b, c\}, \{c\}\}, \{\{c\}, \{c\}\}$  are soft  $\pi g^*b^*$ - closed set but not soft  $\pi g r$  - closed.

**Remark 3.17**

By the above results we have the following diagram



**Fig 1**

- |                                  |                                     |                                  |
|----------------------------------|-------------------------------------|----------------------------------|
| 1. soft $\pi g^*b^*$ -closed set | 5. soft- closed set                 | 9. soft $\pi g b^*$ - closed set |
| 2. soft closed set               | 6. soft $\pi^\wedge g$ - closed set | 10. soft- closed set             |
| 3. soft- closed set              | 7. soft- closed set                 | 11. soft- closed set             |
| 4. soft $g$ - closed set         | 8. soft- closed set                 |                                  |

**4. Soft  $\pi g^*b^*$  - continuous functions**

**Definition 4.1**

Let  $(X, \tau, A)$  and  $(Y, \sigma, B)$  between two soft topological spaces and  $f : (X, \tau, A) \rightarrow (Y, \sigma, B)$  be a function. Then the function is

- (i) soft  $\pi g^*b^*$ - continuous iff  $f^{-1}(G, B)$  is soft  $\pi g^*b^*$ - closed in  $(X, \tau, A)$  for every soft closed set  $(G, B)$  of  $(Y, \sigma, B)$

**Definition 4.2**

Let  $(X, \tau, A)$  and  $(Y, \sigma, B)$  between two soft topological spaces and  $f : (X, \tau, A) \rightarrow (Y, \sigma, B)$

be a function. Then the function is

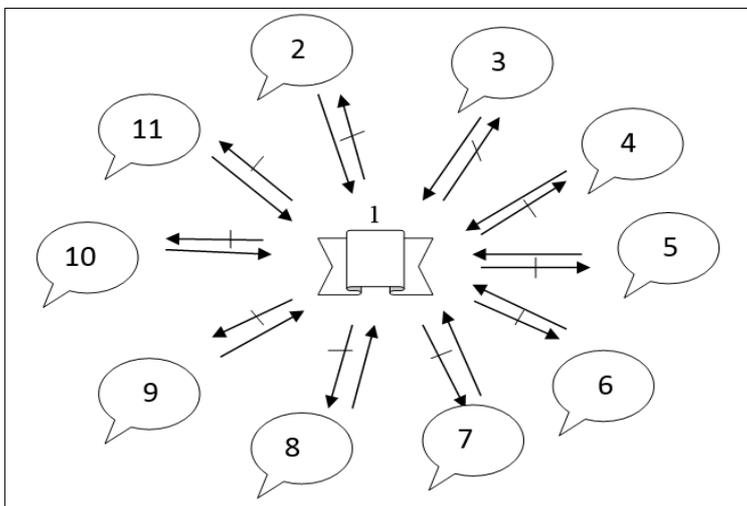
- (i) soft  $\pi g^*b^*$ -continuous iff  $(G,A)$  is soft  $\pi g^*b^*$ -continuous in  $(Y, \sigma, B)$  for every soft continuous set  $(G,A)$  of  $(X, \tau, A)$
- (ii) soft  $\pi g^*b^*$ -closed iff  $(G,A)$  is soft  $\pi g^*b^*$ -closed in  $(Y, \sigma, B)$  for every soft closed set  $(G,A)$  of  $(X, \tau, A)$

**Theorem 4.3**

- i) Every soft continuous is soft  $\pi g^*b^*$ -continuous.
- ii) Every soft  $\alpha$ -continuous is soft  $\pi g^*b^*$ -continuous.
- iii) Every soft  $g$ -continuous is soft  $\pi g^*b^*$ -continuous.
- iv) Every soft  $\pi g$ -continuous is soft  $\pi g^*b^*$ -continuous.
- v) Every soft  $\pi \wedge g$ -continuous set is soft  $\pi g^*b^*$ -continuous.
- vi) Every soft  $\pi^*g$ -continuous is soft  $\pi g^*b^*$ -continuous.
- vii) Every soft  $\pi gb$ -continuous is soft  $\pi g^*b^*$ -continuous.
- viii) Every soft  $\pi gb^*$ -continuous is soft  $\pi g^*b^*$ -continuous.
- ix) Every soft  $\pi gp$ -continuous is soft  $\pi g^*b^*$ -continuous.
- x) Every soft  $\pi gr$ -continuous is soft  $\pi g^*b^*$ -continuous.

**Proof**

Let  $(G,B)$  be a soft closed in  $Y$ .  
 since  $f$  is soft continuous,  $f^{-1}(G,B)$  is soft closed in  $X$ .  
 since every closed set is soft  $\pi g^*b^*$ -closed,  $f^{-1}(G,B)$  is soft  $\pi g^*b^*$ -closed.  
 Hence  $f$  is soft  $\pi g^*b^*$ -continuous. The proof is obvious.



**Fig 2**

- |                                  |                                    |                                |
|----------------------------------|------------------------------------|--------------------------------|
| 1. soft $\pi g^*b^*$ -continuous | 5. soft-continuous                 | 9. soft $\pi gb^*$ -continuous |
| 2. soft continuous               | 6. soft $\pi \wedge g$ -continuous | 10. soft-continuous            |
| 3. soft-continuous               | 7. soft $\pi^*g$ -continuous       | 11. soft-continuous            |
| 4. soft $g$ -continuous          | 8. soft-continuous                 |                                |

**Example 4.4**

Let  $X = \{a,b,c\} = Y$ ,  $E = \{e_1, e_2\}$ . Let  $F_1, F_2, \dots, F_7$  are functions from  $E$  to  $P(X)$  and are defined as follows :

$F_1(e_1) = \{b,c\}, F_1(e_2) = \{X\}, F_2(e_1) = \{a,c\}, F_2(e_2) = \{a,b\}, F_3(e_1) = \{a,b\}, F_3(e_2) = \{b,c\},$   
 $F_4(e_1) = \{c\}, F_4(e_2) = \{a,b\}, F_5(e_1) = \{b\}, F_5(e_2) = \{b,c\}, F_6(e_1) = \{a\}, F_6(e_2) = \{b\},$   
 $F_7(e_1) = \{ \phi \}, F_7(e_2) = \{b\}.$

Then  $\tau_1 = \{ \phi, X, (F_1, E), (F_2, E), \dots, (F_7, E) \}$  is a soft topology and elements in  $\tau_1$  are soft open sets. Let  $G_1, G_2, \dots, G_7$  are functions from  $E$  to  $P(Y)$  and are defined as follows :

$G_1(e_1) = \{X\}, G_1(e_2) = \{b\}, G_2(e_1) = \{X\}, G_2(e_2) = \{b,c\}, G_3(e_1) = \{a\}, G_3(e_2) = \{a,c\},$   
 $G_4(e_1) = \{a,b\}, G_4(e_2) = \{a\}, G_5(e_1) = \{a,b\}, G_5(e_2) = \{a,c\}, G_6(e_1) = \{a,c\}, G_6(e_2) = \{c\},$   
 $G_7(e_1) = \{a,c\}, G_7(e_2) = \{a,c\}$

Then  $\tau_2 = \{ \phi, \tilde{X}, (G_1, E), (G_2, E), \dots, (G_7, E) \}$  is a soft topology on  $Y$ .

Let  $f : X \rightarrow Y$  be an identity map.

Here the inverse image of the soft closed set  $(A, E) = \{ \{a\}, \{a,c\} \}, \{ \{a,b\}, \{a\} \}, \{ \{a,b\}, \{a,c\} \},$

$\{\{a,c\},\{c\}\},\{\{a,c\},\{a,c\}\}$  in  $Y$  is not soft closed, soft  $\alpha$ -closed, soft  $g$ -closed, soft  $\pi$   $g$ -closed, soft  $\pi^\wedge$   $g$ -closed, soft  $\pi * g$ -closed, soft  $\pi g b$ -closed, soft  $\pi g r$ -closed, soft  $\pi g p$ -closed in  $X$ . Hence  $f$  is  $\pi g * b^*$ -continuous but not soft continuous, soft  $\alpha$ -continuous, soft  $g$ -continuous, soft  $\pi$   $g$ -continuous, soft  $\pi^\wedge g$ -continuous, soft  $\pi * g$ -continuous, soft  $\pi g b$ -continuous, soft  $\pi g p$ -continuous, soft  $\pi g r$ -continuous.

**Theorem 4.5**

(a) Let  $f : (X, \tau, A) \rightarrow (Y, \sigma, B)$  be a soft function, then the following statements are equivalent.

- (i)  $f$  is soft  $\pi g * b^*$ -continuous
  - (ii) The inverse image of every soft open set in  $Y$  is also soft  $\pi g * b^*$ -open in  $X$ .
- (b) Iff  $f : (X, \tau, A) \rightarrow (Y, \sigma, B)$  is soft  $\pi g * b^*$ -continuous, then  $f(s \pi g * b^* -cl(A,E)) \tilde{=} s-cl(f(A,E))$  for every subset  $(A,E)$  of  $X$ .

**Proof**

a. (i)  $\Rightarrow$  (ii) Let  $(G,B) \in SO(Y)$ , then  $(Y-(G,B)) \in SC(Y)$ . Since  $f$  is soft  $\pi g * b^*$ -continuous,  $f^{-1}(Y-(G,B)) \in S \pi G * B * C(X)$ . Hence  $X - (f^{-1}(G,B)) \in S \pi G * B * C(X)$ . Then  $f^{-1}(G,E) \in S \pi G * B * O(X)$ .

b. (ii)  $\Rightarrow$  (i) Follows from definition.

Let  $(A,E) \tilde{=} X$ .

Since  $f$  is soft  $\pi g * b^*$ -continuous and  $(A,E) \tilde{=} f^{-1}(s-cl(f(A,E)))$ ,

We obtain  $s \pi g * b^* - cl((A,E)) \tilde{=} f^{-1}(s-cl(f(A,E)))$  and

then  $f(s \pi g * b^* -cl((A,E)) \tilde{=} s-cl(f(A,E)))$ .

**Theorem 4.6**

If a soft function  $f : (X, \tau, A) \rightarrow (Y, \sigma, B)$  is soft  $\pi g * b^*$ -closed,

then  $s \pi g * b^* - cl(f(A,E)) \tilde{=} f(s-cl(f(A,E)))$  for every subset  $(A,E)$  of  $(X, \tau, E)$

**Proof**

Let  $(G,A) \tilde{=} X$ .

Since  $f$  is soft  $\pi g * b^*$ -closed and  $f((G,A) \tilde{=} f(s-cl(G,A)))$ ,

We obtain  $s \pi g * b^* - cl(f(G,A)) \tilde{=} s \pi g * b^* - cl(f(s-cl(G,A)))$

Since  $f(s-cl(G,A))$  is soft  $\pi g * b^*$ -closed in  $(Y, \sigma, B)$ ,

$s \pi g * b^* - cl(f(s-cl(G,A))) = (f(s-cl(G,A)))$  for every soft set  $(A,E)$  of  $(X, \tau, E)$ .

Hence  $s \pi g * b^* - cl(f(G,A)) \tilde{=} f(s-cl(G,A))$  for every soft set  $(G,A)$  of a soft topological space  $(X, \tau, E)$ .

**5. Soft  $\pi g * b^*$ -  $T_{1/2}$  Spaces**

**Definition 5.1**

A soft topological space  $X$  is called

1. Soft  $\pi g * b^*$ - $T_{1/2}$  space if every soft  $\pi g * b^*$ -closed set is soft  $b$ -closed.
2. soft  $\pi g * b^*$ -space if every soft  $\pi g * b^*$ -closed set is soft closed.
3. soft  $T_{\pi g * b^*}$ -space if every soft  $\pi g * b^*$ -closed set is soft  $\pi g$ -closed.

**Theorem 5.2**

For a soft topological space  $(X, \tau, E)$  the following are equivalent

- (i)  $X$  is soft  $\pi g * b^*$ - $T_{1/2}$  space.
- (ii) Every singleton set is either soft  $\pi$ -closed or soft  $b$ -open.

**Proof**

To prove (i)  $\Rightarrow$  (ii) : Let  $X$  is soft  $\pi g * b^*$ - $T_{1/2}$  space.

Let  $(A,E)$  be a soft singleton set in  $X$  and assume that  $(A,E)$  is not soft  $\pi$ -closed.

Then clearly  $X - (A,E)$  is not soft  $\pi$ -open.

Hence  $X - (A,E)$  is trivially a soft  $\pi g * b^*$ -closed.

since  $X$  is soft  $\pi g * b^*$ - $T_{1/2}$  space,  $X - (A,E)$  is soft  $b$ -closed.

Therefore  $(A, E)$  is soft  $b$  – open.

To prove (ii)  $\Rightarrow$  (i): Assume that every singleton of  $X$  is either soft  $\pi g$  -closed or soft  $b$ -open.

Let  $(A, E)$  be a  $\pi g * b *$  -closed set. Let  $(A, E) \in \text{bcl}(A, E)$ .

**Case (i)**

Let the singleton set  $(F, E)$  be soft  $\pi$  -closed.

Suppose  $(F, E)$  does not belong to  $(A, E)$ .

Then  $(F, E) \in \text{bcl}(A, E) - (A, E)$ ,  $(F, E) \in (A, E)$ .

Hence  $\text{bcl}(A, E) \tilde{=} (A, E)$ .

**Case (ii)**

Let the singleton set  $(F, E)$  be soft  $b$ -open.

since  $(F, E) \in \text{bcl}(A, E)$ .

We have  $(F, E) \cap (A, E) \neq \emptyset$  implies that  $(F, E) \in (A, E)$ .

In both the cases  $\text{bcl}(A, E) \tilde{=} (A, E)$  or equivalently  $(A, E)$  is soft  $b$ -closed.

**Theorem 5.3**

For a soft topological space  $(X, \tau, E)$  the following are equivalent

- (i)  $X$  is soft  $\pi g * b *$  - $T_{1/2}$  space.
- (ii) For every soft subset  $(A, E)$  of  $X$ ,  $(A, E)$  is  $\pi g * b *$  -open iff  $(A, E)$  is soft  $b$ -open.

**Proof**

To prove (i)  $\Rightarrow$  (ii) : Let the soft topological space  $X$  be soft  $\pi g * b *$  - $T_{1/2}$  and

Let  $(A, E)$  be a soft  $\pi g * b *$  - open soft subset of  $X$ .

Then  $X - (A, E)$  is soft  $\pi g * b *$  - closed and so  $X - (A, E)$  is soft  $b$ -closed.

Hence  $(A, E)$  is soft  $b$ -open.

Conversely, let  $(A, E)$  be a soft  $b$ -open subset of  $X$ .

Thus  $X - (A, E)$  is soft  $b$ - closed.

Since every soft  $b$ -closed set is soft  $\pi g * b *$  - closed

Then  $X - (A, E)$  is soft  $\pi g * b *$  - closed.

Hence  $(A, E)$  is soft  $\pi g * b *$  - open.

To prove (ii)  $\Rightarrow$  (i) : Let  $(A, E)$  be a soft  $\pi g * b *$  - closed subset of  $X$ .

Then  $X - (A, E)$  is soft  $\pi g * b *$  - open.

By the hypothesis  $X - (A, E)$  is soft  $b$ -open.

Thus  $(A, E)$  is soft  $b$ -closed. Since every soft  $\pi g * b *$  - closed is soft  $b$ -closed,

$X$  is soft  $\pi g * b *$  - $T_{1/2}$  space.

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