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Fuzzy analysis for M/M/C finite capacity queueing model

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Abstract
In this paper we show that queueing theory can accurately model the flow of in-patient in hospital. Further Robust Ranking technique is used to find the expected mean queue length and waiting time in queue. The numerical results for L_q and W_q are calculated under fuzzy environment. Further, the analytical results are also numerically verified for L_q and W_q.

Keywords: Mean queue length, Robust ranking technique, waiting time in queue, fuzzy number

1. Introduction
The theory of fuzzy logic is based on the notation of relative graded membership, as inspired by the processes of human perception and cognition. Lotfi A. Zadeh published his first famous research paper on fuzzy sets in 1965. Fuzzy queueing models have been described by such researchers like Li and Lee (1989) [8]. They investigated analytical results for two fuzzy queues using a general approach based on Zadeh's extention principle. Nagi and Lee (1992) [10] proposed a procedure using α-cut and two variables simulations to analyse fuzzy queues, using parametric programming. Kao *et al* (1993) [7] proposed a general queuing systems in a fuzzy environment based on Zadeh's extension principle. Ritha and Lilly Robert (2009) [14] have discussed Application of fuzzy set theory to queues. Moreover Julia Rose Mary.K and Shanmuha priya (2014) [4] have discussed $FM^{x}_{(m,n)}/G_{sos}/1$ with fuzzy breakdowns and fuzzy multiple vacations. They derived the membership function of total average cost using Zadeh extension principle.

Ranking Technique has been discussed by research like Choobinesh and Li (1993) [2], Yager R.R (1981) [15], and Chen. S.P (2005) [1]. Nagoor Gani. A and V. Ashok Kumar (2009) [12] have analysed bulk arrival fuzzy queues with fuzzy outputs. Kao applied α-cut approach to reduce a fuzzy queue into family of crisp queue.

In literature we have many methods for converting fuzzy into crisp for which Robust ranking technique is the most successful and very convenient method for converting fuzzy to crisp values. Palpandi. B and Geedhamani. G (2013) [13] have analysed evaluations of performance measures of bulk arrival queue with fuzzy using Robust Ranking Technique. Julia Rose Mary. K and Angel Jenitta (2014) [3] have studied the cost analysis for bi level threshold policy and single vacation of an unreliable server with fuzzy parameters using Robust ranking technique successively. Julia Rose Mary. K and Majula Christina (2015) [5] studied Fuzzy parameters on total average cost for $M^{X}_{(m,N)}/M/1/BD/MV$. Recently, Julia Rose Mary. K and Pavithra. J (2016) [6] studied the FM/M (a, b)/1 with multiple working vacations queuing model in Robust Ranking Technique. In this paper we apply Robust Ranking Technique which helps to provide system characteristics of interest in terms of crisp values for bulk arrival queue with fuzzified Poisson arrival rate, service rate, number of parallel servers and finite capacity.

2. Model description
In queueing theory a discipline within the mathematical theory of probability, the M/M/C queue or (Erlang–C model) is a multi-server queueing model. In Kendall's notation it describes a system where arrivals form a single queue and are governed by a Poisson process, there are *c* servers and job service times are exponentially distributed.

It is a generalization of the M/M/1 queue which considers only a single server. The model with infinitely many servers is the M/M/C:N queue.
In an M/M/c/N queue only N customers can queue at any one time (including those in service). Any further arrivals to the queue are considered "lost". We assume that $K \geq c$. The model has transition rate matrix

$$Q=\begin{pmatrix} -\lambda & \lambda & & & & & & & & & \\ \mu & -(\mu+\lambda) & \lambda & & & & & & & & \\ & 2\mu & -(2\mu+\lambda) & \lambda & & & & & & & \\ & & 3\mu & -(3\mu+\lambda) & \lambda & & & & & & \\ & & & & \ddots & & & & & & \\ & & & & C\mu & -(C\mu+\lambda) & \lambda & & & & \\ & & & & & & \ddots & & & & \\ & & & & & & C\mu & -(C\mu+\lambda) & \lambda & & \\ & & & & & & & & \ddots & & \\ & & & & & & & & & c\mu & -c\mu \end{pmatrix}$$

On the state space $\{0, 1, 2...c,...,N\}$. In case where $c = N$, the M/M/c/N queue is also known as the Erlang–B model.This model is same as M/M/C: (∞/FIFO) except that the maximum number in the system is limited to N.
Here we have applied fuzzy Analysis in the above M/M/C:N/FIFO Queueing model particularly to the Mean queue length and Waiting time in the queue. Suppose the arrival rate λ with Poisson distribution, service rate μ for busy period, number of parallel servers c and finite capacity N. Then we represent fuzzy set $\bar{\lambda}, \bar{\mu}, \bar{c}, \bar{N}$

(i.e) $\bar{\lambda} = \left\{t, \theta(t) / \theta_{\bar{\lambda}}(t) / t \in S(\bar{\lambda})\right\}, \bar{\mu} = \left\{x, \theta_{\bar{\mu}}(x) / x \in S(\bar{\mu})\right\}, \ \bar{c} = \left\{d, \theta_{\bar{c}}(d) / u \in S(\bar{c})\right\}, \bar{N} = \left\{w, \theta_{\bar{N}}(w) / w \in S(\bar{N})\right\},$

Here $\theta_a(b)$ and S(a) denote the membership function and support of a where $a = \bar{\lambda}, \bar{\mu}, \bar{c}, \bar{N}$ are fuzzy numbers and b=t,x,d,w are crisp values corresponding to arrival rate, service rate for busy period, number of parallel servers and Finite capacity respectively.

On the basis of the concept of α-cut we develop a mathematical programming approach for deriving the α-cuts of $\bar{\lambda}, \bar{\mu}, \bar{c}, \bar{N}$ as crisp intervals which are given by,

$$\bar{\lambda}(\alpha) = \left\{t \in T / \theta_{\bar{\lambda}}(t) \geq \alpha\right\}, \ \bar{\mu}(\alpha) = \left\{x \in X / \theta_{\bar{\mu}}(x) \geq \alpha\right\}, \ \bar{c}(\alpha) = \left\{d \in D / \theta_{\bar{c}}(d) \geq \alpha\right\}, \ \bar{N}(\alpha) = \left\{w \in W / \theta_{\bar{N}}(w) \geq \alpha\right\}$$

where 0<α≤1. Hence a fuzzy queue can be reduced to a family of crisp queues with difference α-cuts as $\{\lambda(\alpha)/0<\alpha\leq 1\}$, $\{\mu(\alpha)/0<\alpha\leq 1\}$, $\{c(\alpha)/0<\alpha\leq 1\}$, $\{N(\alpha)/0<\alpha\leq 1\}$.

Let the confidence interval of the fuzzy sets $\lambda(\alpha), \mu(\alpha), c(\alpha), N(\alpha)$ be

$[l_\lambda(\alpha) \quad u_\lambda(\alpha)]$, $[l_\mu(\alpha) \quad u_\mu(\alpha)]$,

$[l_c(\alpha) \quad u_c(\alpha)]$ and $[l_N(\alpha) \quad u_N(\alpha)]$.

Then the expected queue length (Lq) is given by,

$$L_q = \frac{(\lambda/\mu)^{c+1} p_0}{c\ c!}\left[\frac{1-\rho^{N-C+1}-(N-c+1)(1-\rho)\rho^{N-c}}{(1-\rho)^2}\right]$$

$$where\ \rho = \frac{\lambda}{c\mu},$$

$$p_0 = \left[\sum_{n=0}^{c-1}\frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^c}{c!}\left(1-\left(\frac{\lambda}{c\mu}\right)^{N-c+1}\left(\frac{c\mu}{c\mu-\lambda}\right)\right)\right]^{-1}$$

In the above formula by applying the fuzzy variable for arrival rate, service rate for busy, number of parallel servers and finite capacity then we get,
The expected queue length (Lq) is given as,

$$L_q = \frac{(t/x)^{d+1} p_0}{d\, d!}\left[\frac{1-\rho^{w-d+1}-(w-d+1)(1-\rho)\rho^{w-d}}{(1-\rho)^2}\right]$$

Similarly, waiting time in the queue as,

$$w = \frac{L_q}{\lambda} = \left[\frac{p_c p_0 \rho\left[1-\rho^{N-C+1}-(N-c+1)(1-\rho)\rho^{N-c}\right]}{\lambda(1-\rho)^2}\right]$$

Now by applying the Robust Ranking Technique to the required formula.

3. Numerical example

In this section numerical results are calculated under fuzzy environment.

3.1 Fuzzy environment

Consider fuzzy M/M/C:N/FIFO queueing system. The corresponding parameters such as arrival rate, service rate for busy, number of parallel servers and finite capacity are fuzzy numbers. Let us consider the parameters us, $\lambda = [0.15, 0.2, 0.25, 0.3]$, $\mu = [0.01, 0.02, 0.03, 0.04]$ whose intervals of confidence are $[0.15+\alpha \quad 0.3-\alpha]$, $[0.01+\alpha \quad 0.04-\alpha]$ respectively, Now we evaluate R(0.15,0.2,0.25,0.3) by applying Robust Ranking Method.

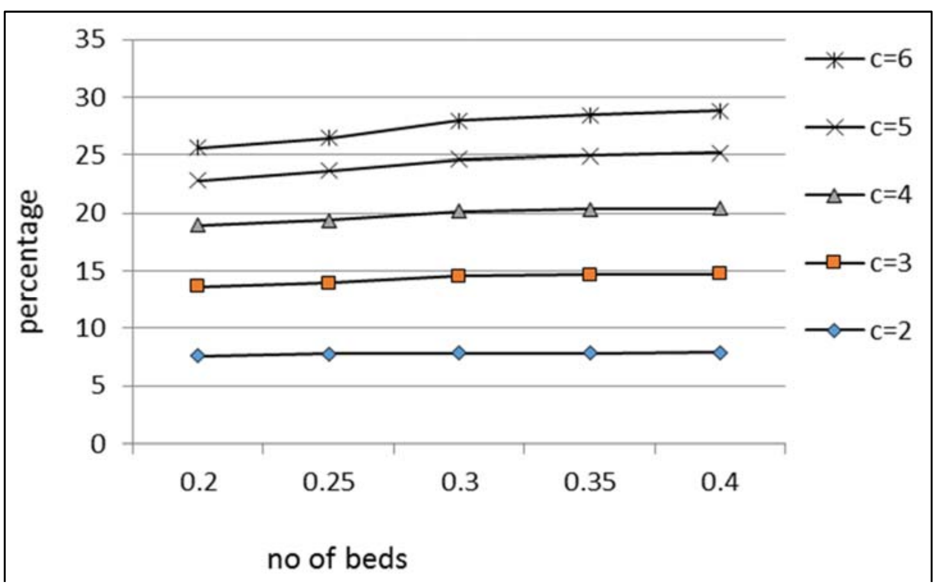
$$R(\bar{\lambda}) = R(0.15, 0.2, 0.25, 0.3) = \int_0^1 0.5(0.15+0.3)d\alpha = \int_0^1 0.5(0.55)d\alpha = 0.2$$

$$R(\bar{\mu}) = R(0.01, 0.02, 0.03, 0.04) = \int_0^1 0.5(0.01+0.04)d\alpha = \int_0^1 0.5(0.05)d\alpha = 0.02$$

By proceeding similarly the Robust Ranking indices for the fuzzy numbers $\bar{\lambda}, \bar{\mu}, \bar{c}, \bar{N}$ are calculated as $R(\bar{\lambda}) = 0.2, R(\bar{\mu}) = 0.02$

Table 1: Mean queue length for M/M/C: N/FIFO model.

λ/c	0.2	0.25	0.3	0.35	0.4
2	7.5683	7.7793	7.8410	7.8554	7.8888
3	6.0812	6.1801	6.6741	6.8060	6.8171
4	5.3345	5.3909	5.6336	5.6735	5.6901
5	3.87	4.3121	4.4964	4.5991	4.7577
6	2.7382	2.8044	3.3393	3.5388	3.6385

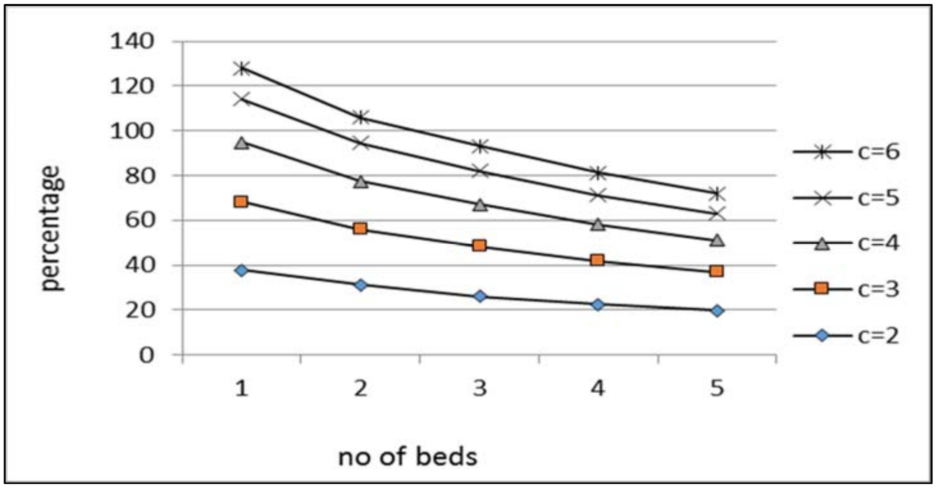


Graph 1: Lq versus (c, λ)

From the table and figure. 1, we conclude that, the mean queue length L_q increases when the arrival rate increases. Also, we find that the mean queue length (L_q) decreases when the number of beds increases.

Table 2: Waiting time in queue for M/M/C:N/FIFO model.

λ / C	0.2	0.25	0.3	0.35	0.4
2	37.84	31.11	26.14	22.44	19.72
3	30.41	24.72	22.25	19.45	17.04
4	26.67	21.56	18.77	16.21	14.23
5	19.35	17.25	14.98	13.14	11.89
6	13.69	11.22	11.13	10.11	9.09



Graph 2: Wq versus (c, λ)

From the table (2) and figure (2) we observe that w_q decreases for increasing the arrival rate and decreases for increasing number of beds.

4. Conclusion

Many of the queuing systems have been analyzed by using the fuzzy set theory which provides wider application in many fields. Thus in this paper by applying the techniques of α-cut and Zadeh's extension principle. The arrival rate, service rate, number of parallel numbers and finite capacity are fuzzy number which are more realistic and general in nature. numerical results are calculated by using Robust Ranking Technique. We find that by applying Robust Ranking Method the solution of fuzzy problem is more efficient.

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