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Financial modeling by ordinary and stochastic differential equations

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Abstract
The valuation and its sensitivity to interest rate change is defined as an ordinary differential equation. In this paper we are the ordinary and stochastic equations in the finance will we described. A noise term is added to ordinary differential equations in order to use them as a powerful mathematical tool for risky assets.

Keywords: Sensitivity, valuation, equations

Introduction
Bonds are one of the instruments that firms and public administrations may use to fund their activities. They are debt instruments which, unlike stocks, do not imply any ownership of a firm on the part of the buyer. Basically, the buyer of a bond lends some money to the issuer, over some time span ending at bond maturity. At maturity the issuer will pay the bond owner an amount of money corresponding to the face value, also called the par value, of the bond. In the simplest bonds, coupons are fixed and expressed as a percentage of face value; coupons are usually paid annually or semi-annually.

There is another class of bonds, which just promise the payment of face value at maturity. They are called zero-coupon bonds and are typically characterized by shorter maturities.

Ordinary differential equations are used for moving (changing) phenomena and by solution of ODEs; it is possible to understand behavior of them. Because of variety of these phenomena, we need different models that some of them are stochastic and need stochastic differential equations to explain their behaviors. To understand importance of differential equations, some financial quantities will be modeled by ordinary or stochastic differential equations [1–5].

Analysis
Profit rate of zero coupon bonds with face value of F, one-year maturity and price
Of P equals to

$$r = \frac{F\ P}{P}$$

Then we have,

$$P = \frac{F}{l + r}$$

This equation is a valuation formula and it is not more than a simple discount formula. Consequently periodic cash flows C1, C2… Cn inn years results in following equation for the present value of bond,

$$PV = \frac{c_1}{1 + r} + \frac{c_2}{(1 + r)^z} + \cdots + \frac{c_n + F}{(1 + r)^z}$$

$$= \sum_{k=1}^{n} \frac{C_k}{(1 + r)^x} + \frac{F}{(1 + r)^x}$$

Where C_k is coupon paid at year number k. If there are m payments per year at regular time intervals, we have

$$PV = \sum_{k=1}^{mn} \frac{C_k^+}{\left(1+\frac{r}{m}\right)^x} + \frac{F}{\left(1+\frac{r}{m}\right)^{mn}}$$

$$= \sum_{k=1}^{n} \frac{C_k^+}{\left(1+\frac{r}{m}\right)^k} + \frac{F}{\left(1+\frac{r}{m}\right)^{mn}}$$

Or,

$$PV = \sum_{k=1}^{mn} \frac{C_k}{\left(1+\frac{r}{m}\right)^x}$$

Where

$$C_k = C_k^+ \; and \; C_m = C_n^+ + F.$$

As we explained, by change of interest rate, r, bond prices, P, will be changed so we can say bond prices are a function of interest rate:
P = P(r)
Now let us define duration of the stream as

$$D = \frac{PV(t_0)t_0 + PvV(t_1)t_1 + \cdots + PV(t_n)t_n}{PV}$$

Where PV is the present value of the whole stream and PV (t_i) is the present value of cash flow C_i occurring at time t_i, i = 0, 1, n. In some sense, the duration looks like a weighted average of cash flow times, where the weights are the present values of the cash flows. Remember that for zero coupon bonds, D is equal to T, where it is time to maturity.
Considering a generic bond and employing the yield as the discount rate for computing the present values develop the following equation

$$t_0 = 0, t_1 = \frac{1}{m}, t_k = \frac{k}{m}$$

$$P(t_k) = \frac{C_k}{\left(1+\frac{1}{m}\right)^k}$$

Then, the derivative of the price with respect to r is compute:

$$\frac{dp}{dr} = \frac{d}{dr}\left(\sum_{k=1}^{n} \frac{C_x}{\left(1+\frac{1}{m}\right)^k}\right)$$

$$= \sum_{k=1}^{n} C_k \frac{d}{dr}\left(\frac{C_x}{\left(1+\frac{r}{m}\right)^k}\right)$$

$$= -\sum_{k=1}^{n} C_k \frac{k}{m} \frac{1}{\left(1+\frac{r}{m}\right)^k} \frac{1}{\left(1+\frac{r}{m}\right)}$$

Defining D_m as $D_m = \frac{D}{1+\frac{r}{m}}$ results

$$\frac{dp}{dr} = -D_m p$$

Which is an ordinary differential equation. MATLAB can compute D_m using cfdur function as follow:

```
>> [Duration, Mod_ Duration] =cfdur (Chash Flow, Yield)
```

This returns both Macauley and modified duration. Now, a numeric estimation for above equation can be estimated by Taylor series expansion

$$p(r_{i+1}) = p(r_i) + (r_{i+1} - r_i)p'(r_i)$$

$$+ \frac{(r_{i+2} - r_i)}{2|} p^n(r_i) + \cdots$$

Using linear approximation results,

$$p(r_{i+1}) \cong p(r_i) + \delta\lambda p^i(r^i)$$

$$p^{\prime(r_i)} - \frac{dp_i}{dr_i}$$

Using linear approximation results,

$$\frac{\delta p_i}{\delta r_i} = -D_i P_i$$

Or

$$\delta p = D_m P_i \delta r$$

In order to obtain a higher order equation higher order derivatives needed to be calculated with respect to r,

$$\frac{d^2p}{dx^2} = -\sum_{k=1}^{n} C_k \frac{k(k+1)}{m^2 \left(1+\frac{r}{m}\right)^m \left(1+\frac{r}{m}\right)^2}$$

Assuming,

$$C = \frac{1}{P\left(1+\frac{r}{m}\right)^2} \sum_{k=1}^{n} \frac{k(k+1)}{m^2} \frac{c_k}{\left(1+\frac{r}{m}\right)^k}$$

Where, C is called convexity by cfconv is MATLAB function can be used to calculate this quantity

```
>> Conv= cfconv(Cash flow, Yield)
```

Second order numerical approximation can be computed by Taylor series expansion:

$$p(r_{i+1}) = p(r_i) + (r_{i+1} - r_i)p'(r_i)$$

$$+ \frac{(r_{i+2} - r_i)}{2|} p^n(r_i) + \cdots$$

By means of first order numerical approximation and above equation we have

$$\delta p \cong -D_m p^{\delta} r + \frac{Pc}{2}(\delta r)^2$$

Which is a second order approximation by duration and convexity.

Stock price changes: stochastic Differential equations

Considering environmental effects as a stochastic parameter in the equation, the result model becomes more realistic. Hence, modeling of stock price changes must be done based on realistic assumptions explaining dynamics of stock price behavior. For instance with a 1$ stock, we have to be able to predict its price distribution in tomorrow, one week later or more. In practice, stock price behavior is similar to a stochastic variable, in general, stochastic process (Fig).

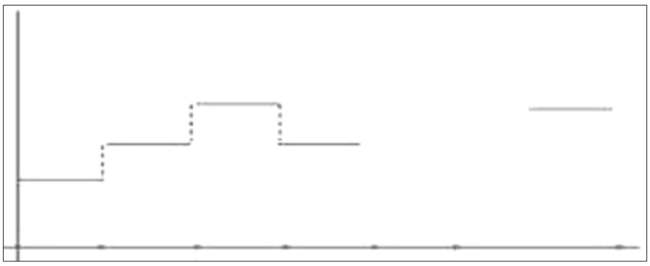

Fig 1: Sample Stock Price behaviour

Numerical results

MATLAB financial toolbox contains various functions for analysis of cash flows, sensitivity of bonds value to interest rate change, ODEs and of course SDEs. Consider for instance the cash flow stream corresponding to a bond maturing in five years, with face value 100 and an 8% coupon rate. We can show its cash flows pattern in MATLAB by following function.

>>cash flow= [0 8 8 8 8 108]

It is obvious that present value of this stream if we discount it by an interest rate 8% is equal to face value, 100$. Now if interest rate increases to 8.8% present value will decrease to 96.8 $(equals to 3.12$ decrease).

>>bond_price1=pvvar(cash_flow,0.088)
bond_price1=96.8721

So we have 3.5$ decrease in bond value. Now we
May compute the modified duration and the convexity

>> [duration modified duration] =cfdur (cash flow, 0.08)
Duration=5.3121

Modified duration =4.9186
>> Convexity=cfconv (cash flow, 0.08)
Convexity=30.1551

>>-modified duration * bond price1*0.01
Ans =-3.8118

>>-modified duration*bond_price1*0.008+ …
0.5*convexity*bond_price1*(0.008) ^ (2)

Ans =-3.7183

As you can see, 3.7 is a good approximation for 3.5$ which is real decrease of bond value.

Conclusion

There is no closed form solution for most of the complicated SDEs and we have to use numerical methods to solve them, which can be second suggestion for future studies.

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