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Inter relationship between the notion of fixed point, Kutasth Vindu and Vedanta philosophy

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Abstract
Fixed point theory is a rich, interesting and exciting branch of mathematics. It is relatively young but fully developed area of research. Study of the existence of fixed points falls within several domains such as functional analysis, operator theory, general topology. Fixed points and fixed point theorems have always been a major theoretical tool in fields as widely apart as topology, mathematical economics, game theory, approximation theory and initial and boundary value problems in ordinary and partial differential equations. Moreover, recently, the usefulness of this concept for applications increased enormously by the development of accurate and efficient techniques for computing fixed points, making fixed point methods a major tool in the arsenal of mathematics. The theory of fixed points is concerned with the conditions which guarantee that a map $T : X \rightarrow X$ of a topological space X into itself. admits one or more fixed points that is, points x in X for which x = Tx. The Sanskrit word kutastha means "that which remains unchanged". Vedanta, the most prominent school of Hinduism, is a philosophical system that is concerned with questions such as 'Who am I?', 'What is this Universe?' 'How am I related to the Universe?' In this paper we discuss fixed point notion, kutastha vindu and Vedanta Philosophy and also to find their interrelationship used in Hindu mythological books.

Keywords: Fixed point, Kutasth Vindu, vedanta

1. Introduction
Mathematics is the science of numbers and space. It is also defined as the science of measurement, quality and magnitude. It is derived from Greek word "máthēma", means knowledge, study and learning of numbers. In Indian mathematics, it is known as Ganita (गणित) which in Sanskrit means is counting and calculation of number. The importance of mathematics in Hindu system can be understood by knowing the following sloka used in Vedanga Jyotisa as:

यथा शिखा मयूराणां नागानां मणयोयथा।
तद्वद् वेदांग शास्त्राणां गणितं मूर्धनि संस्थितम्।। (R-VJ 35; Y – VJ 4)

In English translation, we have

Yatha Sikha Mayuranaam, Naganam Manayo Yatha |
Tatvd Vedanga Shastranam, Ganitham Murdhani Sthitham ||

This means"Like the comb of peacocks and the crest-jewels of the serpents, so does the lore of Jyotisa (Ganita or Computation) stand at the head of all the lores forming the auxiliaries of Vedas". This shows Ganita is the essence of all the sciences.
The theory of fixed points belongs to topology, a part of mathematics created at the end of nineteenth century and makes extensive use of such topological notions as continuity, compactness and the degree of a mapping. The famous French mathematician H. Poincare (1854-1912) was the founder of the fixed point approach. He had deep insight into its future importance for problems of mathematical analysis and took an active part in its development

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Analysis is an important branch of mathematics that deals with continuous change and with certain general types of processes which have emerged from the study of continuous change, such as limits, differentiation, and integration. Since the discovery of the differential and integral calculus by Isaac Newton and Gottfried Wilhelm Leibniz at the end of the 17th century, analysis has grown into an enormous and central field of mathematical research, with applications throughout the sciences and in areas such as finance, economics, and sociology.

It has been classified into linear and nonlinear parts. The linear analysis deals with infinite dimensional topological vector space and their operators acting upon them. Nonlinear analysis since 1960 deals with the extension of linear things to various kinds of nonlinear operators. Nonlinear analysis is characterized by a remarks mixture of analysis, topology and applications.

The theory of fixed points belongs to topology, a part of mathematics created at the end of nineteenth century and makes extensive use of such topological notions as continuity, compactness and the degree of a mapping. The famous French mathematician H. Poincare (1854-1912) was the founder of the fixed point approach. He had deep insight into its future importance for problems of mathematical analysis and took an active part in its development. The metric contraction principles and metric fixed point theory have their origins in the successive approximation approach for demonstrating the existenceand uniqueness of differential equation solutions. This technique is related to the names offamous nineteenth-century mathematicians such as Cauchy, Liouville, Lipschitz, Peano, and most notably, Picard. Actually, the iterative procedure used in the proof of the contraction theorem is due to Picard iterates. Brouwer in [5], proved the following theorem in 1912.

Currently, Fixed Point Theory has been classified into three major areas:
1. Topological Fixed Point Theory [12],
2. Metric Fixed Point Theory, and
3. Discrete Fixed Point Theory

Which are defined respectively by the discovery of following three major theorems:
1. **Brouwer's Fixed Point Theory:** It states that any continuous self mapping f defined on a compact convex set has a fixed point,
2. **Banach Fixed Point Theory:** Let (X, d) be a metric space. Then a contraction mapping T on X has a unique fixed point.
3. **Tarski's Fixed Point Theory:** Let L be a complete lattice and let $f: L \to L$ be an order- preserving function. Then, the set of fixed points of f in L is also a complete lattice.

The earliest fixed point theorem is that of Brouwer [5], proved that a continuous self mapping T of the closed unit ball R^n has at least one fixed point, i.e a point x such that $Tx = x$. Several proof of this historic result can be found in the existing literature.

Another fundamental result after Brouwer's fixed point theorem was given by polish mathematician Banach [2]. He proved a theorem, which ensures under appropriate conditions, the existence and uniqueness of a fixed point. This result is popularly known as "Banach fixed point theorem" or the "Banach Contraction Principle". It states that a contraction mapping of a complete metric space into itself has a unique fixed point. It is the simplest and one of the most versatile results in fixed point theory. Being based on an iteration process, it can be implemented on a computer to find the fixed point of a contractive map, it produces approximations of any required accuracy. Due to its applications in various disciplines of mathematics and mathematical sciences, the Banach contraction principle has been extensively studied and generalized on many settings and fixed point theorems have been established.

For example, consider the equation: $f(x) = x^3 - 2x - 1 = 0$ for which $f(2) = 3$ & $f(-2) = -5$ and $f(2).f(-2) = -15 < 0$. So, using the Initial Value Theorem (IVT) [9], the solution of given equation lies in [—2, 2]. This equation can be written as function equation $x = Tx = \frac{1}{x^2 - 2}$. Then, for $x = -1$, we have $T(-1) = -1$. Therefore, by definition, $x = -1$ is the fixed point of T which is also the solution of given equation.

2. Kutasth Vindu and Vedanta Phylosophy

Gita is the divine discourse spoken by the Supreme Lord Krishna himself and is the most popular and well known of all the sacred scriptures from ancient India. Always being revered as a true source of spiritual knowledge, it reveals the purpose and goal of human existence [1]. On the other hand, "Vedanta" is a combination of two words: "Veda" which means "knowledge" and "anta" which means "the end of" or "the goal of." It is one of the world's most ancient spiritual philosophies and one of its broadest, based on the Vedas, the sacred scriptures of India. It is the philosophical foundation of Hinduism; but while Hinduism includes aspects of Indian culture, Vedanta is universal in its application and is equally relevant to all countries, all cultures, and all religious backgrounds [3]. Vedanta, the most prominent school of Hinduism, is a philosophical system that is concerned with questions such as 'Who am I?', 'What is this Universe?' 'How am I related to the Universe?' The roots of the Vedanta philosophy are in the Upanishads, which are the concluding portions of the Vedas. It literally means the end of the Vedas. There are three main concepts in Vedanta. The first is the Brahman or Ishvara, which is the ultimate reality. The second is Atman or the individual souls. The third is the Prakriti or the physical world. When it comes to the relationship between the three, there are many views in Vedanta, each represented by a school of thought.

We have some following slokas dealing with Kutasth vindu in Gita [3, 11] and Panchadashi as [1]

Sloka 2.1

ये त्वक्षरमनिर्देश्यमव्यक्तं पर्युपासते ।
सवत्रगमचिन्तयज्ञ च कूटस्थमचलं ध्रुवम् ॥ [Bhagvat Gita /12 / 3]

In English translation, we have

Ye tvaksaramanirdesyamavyaktam paryupasate /
sarvatra-gam acintyan ca kutasthamacalam dhruvam//

That means, All country, time, objects, persons have ever living, never destructive, always determined, constant and never changing, that is constant element, knowledge, Kutastha. This is the same heavy metal remains the same through many things are reshaped and changed on this same metal. The universe gets everything from the same source, knowledge that is unchangeable.

Sloka 2.2

द्राविमौ पुरुषौ लोके क्षरश्चाक्षर एव च ।
क्षर: सर्वाणि भुतानी कूटस्थोऽक्षर उच्यते ॥ [Bhagvat Gita/15/ 16]

In English translation, we have

Dvvimau purusau loke / karackara eva ca /
Kasarh sarvibhtni bhutani kutassthokara ucyate //

That means, while describing destroyable and undestroyable men, undestroyable person is always the same, constant and becomes cause of creativity of destroyable person. All living things are destroyable but never changing and ever living soul is undestroyable element inside those living things. This soul is called Kutasth. The constant point, which we have raised as an issue, is exactly the same like the soul. The constant point is Kutastha point. Constant point is generally common and simple but Kutastha point includes conscience. Similarly Panchadashi is a simple yet comprehensive manual of Advaita Vedanta written in the fourteenth century A.D. Pancha means five and dasi means ten, are the total fifteen chapters divided into three quintets the three aspects of Brahman, Sat (Vivek), chit (Consciousness) and Ananda (Bliss) aspects of Reality. It elaborates Advaita (view of Brahman and its derivatives), Consciousness, Jiva, Maya, Prakriti (prakṛti, primary substance, Mahat (Omnipotent, Matter), Buddhi (Intellect), Ahamkara (Ego), Avidya (Ignorance), and ānanda (Bliss) [6].
This is very much like the three aspects of Brahman – sat (existence), cit (consciousness) and ananda (bliss), respectively.

Sloka 2.3

द्विगुणीकृतचैतन्ये जन्मनाशानुभूतित: ।
अकूटस्थं तदन्यक्तुम कुटस्थमविकारत:॥ [Panchadashi /8/24]

In English translation, we have

Dviguniktitchaitanye janmanashanubhutihah /
Akutastham tadanyantu kutasthamvikartah //

That means, Conscience (chidavas) changes in awakening stage and sleeping stage but self-consciousness (Kutastha) never changes. It neither dies nor changes. So it is undividable, unchangeable and immortal.

Sloka 2.4

सामानाधिकरण्यस्य बाधार्थत्वं निराकृतम्।
प्रयत्नतो विवरणे कूटस्थस्य विवक्षया॥ [Panchadashi /8/46]

In English translation, we have

Samanadhikrnysy badharthtvng nirakritam /
Prayatnato vivrane kutasthasy vivikshaya //

That means, Many Vedanta Acharyas believe that the ultimate aim of Vedanta is to know 'All the world as Brahma (Kutastha) or Me myself is Brahma (Kutastha). If we accept this theory then Kutastha can be changeable as man and the world are changeable. So to clarify this confusion the writer of Vivarana scripture, Shreeprakasatmayati, one of the Vedanta philosophers criticizes the theory of obstacle for the sameness (Vadhasamanadhikaranya- I=You=Brahma (Kutastha)). He opines that Aham (I) means a conscious being or Chidavas. Therefore I cannot be Brahma/Atma/Kutastha. He says that I is not the Kutastha but absence of I or unification of I with Brahma is the Kutastha.

3. Conclusion

The classical notion of fixed point introduced by Joseph Liouville in 1837, Charles Emile Picard in 1890 and S. Banach in 1922 plays very important role to provide solutions of various functional equations in applied mathematics. Also, the notion Kutastha Vindu as mathematical philosophical terminology has been used long ahead in Hindu mythological books, by Hindu gurus and different Vedanta philosophy. Our study indicates that both the terms "fixed point" and "Kutasth Vindu" are invariant under self-mapping. Similarly, Vedanta asserts that the goal of life is to realize and to manifest our own divinity. This divinity is our real nature, and its realization is our birth right. We are moving towards this goal as we grow with knowledge and life experiences. It is inevitable that we will eventually, either in this or in future lives, discover that the greatest truth of our existence is our own divine nature. Recently, Rani *et al.* [13] has beautifully applied this approach to find Vedic fractals. This also indicates the interrelationship between these two important notions. The topological structure of sudrashan chakra and Chakrabueha in Mahabharatha shows the iteration pattern like in classical mathematics and so we have the concept of fixed point.

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