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A study on contra NWG-closed and NWG-open maps

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Abstract
This paper introduces a weaker form of closed map called Contra Nwg-closed map in Nano topological spaces. A few of its properties are analyzed. The equivalent condition for a function to be a contra Nwg-closed map is obtained. Further Contra Nwg-open map is introduced and some of its characteristics are investigated.

Keywords: Nano topological space, Nano continuous function, Nano contra continuous function, Contra Nwg-closed map and Contra Nwg-open map

1. Introduction

In 1994 J. Dontchev [5] introduced contra-continuity which is a stronger form of LC-continuity in general topological space. C.W Baker [1] introduced the notion of Contra open function and Contra closed function. Caldas *et al.* [3] studied contra pre semi open maps. Dhanis Arul Mary [4] analysed contra nano generalized b-closed maps in Nano topological space. Zdzislaw Pawlak [10] discussed the applications of rough set theory with an example. Lellis Thivagar *et al* [6] defined a topology called Nano topology in terms of approximations and boundary region of a universal set using equivalence relation on it. He [7] established Nano continuity using Nano interior. Nagaveni and Bhuvaneswari [8, 2] applied the concept of weakly generalization in Nano topology and studied Nwg-closed map. In this paper contra Nwg-closed map is introduced and few of its properties are studied. Also Contra Nwg-open map is analysed.

Throughout this paper $(U, \tau_R(X))$ is a Nano Topological space with respect to X Where $X \subseteq U$, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R. $(V, \tau_{R'}(Y))$ is a Nano Topological space with respect to Y Where $Y \subseteq V$, R' is an equivalence relation on V, V/R' denotes the family of equivalence classes of V by R'. $(W, \tau_{R''}(Z))$ is a Nano Topological space with respect to Z Where $Z \subseteq W$, R'' is an equivalence relation on W, W/R'' denotes the family of equivalence classes of W by R''.

2. Preliminaries

This section is to recall some definitions and properties which are useful in this study.

Definition: 2.1 [6] Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another.The pair (U,R) is said to be the approximation space. Let $X \subseteq U$,

1. The Lower approximation of X with respect to R is the set of all objects,which can be for certain classified as X with respect to R and is defined by $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$. Where R(x) denotes the equivalence class determined by x

2. The upper approximation of X with respect to R is the set of all objects,which can be possibly classified as X with respect to R and is defined by $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects,which can be classified neither as X nor as not- X with respect to R and is defined by $B_R(X) = U_R(X) - L_R(X)$.

Definition: 2.2 [6] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms.

1. U *and* $\phi \in \tau_R(X)$.
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ forms a topology on U called as the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets.Elements of $[\tau_R(X)]^c$ are called Nano closed sets with $[\tau_R(X)]^c$.

Definition: 2.3 [6] If $(U, \tau_R(X))$ is a Nano Topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$ then the Nano interior of A is defined as the union of all Nano open subsets of A and it is denoted by NInt(A).Nano interior is the largest Nano open subset of A.

Definition: 2.4 [6] The Nano closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by Ncl(A).It is the smallest Nano closed set containing A.

Definition: 2.5 [6] Let $(U, \tau_R(X))$ be a Nano Topological space with respect to X and $A \subseteq U$. Then A is said to be

(i) Nano semi- open if $A \subseteq NCl(NInt(A))$
(ii) Nano pre- open if $A \subseteq NInt(NCl(A))$
(iii) Nano α - open if $A \subseteq NInt(NCl(NInt(A)))$
(iv) Nano Regular open if $A = NInt(NCl(A))$

Definition:2.6 [8] Let $(U, \tau_R(X))$ be a Nano Topological space. A subset A of $(U, \tau_R(X))$ is called Nano weakly generalized closed (briefly Nwg-closed) set if $Ncl(N\,\text{int}(A)) \subseteq V$ where $A \subseteq V$ and V is Nano open.The complement of Nano weakly generalized closed set is Nano weakly generalized open set.

Definition: 2.7 The map $f : (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ is called

(i) Nwg-open map [2] if the image of every Nano open set in U is Nano weakly generalized open in V.
(ii) Nwg-closed map [2] if the image of every Nano closed set in U is Nano weakly generalized closed in V.

Definition: 2.8 Nano weakly generalized interior of a subset A is the union of all the Nwg-open sets contained in A. $\text{NInt}_{wg}(\text{A}) = \cup \{\text{B : B is Nwg - open sets such that B} \subseteq \text{A}\}$

Definition: 2.9 The intersection of all the Nwg-closed set containing A is called Nwg-closure of A. $\text{NCl}_{wg}(\text{A}) = \cap \{\text{B : B is Nwg - closed sets such that A} \subseteq \text{B}\}$

3. Contra Nwg- closed Maps

In this section we define the map called Contra Nwg- closed map and study some of its properties.

Definition: 3.1 The map $f : (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ is Contra Nwg-closed on U if the image of every Nano closed set in U is Nano weakly generalized open set in U.

Example: 3.2 Let $U = \{a, b, c, d, e\}$ with $U / R = \{\{a\}, \{b\}, \{c\}\{d\}, \{e\}\}$ and $X = \{a, b\}$. Then the Nano topology is $\tau_R(X) = \{U, \phi, \{a, b\}\}$.

Let $V = U = \{a, b, c, d, e\}$ with $V / R^{'} = \{\{d, e\}, \{a, c\}, \{b\}\}$ and $Y = \{a, b, c\}$. Then the Nano topology is $\tau_R^{'}(Y) = \{V, \phi, \{a, b, c\}\}$.

Define $f:(U,\tau_R(X)) \to (V,\tau_R{}'(Y))$ as $f(a)=d, f(b)=e, f(c)=a, f(d)=b, f(e)=c$. f is contra Nwg-closed map.

Remark: 3.3 Composition of two contra Nwg closed map need not be contra Nwg closed as shown in the following example.

Example: 3.4 Let $U=\{a,b,c,d,e\}$ with $U/R=\{\{a,c\},\{b\},\{d\},\{e\}\}$ and $X=\{a,b\}$. Then the Nano topology is $\tau_R(X)=\{U,\phi,\{b\},\{a,c\},\{a,b,c\}\}$.

Let $V=\{a,b,c,d,e\}$ with $V/R'=\{\{a,b\},\{c,e\},\{d\}\}$ and $Y=\{a,b\}$. Then the Nano topology is $\tau_R{}'(Y)=\{V,\phi,\{a,b\}\}$.

Let $W=\{a,b,c,d,e\}$ with $W/R''=\{\{a,c\},\{e\},\{b,d\}\}$ and $Z=\{c,e\}$. Then the Nano topology is $\tau_R{}'(Z)=\{W,\phi,\{e\},\{a,c\},\{a,c,e\}\}$.

Define $f:(U,\tau_R(X)) \to (V,\tau_R{}'(Y))$ as $f(a)=c, f(b)=d, f(c)=e, f(d)=a, f(e)=b$ and $g:(V,\tau_R{}'(Y)) \to (W,\tau_{R''}(Z))$ as $g(a)=b, g(b)=d, g(c)=c, g(d)=e, g(e)=a$. Here f and g are contra Nwg-closed maps but their composition is not contra Nwg-closed map $g(f(\{a,c,d,e\}))=g(\{a,b,c,e\})=\{a,b,c,d\}$ which is not Nwg-open.

Remark: 3.5 Contra Nwg-closed map and Nwg-closed map are independent.

Example: 3.6 Let $U=\{a,b,c,d,e\}$ with $U/R=\{\{a,c\},\{b\},\{d\},\{e\}\}$ and $X=\{a,b\}$. Then the Nano topology is $\tau_R(X)=\{U,\phi,\{b\},\{a,c\},\{a,b,c\}\}$.

Let $V=\{a,b,c,d,e\}$ with $V/R'=\{\{a\},\{b,d\},\{c,e\}\}$ and $Y=\{c,e\}$. Then the Nano topology is $\tau_R{}'(Y)=\{V,\phi,\{e\},\{a,c\},\{a,c,e\}\}$.

Define $f:(U,\tau_R(X)) \to (V,\tau_R{}'(Y))$ as $f(a)=c, f(b)=e, f(c)=a, f(d)=d, f(e)=b$. Then f is Nwg- closed map but not Contra Nwg-closed map since $f(\{b,d,e\})=\{b,d,e\}$ is not Nwg-open set in V.

Example: 3.7 Let $U=\{a,b,c,d,e\}$ with $U/R=\{\{a,c\},\{b\},\{d\},\{e\}\}$ and $X=\{a,b\}$. Then the Nano topology is $\tau_R(X)=\{U,\phi,\{b\},\{a,c\},\{a,b,c\}\}$.

Let $V=\{a,b,c,d,e\}$ with $V/R'=\{\{a\},\{b\},\{c,d\},\{e\}\}$ and $Y=\{a,b\}$. Then the Nano topology is $\tau_R{}'(Y)=\{V,\phi,\{a,b\}\}$.

Let $f:(U,\tau_R(X)) \to (V,\tau_R{}'(Y))$ be a function defined as $f(a)=c, f(b)=d, f(c)=e, f(d)=a, f(e)=b$, then f is Contra Nwg-closed map. Since $f(\{d,e\})=\{a,b\}$ is not Nwg-closed in V f is not Nwg-closed function.

Theorem: 3.8 Let f be a function from Nano topological space U to Nano topological space V and if the only Nano open set containing the complement of image of every Nano closed set of U is V, then f is contra Nwg-closed map.

Proof: Let A be a Nano closed set in U and V is the only Nano open set such that $(f(A))^c \subseteq V$. Then $Ncl\left(N\text{int}\,(f(A))^c\right) \subseteq V$. (i.e) $(f(A))^c$ is Nwg-closed in V. f(A) is Nwg-open set. Then f is Contra Nwg-closed map.

Theorem: 3.9 Let f be a function defined from Nano topological space $(U,\tau_R(X))$ to Nano topological space $(V,\tau_R{}'(Y))$ and $N\text{int}\left((f(A))^c\right)=\phi$ for every Nano closed set A of U then f is Contra Nwg-closed map.

Theorem: 3.10 Let f be a contra Nwg-closed map then $\text{NInt}_{wg}(f(A)) \subseteq f(NCl(A))$ for every subset $A \subseteq U$.

Proof: Let $A \subseteq U$ then $NCl(A))$ is a Nano closed set in U. Since f is contra Nwg-closed map $f(NCl(A))$ is Nwg-open set in U and $\text{NInt}_{wg}(f(NCl(A)))=f(NCl(A))$. $A \subseteq NCl(A)$, $f(A) \subseteq f(NCl(A))$, $\text{NInt}_{wg}(f(A)) \subseteq \text{NInt}_{wg}(f(NCl(A)))=f(NCl(A))$, $\text{NInt}_{wg}(f(A)) \subseteq f(NCl(A))$.

Remark: 3.11 The converse of the above theorem need not be true as shown in the following example.

Example: 3.12In example: 3.6 $NInt_{wg}(f(A)) \subseteq f(NCl(A))$ but f is not contra Nwg-closed map.

Corollary: 3.13 Let $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ be a Contra Nwg-closed map then $NInt_{wg}(A) \subseteq f(NCl(f^{-1}(A)))$ for every subset $A \subseteq V$.

Remark: 3.14 The converse of the above need not be true as given in the following example.

Example: 3.15In example: 3.6 $NInt_{wg}(A) \subseteq f(NCl(f^{-1}(A)))$ but f is not contra Nwg-closed map.

4. Contra Nwg- open maps

In this section we define the function called Contra Nwg- open map and study some of its properties.

Definition: 4.1 The map $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ is Contra Nwg-open map on U if the image of every Nano open set of U is Nano weakly generalized closed set in V.

Example: 4.2 Let $U=\{a,b,c,d,e\}$ with $U/R=\{\{a\},\{b\},\{c\}\{d\},\{e\}\}$ and $X=\{a,b\}$. Then the Nano topology is $\tau_R(X)=\{U,\phi,\{a,b\}\}$.

Let $V=U=\{a,b,c,d,e\}$ with $V/R^{'}=\{\{d,e\},\{a,c\},\{b\}\}$ and $Y=\{a,b,c\}$. Then the Nano topology is $\tau_R^{'}(Y)=\{V,\phi,\{a,b,c\}\}$.

Define $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ as $f(a)=d,\ f(b)=e,\ f(c)=a,\ f(d)=b,\ f(e)=c$. f is contra Nwg-open map.

Remark: 4.3 Composition of two contra Nwg open map need not be contra Nwg open map as shown in the following example.

Example: 4.4 Let $U=\{a,b,c,d,e\}$ with $U/R=\{\{a,c\},\{b\},\{d\},\{e\}\}$ and $X=\{a,b\}$. Then the Nano topology is $\tau_R(X)=\{U,\phi,\{b\},\{a,c\},\{a,b,c\}\}$.

Let $V=\{a,b,c,d,e\}$ with $V/R^{'}=\{\{a,b\},\{c,e\},\{d\}\}$ and $Y=\{a,b\}$. Then the Nano topology is $\tau_R^{'}(Y)=\{V,\phi,\{a,b\}\}$.

Let $W=\{a,b,c,d,e\}$ with $W/R^{''}=\{\{a\},\{b\},\{c\}\{d\},\{e\}\}$ and $Z=\{a,c,e\}$. Then the Nano topology is $\tau_R^{'}(Z)=\{W,\phi,\{a,c,e\}\}$.

Define $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ as $f(a)=c,\ f(b)=d,\ f(c)=e,\ f(d)=a,\ f(e)=b$ and $g:(V,\tau_R^{'}(Y)) \to (W,\tau_R^{''}(Z))$ as $g(a)=b,\ g(b)=d,\ g(c)=c,\ g(d)=e,\ g(e)=a$. Here f and g are contra Nwg closed map but their composition is not contra Nwg-closed map since $g(f(\{d,e\}))=g(\{a,b\})=\{b,d\}$ is not Nwg-open.

Remark: 4.5 Contra Nwg-open map and Nwg-open map are independent.

Example: 4.6 Let $U=\{a,b,c,d,e\}$ with $U/R=\{\{a,c\},\{b\},\{d\},\{e\}\}$ and $X=\{a,b\}$. Then the Nano topology is $\tau_R(X)=\{U,\phi,\{b\},\{a,c\},\{a,b,c\}\}$.

Let $V=\{a,b,c,d,e\}$ with $V/R^{'}=\{\{a\},\{c\}\{b,d\},\{e\}\}$ and $Y=\{a,c\}$. Then the Nano topology is $\tau_R^{'}(Y)=\{V,\phi,\{a,c\}\}$.

Define $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ as $f(a)=c,\ f(b)=e,\ f(c)=a,\ f(d)=d,\ f(e)=b$. The image of Nano open sets are $f(\{b\})=\{e\},\ f(\{a,c\})=\{a,c\},\ f(\{a,b,c\})=\{a,c,e\}$ where $\{e\},\{a,c\},\{a,c,e\}$ are Nano open. Hence f is Nwg- open map. But it is not contra Nwg-open since $f(\{a,c\})=\{a,c\}$ which not Nwg-open set in V.

Example: 4.7 Let $U=\{a,b,c,d,e\}$ with $U/R=\{\{a,c\},\{b\},\{d\},\{e\}\}$ and $X=\{a,b\}$. Then the Nano topology is $\tau_R(X)=\{U,\phi,\{b\},\{a,c\},\{a,b,c\}\}$.

Let $V = \{a, b, c, d, e\}$ with $V / R^{'} = \{\{a\}, \{b\}, \{c, d\}, \{e\}\}$ and $Y = \{a, b\}$. Then the Nano topology is $\tau_R^{'}(Y) = \{V, \phi, \{a, b\}\}$.

Let $f : (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ be a function defined as $f(a) = c,\ f(b) = d,\ f(c) = e,\ f(d) = a,\ f(e) = b$. The image of Nano open sets are $f(\{b\}) = \{d\},\ f(\{a, c\}) = \{c, e\},\ f(\{a, b, c\}) = \{c, d, e\}$. Since $\{d\}, \{c, e\}, \{c, d, e\}$ are Nwg closed f is Contra Nwg open map. But not Nwg-open map because $\{c, d, e\}$ is not Nwg open in V.

Theorem:4.8 Let f be a function from Nano topological space U to Nano topological space V and if the only Nano open set containing the image of every Nano open set of U is V, then f is contra Nwg-open map.

Proof: Let A be a Nano open set in U and V is the only Nano open set such that $(f(A)) \subseteq V$. Then $Ncl\ (N\ \text{int}\ (f(A))) \subseteq V$. (i.e) $(f(A))$ is Nwg-closed in V. Then f is Contra Nwg-open map.

Corollary: 4.9 Let f be a function defined from Nano topological space $(U, \tau_R(X))$ to Nano topological space $(V, \tau_R^{'}(Y))$ and $N\ \text{int}\ ((f(A))) = \phi$ for every Nano closed set A of U then f is Contra Nwg-open map.

Theorem: 4.10 Let $f : (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ be a contra Nwg-open map then $f^{-1}(NIntA) \subseteq NCl_{wg}(f(A))$ for every subset $A \subseteq U$.

Proof: Let $A \subseteq U$. then $NIntA$ is a Nano open set in U. Since f is contra Nwg-closed $f(NIntA)$ is Nwg- set closed set in V and $NCl_{wg}(f(NIntA)) = f(NIntA)$. $NIntA \subseteq A$, $f(NIntA) \subseteq f(A)$, $NCl_{wg}(f(NIntA)) \subseteq NCl_{wg}(f(A))$, $NCl_{wg}(f(NIntA)) = f(NIntA) \subseteq NCl_{wg}(f(A))$, $NIntA \subseteq f^{-1}(NCl_{wg}(f(A)))$

Remark: 4.11 The converse of the above theorem need not be true as shown in the following example.

Example: 4.12 In example: 4.6, f is not Contra Nwg- closed map, but $f^{-1}(NIntA) \subseteq NCl_{wg}(f(A))$ for every subset $A \subseteq U$.

Corollary: 4.13 Let $f : (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ be a contra Nwg-open map then $f^{-1}(NInt\ (f^{-1}(A))) \subseteq NCl_{wg}(A)$ for every subset $A \subseteq V$.

Remark: 4.14 The converse of the above theorem need not be true as shown in the following example.

Example: 4.15 In example: 4.6, f is not Contra Nwg- closed map, but $f^{-1}(NInt\ (f^{-1}(A))) \subseteq NCl_{wg}(A)$ for every subset $A \subseteq V$.

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