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New compactness and connectedness in topological spaces

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Abstract
In this paper, we introduce and investigate topological spaces called pgprw-compactness Spaces and pgprw-connectedness space and we get several characterizations and some of their properties. Also we investigate its relationship with other types of functions.

Keywords: pgprw-open set, pgprw-closed sets, pgprw-compact spaces, pgprw-connectedness

1. Introduction
The notions of compactness and connectedness are useful and fundamental notions of not only general topology but also of other advanced branches of mathematics. Many researchers have investigated the basic properties of compactness and connectedness. The productivity and fruitfulness of these notions of compactness and connectedness motivated mathematicians to generalize these notions. In the course of these attempts many stronger and weaker forms of compactness and connectedness have been introduced and investigated. D. Andrijevic [1] introduced a new class of generalized open sets in a topological space called b-open sets.

The class of b-open sets generates the same topology as the class of b-open sets. Since the advent of this notion, several research paper with interesting results in different respects came into existence. M. Ganster and M. Steiner [5] introduced and studied the properties of gb-closed sets in topological spaces. The aim of this paper is to introduce the concept of pgprw-compactness and pgprw-connectedness in topological spaces and is to give some characterizations of pgprw-compact spaces in terms of nets and filter bases.

2. Preliminary Notes
Throughout this paper (X,τ), (Y,σ) are topological spaces with no separation axioms assumed unless otherwise stated. Let $A \subseteq X$. The closure of a and the interior of A will be denoted by Cl (A) and Int (A) respectively.

Definition 2.1: A subset A of X is said to be b-open [1] if $A \subseteq$ Int (Cl (A)) $\cup$Cl (Int (A)). The complement of b-open set is said to be b-closed. The family of all b-open sets (respectively b-closed sets) of (X, τ) is denoted by Bo (X,τ) [respectively bCL (X,τ)].

Definition 2.2: Let A be a subset of X. Then
(i) b-interior [1] of A is the union of all b-open sets contained in A.
(ii) b-closure [1] of A is the intersection of all b-closed sets containing A. The b-interior [respectively b-closure] of A is denoted by b-Int (A) [respectively b-Cl (A)].

Definition 2.3: Let A be a subset of X. Then A is said to be pgprw-closed [12] If pcl (A) $\subseteq$U whenever A $\subseteq$U and U $\in rg\alpha$ (X,τ). The complement of pgprw-closed [12] set is called pgprw-open. The family of all pgprw-open [respectively pgprw-closed] sets of (X, τ) is denoted by pgprwO (X, τ) [respectively, pgprw-CL (X, τ)].

Definition 2.4: The pgprw-closure [14] of a set A, denoted by pgprw-Cl (A), is the Intersection of all pgprw-closed sets containing A.

Definition 2.5: The pgprw-interior [14] of a set A, denoted by pgprw-Int (A), is the Union of all pgprw-open sets contained in A.

Remark 2.6: Every pre-closed set is pgprw-closed.

3. Pgprw-Compactness

Definition 3.1: A collection $\{A_i: i \in \wedge\}$ of pgprw-open sets in a topological space X is called a pgprw-open cover of a subset B of X if $B \subset \{Ai: i \in \wedge\}$ holds.

Definition 3.2: A topological space X is pgprw-compact if every pgprw-open cover of X has a finite sub-cover.

Definition 3.3: A subset B of a topological space X is said to be pgprw-compact relative to X if, for every collection $\{A_i: i \in \wedge\}$ of pgprw-open subsets of X such that $B \subset \cup \{A_{i:}$ $i \in \wedge\}$ there exists a finite subset $\wedge_0$ of $\wedge$ such that $B \subseteq \cup \{A_i: i \in \wedge_0\}$.

Definition 3.4: A subset B of a topological space X is said to be pgprw-compact if B is pgprw-compact as a subspace of X.

Theorem 3.5: Every pgprw-closed subset of a pgprw-compact space is pgprw-compact relative to X.

Proof. Let A be pgprw-closed subset of pgprw-compact space X. Then A^C is pgprw-open in X. Let $M = \{G_\alpha: \alpha \in \wedge\}$ be a cover of A by pgprw-open sets in X. Then $M* = M \cup A^c$ is a pgprw-open cover of X. Since X is pgprw-compact M∗ is reducible to a finite subcover of X, say $X = G\alpha_1 \cup G\alpha_2 \cup \cdot\cdot\cdot \cup G\alpha_m \cup A^C$, $G\alpha_k \in M$. But A and A^C are disjoint hence $A \subset G\alpha_1 \cup \cdot\cdot\cdot \cup G\alpha_m$, $G\alpha_k \in M$, which implies that any pgprw-open cover M of A contains a finite sub-cover.
Therefore A is pgprw-compact relative to X. Thus every pgprw-closed subset of a pgprw-compact space X is pgprw-compact.

Definition 3.6: A function f: X → Y is said to be pgprw-continuous [5] if $f^{-1}(V)$ is pgprw-closed in X for every closed set V of Y.

Definition 3.7: A function f: X → Y is said to be pgprw-irresolute [5] if $f^{-1}(V)$ is pgprw-closed in X for every pgprw-closed set V of Y.

Theorem 3.8: A pgprw-continuous image of a pgprw-compact space is compact

Proof. Let f: X → Y be a pgprw-continuous map from a pgprw-compact space X onto a topological space Y. Let $\{A_i: i \in \wedge\}$ be an open cover of Y. Then $\{f^{-1}(A_i): i \in \wedge\}$ is a pgprw-open cover of X. Since X is pgprw-compact it has a finite sub-cover say $\{f^{-1}(A_1), f^{-1}(A_2), \cdot\cdot\cdot, f^{-1}(A_n)\}$. Since f is onto $\{A_1, \cdot\cdot\cdot, an\}$ is a cover of Y, which is finite. Therefore Y is compact.

Theorem 3.9: If a map f: X → Y is pgprw-irresolute and a subset B of X is pgprw-compact relative to X, then the image f(B) is pgprw-compact relative to Y.

Proof. Let $\{A_\alpha: \alpha \in \wedge\}$ be any collection of pgprw-open subsets of Y such that $f(B) \subset \cup\{A_\alpha: \alpha \in \wedge\}$. Then $B \subset \cup\{f^{-1}(A_\alpha): \alpha \in \wedge\}$ holds. Since by hypothesis B is pgprw-compact relative to X there exists a finite subset $\wedge_0$ of $\wedge$ such that $B \subset \cup \{f^{-1}(A_\alpha): \alpha \in \wedge_0\}$ Therefore we have $f(B) \subset \cup\{A_\alpha: \alpha \in \wedge_0\}$, which shows that f (B) is pgprw compact relative to Y.

4. Pgprw-Connectedness

Definition 4.1: A topological space X is said to be pgprw-connected if X cannot be expressed as a disjoint union of two non-empty pgprw-open sets. A subset of X is pgprw-connected if it is pgprw-connected as a subspace.

Example 4.2: Let X = {a, b} and let τ = {X, φ, {a}}. Then it is pgprw-connected.

Remark 4.3: Every pgprw-connected space is connected but the converse need not be true in general, which follows from the following example.

Example 4.4: Let X = {a, b} and let τ = {X, φ}. Clearly (X, τ) is connected. The pgprw-open sets of X are {X, φ, {a}, {b}}. Therefore (X, τ) is not a pgprw-connected space, because X = {a} ∪ {b} where {a} and {b} are non-empty pgprw-open sets.

Theorem 4.5: For a topological space X the following are equivalent.
(i) X is pgprw-connected.
(ii) X and φ are the only subsets of X which are both pgprw-open and pgprw-closed.
(iii) Each pgprw-continuous map of X into a discrete space Y with at least two Points is a constant map.

Proof: (i) ⇒ (ii): Let O be any pgprw-open and pgprw-closed subset of X. Then O^C is both pgprw-open and pgprw-closed. Since X is disjoint union of the pgprw-open sets O and O^C implies from the hypothesis of (i) that either O = φ or O = X.

(ii) ⇒ (i): Suppose that X = A ∪ B where A and B are disjoint non-empty pgprw-open subsets of X. Then A is both pgprw-open and pgprw-closed. By assumption A = φ or X. Therefore X is pgprw-connected.

(ii) ⇒ (iii): Let f: X → Y be a pgprw-continuous map. Then X is covered by pgprw-open and pgprw-closed covering $\{f^{-1}(Y): y \in (Y)\}$. By assumption $f^{-1}(y) = \varphi$ or X for each $y \in Y$. If $f-1(y) = \varphi$ for all $y \in Y$, then f fails to be a map. Then there exists only one point $y \in Y$ such that $f^{-1}(y) \neq \varphi$ and hence $f^{-1}(y) = X$. This shows that f is a constant map.

(iii) ⇒ (ii): Let O be both pgprw-open and pgprw-closed in X. Suppose O ≠ φ. Let f: X → Y be a pgprw-continuous map defined by f (O) = y and $f(O^C) = \{w\}$ for some distinct points y and win Y.
By assumption f is constant. Therefore we have O = X.

Theorem 4.6: If f: X → Y is a pgprw-continuous and X is pgprw-connected, then Y is connected.

Proof: Suppose that Y is not connected. Let Y = A∪B where A and B are disjoint non-empty open set in Y. Since f is pgprw-continuous and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty pgprw-open sets in X. This contradicts the fact that X is pgprw-connected. Hence Y is connected.

Theorem 4.7: If f: X → Y is a pgprw-irresolute surjection and X is pgprw-connected, then Y is pgprw-connected.

Proof: Suppose that Y is not pgprw-connected. Let Y =A∪B where A and B are disjoint non-empty pgprw-open set in Y. Since f is pgprw-irresolute and onto, X = f^{-1} (A) ∪ f^{-1} (B) where f^{-1} (A) and f^{-1} (B) are disjoint non-empty pgprw-open sets in X. This contradicts the fact that X is pgprw-connected. Hence Y is connected.

Theorem 4.8: In a topological space (X, τ) with at least two points, if pgprw-O(X, τ) = pgprw-CL(X, τ) then X is not pgprw-connected.

Proof: By hypothesis we have pgprw-O(X, τ) = pgprw-CL(X, τ) and by Remark 2.6 we have every pre-closed set is pgprw-closed, there exists some non-empty proper subset of X which is both pgprw-open and pgprw-closed in X. So by last Theorem 4.5 we have X is not pgprw-connected.

Definition 4.9: A topological space X is said to be T_{pgprw}-space if every pgprw-closed
Subset of X is closed subset of X.

Theorem 4.10: Suppose that X is a T_{pgprw}-space then X is connected if and only if it is pgprw-connected.

Proof: Suppose that X is connected. Then X can not be expressed as disjoint union of two non-empty proper subsets of X. Suppose X is not a pgprw-connected space. Let A and B be any two pgprw-open subsets of X such that X = A∪B, where A ∩ B = φ and A ⊂ X, B ⊂ X. Since X is T_{pgprw}-space and A, B are pgprw-open, A, B are open subsets of X, which contradicts that X is connected. Therefore X is pgprw-connected. Conversely, every open set is pgprw-open. Therefore every pgprw-connected space is connected.

Theorem 4.11: If the pgprw-open sets C and D form a separation of X and if Y is pgprw-connected subspace of X, then Y lies entirely within C or D.

Proof: Since C and D are both pgprw-open in X the sets C∩Y and D∩Y are pgprw-open in Y these two sets are disjoint and their union is Y. If they were both non-empty, they would constitute a separation of Y. Therefore, one of them is empty. Hence Y must lie entirely in C or in D.

Theorem 4.12: Let A be a pgprw-connected subspace of X. If A⊂B⊂pgprw-Cl (A) Then B is also pgprw-connected.

Proof: Let A be pgprw-connected and let A⊂B⊂pgprw-Cl (A). Suppose that B = C∪D is a separation of B by pgprw-open sets. Then by Theorem 4.11 above A must lie entirely in C or in D. suppose that A ⊂ C, then pgprw-Cl (A) ⊆ pgprw-Cl (C). Since pgprw-Cl (C) and Dare disjoint, B cannot intersect D. This contradicts the fact that D is non-empty subset of B. So D = φ which implies B is pgprwconnected.

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