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Analysis of differential equations in economics

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Abstract
In this paper, I introduce Basic concepts, theorems and methods in differential equation theory which are widely used in contemporary economic analysis and provides simple as well as comprehensive applications to different fields in economics.

Keywords: Parental attitude, participation, sports, girls

Introduction
Recently, differential equations are used in modeling motion and change in all areas of science. It has become an essential tool of economic analysis. The main objective in this paper is to explains about application of differential equations in business and industry.

Analysis
Ordinary differential equations are differential equations whose solutions are functions of one independent variable, which we usually denote by t. The variable t often stands for time, and solution we are looking for, $x(t)=t^2x(t)$ is an ordinary differential equation. Ordinary differential equations are classified as autonomous and nonautonomous. The equation

$$\dot{x}(t) = ax(t) + b,$$

with a and b as parameters is an autonomous differential equation because the time variable t does not explicitly appear. If the equation specially involves t, we call the equation nonautonomous or time-dependent. For instance,

$$\dot{x}(t) = x(t) + \sin t,$$

is a non-autonomous differential equation. In this book, we often omit "ordinary", "autonomous" or "non-autonomous" in expression. If an equation involves derivatives up to and includes the ith derivative, it is called an ith order differential equation. The equation $\dot{x}(t) = ax(t) + b,$ with a and b as parameters is a first order autonomous differential equation. The equation

$$\ddot{x} = 3\dot{x} - 2x + 2$$

Is a second order equation, where the second derivative, $\ddot{x}(t)$, is the derivative of $\dot{x}(t)^1$ (t)1 As shown late, the solution is $x(t) = A_1e^{2t} + A_2e^t + 1$, Where A_1 and A_2 are two constants of integration. The first derivative $\dot{x}$ is the only one that can appear in a first order differential equation, but it may enter in various powers: $\dot{x}$, $\dot{x}^2$ and so on. The highest power attained by the derivative in the equation is referred to as the degree of the differential equation. For instance,

$$3\dot{x}^2 - 2x + 2 = 0$$

Is a second-degree first-order differential equation.

Euler-Lagrange equation

One of the powerful tools applied in optimization problems in finance and economic research is the *Euler-Lagrange equation* (Lagrange's equation). The Euler-Lagrange equation was developed by Leonhard Euler and Joseph-Louis Lagrange in the 1750s. It is used to solve for functions that optimize a given cost or profit function. It is analogous to the result from calculus that when a smooth function attains its extreme values its derivative goes to zero. The Euler-Lagrange equation is an equation satisfied by a function f(t), which maximizes (minimizes) the function

$$J = \int_a^b F(t, f(t), f'(t))dt$$

where function F(t, x, y) $\in$ C' (R × X × Y).

The Euler-Lagrange equation is the differential equation

$$F_x(t, f(t), f'(t)) - \frac{d}{dt} F_y (t, f(t), f'(t)) = 0$$

Where F_x and F_y denote the partial derivatives of F(t, x, y) with respect to x, y.

Gross Domestic Product (GDP)

A differential equation expresses the rate of change of the current state as a function of the current state. A simple illustration of this type of dependence is changes of the Gross Domestic Product (GDP) over time. Consider state x(t) of the GDP of the economy at time t. The rate of change of the GDP is proportional to the current GDP; that is,

$$\dot{x}(t) = gr(t),$$

where $\dot{x}(t) = \frac{\partial}{\partial t} x(t)$ and the growth rate g is a given constant at any time t. Then the GDP at time t can be obtained via solving the differential equation and that can be written as

$$\frac{\dot{x}(t)}{x(t)} = \frac{\partial}{\partial t} \ln x(t) = g$$

Therefore, In x(t) = gx + c, where c is constant. Equivalently, x(t) = $e^C e^{gt}$. Furthermore, if we set t = 0, then we have x(0) = e^C. Finally, we have the solution

$$P(t)+P(t)g(t) = e(t) \exp\left(-\int_{x_0}^t g(s)ds\right).$$

By above equations, we have q(t) = e(t) exp($-\int_{x_0}^t g$ (s)ds), or equivalently

$$e(t) = q(t) \exp\left(\int_{x_0}^t g(s)ds\right).$$

Thus,

$$e(t) = C + \int_{x_0}^t q(x) \exp\left(\int_{x_0}^t g(s)ds\right) dx.$$

Now solution is given as

$$I(T) = -I_0 \exp\left\{-\int_t^{t_0} [r(\xi)+\delta]d\xi\right\} + \int_t^{t_0} f(s) \exp\left\{-\int_t^{t_0} [r(\xi)+\delta]d\xi\right\} ds$$

where the solution passes through (t_0, I_0). Since we assume an infinite time horizon, the specific solution which maximizes V is

$$\bar{I}(s) = s\overline{K}(s) - K(0) + \delta\overline{K}(s)$$

Recall that I(t) = K + δK. Taking Laplace transformation on both sides, we have

$$\overline{K}(s) = \frac{\bar{I}(s) + K(0)}{s+\delta}$$

After rearranging above equation, we further have

(s) = .4s)+ K(0)

By taking inverse Laplace transformation, we finally have the capital input function

$$K(t) = I(t) * e^{\delta t} + K(0)e^{-\delta t}$$

where $I(t) * e^{-\delta t}$ is the convolution product of 1(t) and $e^{\delta t}$

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