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Study of MATLAB, calculus and linear algebra

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Abstract
This paper presents the appropriateness of the time allocated to linear algebra; students' background and readiness in regards to objects, language, and ideas that are unique to linear algebra, and students' background and readiness in regards to the concept of proof.

Keywords: Calculus and Linear algebra

Introduction
The educational benefit of incorporating technology in the teaching of mathematics has become, through both research in mathematics education and informal observations, apparent. In this regard, the LACSG recommendation that technology be incorporated into beginning linear algebra courses is expected. I suggest to go a step further and incorporate a matrix package like NIATLAB in the teaching of calculus. This, as I will explain below, would benefit students in both linear algebra and calculus.

There are many computer packages that may well serve the purpose discussed in this section-XMath, Mathematicia, Maple, Derive, MathCad, and Xplore, to mention a few. My own experience, however, is primarily with MATLAB, which I have found to be a very adequate. I will report on my experience with this package.

MATLAB is an interactive system and programming language for scientific and technical computation. It boasts features that make it an excellent tool for achieving the double purpose of strengthening students' understanding of concepts in calculus and acquainting students with the linear algebra environment. Of particular importance is that MATLAB'S basic data element is a matrix. This requires that solutions to problems be approached vectorally. The extent to which the vectorization feature of MATLAB is utilized varies with experience. However, with direction and encouragement, students learn to gradually utilize this important feature. The pedagogical benefit is that in writing MATLAB programs, students learn to represent problems and design and interpret their solutions in the language of vectors and matrices. In the course of doing so, they must imagine the actions carried out by the computer in response to their commands and anticipate the outcomes of these actions. These mental activities of imagination and anticipation involve mental manipulations of vectors and matrices that constitut'e an important component of the linear algebra environment. The best way for one to appreciate this pedagogical value is to write several MATLAB programs oneself and reflect. In the course of this exercise, on one's mental activities. For this, the book Matrices and Matlab, by Marvin Marcus, is an excellent source.

Since I am suggesting that MATLAB be incorporated in calculus, I should add that there exists anecdotal evidence of the pedagogical benefit of MATLAB in the learning of calculus and linear algebra. Allen Weitsman from Purdue University is one person who has been incorporating MATLAB into his calculus classes for several years. I have taken a close look at Weitsman's use of MATLAB in a sequence of three consecutive courses: Calculus of One Variable, Multivariable Calculus, and Linear Algebra and Differential Equations. These were small-class courses taken by the same students. I have recognized through these observations the benefit of MATLAB in strengthening students' calculus and linear algebra concepts. Space does not permit to describe in detail the benefit of his approach to the conceptualization of calculus ideas. However, based on my own analysis of the projects he assigned to his students, detailed observations of one of Weitsman's students, and many conversations with Weitsman, I can point to some pedagogical values.

These include the understanding of

a) the idea of Riemann sum in one- and two-variable functions,
b) three–dimensional graphing,
c) methods of approximation, including Taylor series for two-variable functions and numerical solutions to differential equations and
d) the concept of limit.

With regard to the benefit of MATLAB to the learning of linear algebra, it is interesting to note that despite the fact that the topic "block matrices" was not covered in any of these courses, Weitsman's students developed a tendency to approach, and successfully solve, problems by matrix partitioning. This may be attributed to the experience these students have had with MATLAB, which helped them to develop the spatial symbol manipulation ability I have discussed earlier.

Some of the features which one needs in a software package so that it is good for both calculus and linear algebra include the following and MATLAB has these features. The following remarks are from.

1. MATLAB is currently used in nearly every engineering school in the United States.
2. The superb built-in Help facility in MATLAB makes it simple and nearly instantaneous, for the reader to look up any available command or structure.
3. Programming in MATLAB should be relatively transparent to anyone who has done a month of BASIC programming in high school.
4. It is a sophisticated numerical package with dozens of special built-in commands and functions.
5. It provides the student with a powerful interactive tool to examine significant examples, to strengthen intuitive insight, and to formulate and study plausible conjectures. These are "mathematical" rather than "computing" activities, but they cannot be carried out in any but the most trivial situations without an effective scientific computing package.

Because of these features especially the last one, the incorporation of MATLAB into the first course in linear algebra is most natural and meets the LACSG recommendation to utilize technology.

There is a need however, to further experiment with the idea of incorporating MATLAB or other similar software packages, into calculus and later into linear algebra, and investigate:

1. The benefit to the learning of calculus.
2. The extent to which the use of MATLAB in calculus prepares students for their first course in linear algebra.
3. The success of MATLAB in helping build effective concept images in the first course in linear algebra.

Instructional approach to proof

Students should take an active part in the construction of relations between ideas and in the production of their justifications. Instructors should set the tone for this approach right at the beginning of the course and continue throughout. At first, students may resist this approach because it does not meet their expectation and definition of a class-session. For this reason, it is advised to begin with instructional activities that are elementary enough that all or most students can participate. For example students could be assigned the homework problem:

Let

$$A = \begin{pmatrix} 3 & -6 & 2 & -1 \\ -2 & 4 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 1 & -6 & 1 & 0 \end{pmatrix}$$

Find for which (y_1, y_2, y_3, y_4) the system of equations AX = Y has a solution. You are encouraged to use MATLAB.

There will be students who would use a direct approach to solve this problem by row–reducing the matrix [A Y] into [A' Y'] and solving for Y'. (In this approach, there is, obviously, no gain from using MATLAB.) The answer to the problem (if it is derived from a row-reduced-echelon matrix) is: Y is a linear combination of Y_1 = (–1, 3, 1, 0) and Y_2 = (3, –2, 0, 1).

There will be other students who would use a different method. Using MATLAB, these students may find a basis for the column space by applying the rref command to A^T. Their answer would be: Y is a linear combination of Z_1 = (1,0,0.2857,0.4286) and Z_2 = (0,1,0.4286,0.1429).

In the class session these solution approaches would be presented by the students (if the students had not chosen the second approach, it would be presented by the instructor), and a class discussion would follow. In addition, the relation rreff ([Y_1 Y_2]) = [Z_1 Z_2] may be brought up either by one of the students and, if not, by the instructor. (If a computer display system is available, this can be displayed instantly.)

The learning opportunities that this activity can offer to students are apparent and valuable, I believe. They are valuable not only because students would learn a specific mathematical connection (the connection between a system AX = Y and the column-space of A) or a specific mathematical property (the property that a subspace has more than one spanning set), but also because the student took an active part in discovering these ideas.

Students should be helped to build proofs on their intuitions. Consider, the following example: After running several examples in MATLAB, students can easily conjecture the relation:

For two matrices A and B, rank(AB) 7 min [rank(A), rank(B)].

This relation can be quite intuitive to the students if they are encouraged to think of a matrix as a linear transformation and consider the geometrical meaning of rank, as shown in the following figure 1:

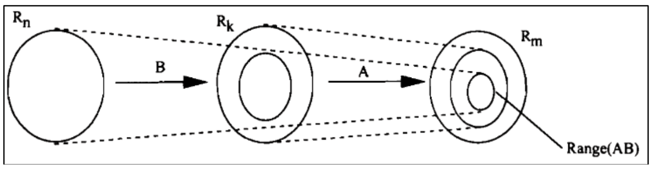

Fig 1: Geometrical meaning of rank

Occasionally, textbooks and instructors present a geometrical explanation of this kind, and succeed, as a result, to convince the student in the truth of the relation. Rather than building on this understanding by expanding it into a more formal proof, they often abandon it and start from scratch an analytic proof. Even when students understand all the steps of the analytic proof, it often happens that they do not see a connection between this proof and the geometric explanation. Once the student is convinced of the truth of a relation through an intuitive explanation, with the help of the instructor if necessary, the challenge is to refine this explanation into an analytic proof.

Students should be encouraged to read proofs. I don't know of any study about undergraduate students' ability to read mathematical texts, but based on my own and some of my colleagues' observations, it does not seem that we are giving enough attention to this important goal. Carl Cowen has suggested an approach of how to encourage students to read mathematics by including on the exam new theorems, together with their proofs, followed by questions that can be answered only by those who read and understood the theorems and proofs. This idea, I believe, should be extended to homework as well as classroom activities. One of Cowen's example is from linear algebra. I will present it here in its entirety.

This example comes from a linear algebra course for engineers. At the time of the test, the students had studied the spectral mapping theorem for polynomials which implies that the positive integer powers of a diagonalizable matrix are similar to the powers of the diagonal matrix, but had not seen the theorem on square roots of positive definite matrices.

Theorem: If A is a diagonalizable matrix all of whose eigenvalues are nonnegative, then there is a matrix B with non-negative eigenvalues such that $B^2 = A$.

Proof: Since A is diagonalizable, there is an invertible matrix S such that $L = S^{-1}AS$ is diagonal. The diagonal entries $\lambda_1, \lambda_2 \ldots \lambda_n$ of L (which are the eigenvalues of A) are non-negative by hypothesis. Let, $\mu_j = \sqrt{\lambda_j}$ be the nonnegative square roots of the eigenvalues and let M be the diagonal matrix with diagonal entries, $\mu_l, \mu_2 \ldots \mu_n$. Clearly, $M^2 = L$. Since similar matrices have the same eigenvalues, the matrix $B = SMS^{-1}$ has eigenvalues, $\mu_1, \mu_2 \ldots \mu_n$, which are non-negative. Moreover,

$$B^2 = (SMS^{-1})^2 = SMS^{-1}\,SMS^{-1} = SM^2S^{-1} = SLS^{-1} = A$$

Problem: The matrix

$$A = \begin{pmatrix} 10 & -9 \\ - & -5 \end{pmatrix}$$

has eigenvalues 1 and 4. Find a matrix S as above and use it to find a matrix B with positive eigenvalues such that $B^2 =$ A.

In this question, I expect the students to understand from the proof of the theorem that the problem is to be solved by diagonalizing A and solving the corresponding problem for the diagonal matrix. In particular, I expect them to use the given eigenvalues of A to find a basis of eigenvectors, and thereby to construct S, L, M, and B.

In the standard approach, students are asked to practice the application of theorems' results, not theorems' proofs. In this respect, Cowen's idea is refreshing.

Another crucial contributor to students' ability (or inability) to read mathematical texts is the way textbooks are written and structured. Many textbooks are written in a style that is more appropriate for a research paper than a text that is intended for sophomore or junior mathematics majors. The textbooks' role is not just to present their material logically and succinctly, but also to share with the students the thought processes behind the choice of definitions, the intellectual need for the theorems, and the motivations for the proofs.

Other textbooks have the tendency to redundantly multiply theorems. By formulating many results as theorems they believe students would have an easy access to the results that are needed to solve problems. I believe that this approach can handicap students' learning. For in so doing these textbooks can end up training students to become theorem-searchers and theorem-users rather than theorem analyzers. In addition, it is difficult for a beginning student who uses such a text to see the trees from the forest. Textbooks should help students to develop the sense that elementary linear algebra can be structured around a few central and unifying ideas. Such a sense can be built only by a strong emphasis on proofs where students would come to realize the centrality of ideas such as linear independence, spanning. Elementary operations, vector-space, and linear transformation.

Students should learn that understanding a proof is more than understanding each of the proof's steps. Following Cowen's suggestion, we can give problems that require a deep understanding of the proof they are asked to read. For example, to solve the following problem one must understand the core idea behind the proof presented in (3):

Problem: A is similar to D and f is a polynomial for which the equation f (X) = A has a solution. Prove that f (X) = D has a solution.

This does not mean that the promotion of this goal should be done only with complex problems. On the contrary, the insistence on understanding the ideas behind the proof should begin with simple proofs, such as in the following example: Students are presented with the definitions of a left inverse and a right inverse of a matrix, followed by the following theorem and its proof.

Theorem: If A has a left inverse B and a right inverse C, then B = C

Proof: Suppose BA = I and AC = I. Then B = BI = B(AC) = (BA)C = IC = C.

The statement of this theorem is simple but somehow surprising, especially to one for whom matrix multiplication is still a peculiar operation. The proof too is simple and its steps can be understood easily by most students, but would it demystify the theorem for the students? The challenge, I must emphasize, is not to look for a "better" proof, which in some cases is advisable. The challenge is to bring students to (a) question the expectedness of a theorem and (b) insist to be convinced by its proof, not just understand each of its steps. As teachers, we would be delighted to see students complaining that they don't understand how the use of associativity of matrix multiplication is what is necessary to prove the theorem and then see them turning to a general 2 × 2 case to examine why the proof proves. Unfortunately, because of the reasons I mentioned earlier, students are usually passive toward theorems and proofs, so it is unlikely that they would raise any questions at all.

Conclusion

I have also suggested incorporating MATLAB (or any other similar software package) in the teaching of calculus for the double purpose of strengthening students' understanding of concepts in calculus and acquainting students with the linear algebra environment. In particular, because MATLAB's basic data element is a matrix, programming in MATLAB can help students make n-tuples and matrices concrete. This would prepare students for a matrix-oriented course as was suggested in Recommendation 1. Also, I have indicated, as other people have already done, that the use of MATLAB in the teaching of linear algebra is most natural. These suggestions are in line with Recommendation 4, which calls for a utilization of technology in linear algebra course.

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