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On complementary distance pattern uniform graphs

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Abstract

It was Koshy (2010) who introduced and investigated the concept Complementary Distance Pattern Uniform (CDPU) sets in a connected graph. In this paper, the researcher introduces one variety of Complementary Distance Pattern Uniform (CDPU) graphs which is α Complementary Distance Pattern Uniform (αcdpu) graphs.

A couple of results are generated in this study. Some of which are the following: $\alpha(G + H) \leq \min\{|V(G)|,|V(H)|\}$ where G and H be connected graphs. Let G be a connected graph and H be a disconnected graph. Then $\alpha(G + H) \leq |V(G)|$. Let G be a self-centered graph and H be any graph. Then $\alpha(G \circ H) = |V(G)|$. Let $K_n$be a complete graph of order n and H be any graph. Then $\alpha(K_n \circ H) = n$. Let C_n be a cycle of order n and H be any graph. Then $\alpha(C_n \circ H)=n$. Let G and H be graphs with isolated vertices $u \in V(G)$ and $v \in V(H)$. Then $\alpha_u(G+H) = 2$. Let $K_{1,n,}$ and $K_{m,n}$be a star of order n +1 and a complete bipartite graph of order n + m, respectively.

Keywords: CDPU graphs, distance, disjoint, join, corona

Introduction

Graph Theory has a relatively long history in classical mathematics. In 1736, Euler solved the problem of Konigsberg bridge in Russia. The river divided the city into four separate landmasses, including the island of Kneiphopf. These regions were linked by seven bridges. Residents of the city wondered if it were possible to leave home, cross each of the seven bridges exactly once, and return home. The Swiss mathematician Leonhard Euler solved the problem without crossing any bridge twice and go back to the starting point and this is time that graph theory was invented (Dickson, 2006).

The concept in this study has important applications in the field of Chemistry. As mentioned by Koshy *et al*. (2010) [6, 7], they introduced and investigated the concept of Complementary Distance Pattern Uniform (CPDU) sets in a connected graph. Koshy stressed that there are strong indications in the literature of Natarajan (2009) [9] that the notion of Complementary Distance Pattern Uniform (CPDU) sets could be used to design a class of Topological Indexes that represent certain stereochemical properties of the molecule wherein the chemical structure of a molecules can be represented as a molecular graph, in which vertices corresponds to atoms, and edges represent covalent bonds between 2 atoms.

In this study, introduces one of the varieties of Complementary Distance Pattern Uniform graphs and this is α Complementary Distance Pattern Uniform (αcdpu) graphs. Let G be a graph. We call G a αcdpu graph if there exist set $M \subseteq V(G)$ such that $f_M(u) = f_M(v)$, where $f_M(u) = \{d(u,v): v \in M\}$, where d(u,v) denotes the usual distance between vertices u and v in G for all $u,v \in V(G)\backslash M$, and $\langle M \rangle$ is a connected graph. The set M is called the α*cdpuset. An α*cdpu set of least cardinality is called an αcdpu set of G. The cardinality of an αcdpu set of G, denoted by α(G), is called and αcdpu number of G.

Objectives of the Study

This study has the following objectives:

1. To introduced αcdpu graphs.
2. To obtain bounds for the αcdpu number of graphs resulting from the join of a complete graph and any graphs.
3. To obtain bounds for the αcdpu number of graphs resulting from the join of graphs.
4. To obtain bounds or exact values for the αcdpu number of fans, wheels, generalized fans, and generalized wheels.

5. To obtain bounds for the αcdpu number of graphs resulting from the corona of graphs.

Research Methodology

The following steps were followed in conducting this study. First, the concepts αcdpu set, αcdpu number, αcdpu graph were introduced. Second, these concepts were discussed thoroughly by giving examples. Third, conjectures regarding the properties of concepts were presented. Finally, these conjectures were shown whether they are true or not.

Results

The following present the main results of this study:

1. αcdpu Graphs

Theorem 1: A CDPU set may not be an α*cdpu set; but an α*cdpu set is always a CDPU set.

Theorem 2: Every connected graph has an α*cdpuset.
Proof: Let G be a connected of order n ≥ 2. Let u ∈ V (G) such that hV (G)\{u}i is a connected graph. Consider M = V (G)\{u}. Then, M is a αcdpu set of G (since u is the only vertex in V (G)\M).

Corollary 3: All connected graphs are αcdpu graphs.

Lemma 4: If G is a self-centered graph with rad(G) > 1, then there exists u, v ∈ V (G) such that uv∈ E(G), degG(u) > 1, and degG(v) > 1. Proof: Let G be a self-centered graph with rad(G) > 1. Suppose that for all u, v ∈ V (G) such that uv∈ E(G), degG(u) = 1 or degG(v) = 1. Then G ∼= K1,n. This is a contradiction since G is a self-centered graph. Hence, 26 there exists u, v ∈ V (G) such that uv∈ E(G), degG(u) > 1, and degG(v) > 1.

Theorem 5: Every self-centered graph, of order n, has a α-set M with |M| ≤ n − 2. Proof: Let G be a self-centered graph. Then e(v) = rad(G), for all v ∈ G. Moreover by Lemma 3.1.4, there exists u, v ∈ V (G) with uv∈ E(G), degG(u) > 1 and degG(v) > 1. Observe that $f_M(u)$ = {1, 2,..., rad(G)} and $f_M(v)$ = {1, 2,..., rad(G)}. Hence, M = V (G)\{u, v} is an α set in G. Therefore, G has an α set M with |M| = |V (G)| − 2 = n − 2.

Corollary 6: For a self-centered graph G, max$f_M(v)$ = rad(G), for every v ∈ V (G)\M.

2. αcdpu Number in the Join of Graphs

Theorem 2.1: Let G and H be connected graphs. Then α(G + H) ≤ min{|V (G)|, |V (H)|}.
Proof: Let G and H be connected graphs. Without loss of generality, suppose |V (G)| ≤ |V (H)|. Let M = V (G) and let u ∈ V (G + H)\V (G). Then $f_M(u)$ = {1}. Since u is arbitrary, $f_M(u)$ = {1} for all u ∈ V (G + H)\M. Hence, M is an α-set. Therefore, α(G + H) ≤ |V (G)| = min{|V (G)|, |V (H)|}.

Theorem 2.2: Let G be a connected graph and H be a disconnected graph. Then α(G + H) ≤ |V (G)|. Proof: Let G be connected graph and H be a disconnected graph. Let M = V (G) and let u ∈ V (G + H)\V (G). Then $f_M(u)$ = {1}. Since u is arbitrary, $f_M(u)$ = {1} for all u ∈ V (G + H)\M. Hence, M is an α-set. Therefore, α(G + H) ≤ |V (G)|.

Corollary 2.3: Let F_n, W_n, $K_{1,n}$, $F_{m,n}$, and $W_{m,n}$ be a fan of order n + 1, a wheel of order n + 1, a star of order n + 1, a generalized fan of order n + m, and a generalized wheel of order n + m, respectively. Then 1. α(F_n) = 1; 28 2. α(W_n) = 1; 3. α($K_{1,n}$) = 1; 4. α($F_{m,n}$) ≤ min{m, n}; 5. α($W_{m,n}$) ≤ min{m, n}.

3. αcdpu Number in the Corona of Graphs

Theorem 3.1: Let G be a self-centered graph and H be any graph. Then α(G ◦ H) = |V (G)|. Proof: Let G be a self-centered graph with rad(G) = n. Then e(v) = rad(G), for all v ∈ V (G). Let M = V (G) and let u ∈ V (G ◦H)\M. Then $f_M(u)$ = {1, 2,..., n + 1}. Since u is arbitrary, $f_M(u)$ = {1, 2,..., n + 1} for all u ∈ V (G ◦ H)\M. Hence, M is an α-set. Therefore, α(G ◦ H) ≤ |V (G)|. Suppose α(G ◦ H) < |V (G)|. Let M be an α-set of G ◦ H with |M| = α(G ◦ H). Then by the Pigeon Hole Principle, there exists u, v ∈ V (G) such that V (u + Hu)∩ M = ∅ and V (v + H_V)∩ M 6= ∅. Let x ∈ V (H_U) and y ∈ V (H_V) with y /∈ M. Then 1 ∈/ $f_M(x)$ and 1 ∈$f_M(y)$ (since h_{Mi} is connected); that is, $f_M(x)$ 6= $f_M(y)$. This is a contradiction since M is an α-set. Therefore, α(G ◦ H) = |V (G)|. 29

Corollary 3.2: Let K_n be a complete graph of order n and H be any graph. Then α(K_n◦ H) = n.

Theorem 3.3: Let Cn be a cycle of order n and H be any graph. Then α(Cn ◦ H) = n. Proof: Let Cn be a cycle of order n and H be any graph. Let M = V (Cn) and let u ∈ V (Cn ◦H)\M. Then $f_M(u)$ = {1, 2,..., bn/2c}. Since u is arbitrary, $f_M(u)$ = {1, 2,..., bn/2c} for all u ∈ V (Cn ◦ H)\M. Hence, M is an α-set. Therefore, α(Cn ◦ H) ≤ n. Suppose α(Cn ◦ H) < n. Let M be an α-set of Cn ◦ H with |M| = α(Cn ◦ H). Then by the Pigeon Hole Principle, there exists u, v ∈ V (Cn) such that V (u + Hu)∩ M = ∅ and V (v + H_V) ∩ M 6= ∅. Let x ∈ V (Hu) and y ∈ V (H_V) with y /∈ M. Then 1 ∈/ $f_M(x)$ and 1 ∈$f_M(y)$ (since h_{Mi} is connected); that is, $f_M(x)$ 6= $f_M(y)$. This is a contradiction since M is an α-set. Therefore, α(Cn ◦ H) = n.

Summary and Recommendations

This study encapsulates all the results of this study including particular cases. Moreover, since some results point to very interesting conjectures and very natural generalizations, we also express some recommendations.

Summary

1. αcdpu Graphs
1.1 A CDP U set may not be an α-set; but an α-set is always a CDP U-set. Theorem 3.1.1
1.2 Every connected graph has an α set. Theorem 3.1.2 3. All connected graphs are α-CDP U. Corollay 3.1.3
1.3 If G is a self-centered graph with rad(G) > 1, then there exists u, v ∈ V (G) such that uv∈ E(G), degG(u) > 1, and degG(v) > 1. Lemma 3.1.4
5. Every self-centered graph, of order n, has a α-set M with |M| ≤ n − 2. Theorem 3.1.5
6. For a self-centered graph G, maxfM(v) = rad(G), for every v ∈ V (G)\M. Corollary 3.1.6
4.1.2 αcdpu Number in the Join of Graphs 1. Let G and H be connected graphs. Then α(G + H) ≤ min{|V (G)|, |V (H)|}. Theorem 3.4.1 2. Let G be a connected graph and H be a disconnected graph. Then α(G + H) ≤ |V (G)|. Theorem

3.2.2 3. Let Fn, Wn, K1,n, Fm,n, and Wm,n be a fan of order n + 1, a wheel of order n + 1, a star of order n + 1, a generalized fan of order n + m, and a generalized wheel of order n + m, respectively. Then • α(Fn) = 1; • α(Wn) = 1; 33 • α(K1,n) = 1; • α(Fm,n) ≤ min{m, n}; • α(Wm,n) ≤ min{m, n}. Corollary 3.2.3 4.1.3 αcdpu Number in the Corona of Graphs 1. Let G be a self-centered graph and H be any graph. Then α(G ◦ H) = |V (G)|. Theorem 3.3.1 2. Let Kn be a complete graph of order n and H be any graph. Then α(Kn◦ H) = n. Corollary 3.4.2 3. Let Cn be a cycle of order n and H be any graph. Then α(Cn ◦ H) = n. Theorem 3.3.3

Recommendations

It is recommended that the results in this study be used in endeavors that require the concepts α-CDPU sets and ubat-sets. Moreover, since the concepts α-CDPU sets and ubat-sets are a broad topics in graph theory, there are still a lot of open problems to research on. Some of these includes, finding some bounds or the exact value of the α-CDPU number and ubat-number of the 1. Cartesian product of two arbitrary graphs, 2. Kronecker product of two arbitrary graphs, 3. Composition of two arbitrary graphs

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