International Journal of Applied Research 2019; 5(5): 310-312

International Journal of Applied Research

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2019; 5(5): 310-312
www.allresearchjournal.com
Received: 21-03-2019
Accepted: 24-04-2019

MD Alam
Village-Post, Belhwar, PS, Rajnagar, District, Madhubani, Bihar, India

Correspondence
MD Alam
Village-Post, Belhwar, PS, Rajnagar, District, Madhubani, Bihar, India

Mathematical modeling in social and behavioral sciences

MD Alam

Abstract

Mathematics is an integral part of contemporary social and behavioral sciences. Many of today's profound insights into human behaviors could hardly to without the help of mathematics. It may be said that the main advance in modern social and behavioral science is characterized by applying mathematics to various social and behavioral problems. This paper presents some application of mathematics in social and behavioral sciences. It provides a general overview of mathematical approaches to different social and behavioral problems.

Keywords: Mathematical modeling, contemporary, profound insights

Introduction

Many of today's profound insights into human behavior could hardly be obtained without the help of mathematics. It may be said that the main advance in modern social and behavioral sciences is characterized by applying mathematics to various social and behavioral problems. The concepts of equilibrium versus non-equilibrium, stability versus instability, and steady states versus chaos in the contemporary literature are difficult to explain without mathematics. In some sense, one can hardly properly appreciate achievements of contemporary social and behavioral sciences without a thorough training in mathematics.

The purpose of this paper is to present some mathematical models that support the field of social and behavioral sciences. It provides a general overview of mathematical approaches to different social and behavioral problems. The variety of models as well as the number of recent contributions is quite impressive. Since the literature on the topic is vast, we will provide only a few areas of the applications in this paper.

Different jobs, such as officer, police, university professor, factory worker, are associated with different social status and amenities. Differences in amenities give rise to compensating wage differentials. Assuming that different jobs bring about different amenities and disamenities, Lundborg (1995) [6] builds some models to explain why some people would have multiple occupations. We now introduce Lundborg's general model of amenities and moonlighting to show how traditional optimization theory can be applied to explain behavior of workers.

Analysis

We assume two sectors, one with jobs with some amenity and one traditional manufacturing sector with no amenity, denoted as the a-sector and the m-sector, respectively. We express atypical individual's utility function as $U = U(X, a, l)$, where X is his commodity consumption, a is the share of his total available time spent for work in the amenity sector and l is the share of time for leisure. Assume that the total time T is divided between leisure time L, work time in the a-sector, A, and work time in manufacturing, M so that

$$T = L + A + M$$

We thus have

$$l + a + m = 1 \quad (1)$$

Where
l ≡ L /T, a ≡ A/T, and m ≡ M /T. Assume that U rises in all three arguments. Consumption X depends on the individual's earnings such that

X = w (1 − a − l) + wSa (2)

Where we use (1), m w is the individual's wage in manufacturing work, w is his wage in the a-sector and S ≡ (1 + s) where s is a subsidy rate provided by the government. We assume that m w and we are fixed for the individual and m w > Sw. Since the job with amenity provides more pleasure than the job with no amenity, it is reasonable to require that the wage rate in the sector with no amenity is higher than the "net wage rate", Sw, in the sector with amenity. Otherwise, all the time available for work would be spent in the sector with amenity. The individual's rational behavior is described by the following maximization problem

Maximize U=U(X, a, l) (3a)

X, a, l

Subject to

$w_m = X + (w_m - wS)\, a + w\, l,$ (3b)

0≤a, l≤1. (3c)

There are then three goods, X, a and l, with prices 1, m (w − Sw), and m w respectively. Define the Lagrange function Γ

$$\Gamma(X,a,l,\lambda) \equiv U(X,a,l) + \lambda[w_m - X - (w_m - wS)a - w_m l]$$

Where
λ is a Lagrangian multiplier. The first-order conditions for the maximization problem are obtained as

$$\frac{\partial \Gamma}{\partial X}\frac{\partial U}{\partial X} - \lambda = 0, \quad (4)$$

$$\frac{\partial \Gamma}{\partial X}\frac{\partial U}{\partial X} - \lambda(w_m - wS) = 0, \quad (5)$$

$$\frac{\partial \Gamma}{\partial X}\frac{\partial U}{\partial X} - \lambda w_m = 0, \quad (6)$$

$$\frac{\partial \Gamma}{\partial X} = w_m - X - (w_m - wS)a - w_{ml} = 0. \quad (7)$$

Calculating the total differentials of Eqs. (4)- (7) With mw, w and S as parameters and expressing the results in matrix form yields [This form is commonly used in social sciences. See Chiang (1984)],

$$D = \begin{bmatrix} dx \\ da \\ dl \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda \mathrm{dw_m} - \lambda \mathrm{Sdw} - \lambda \mathrm{SdS} \\ \lambda \mathrm{dw_m} \\ \lambda_1 \mathrm{dw_m} - \mathrm{Sadw} - \mathrm{wadS} \end{bmatrix} \quad (8)$$

Where $a_1 \equiv l + -l \; and$

$$D \equiv \begin{bmatrix} U_{xx}\, U_{xa}\, U_{xl} & -1 \\ U_{xa}\, U_{aa}\, U_{al} & -w_0 \\ U_{IX}\;\; U_{la}\, U_{ll} & -w_m \\ -1 \; -w_0 \; -w_n & 0 \end{bmatrix}$$

Where the variables with double suffixes are second (partial) derivatives with the variable(s) and 0 m w ≡ w − wS. It is known that a sufficient condition for a solution of the first-order conditions to be a maximum is that the determinant D of the matrix is negative. This is guaranteed if the utility function is strongly concave in X, a, and l. We require this condition to be satisfied in the remainder of this

$$\text{Section.} D \begin{bmatrix} \frac{dX}{da} \\ \frac{da}{dw_m} \\ \frac{dl}{dw_m} \\ \frac{d\lambda}{dw_m} \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda \\ \lambda \\ \mathrm{a_1} \end{bmatrix}$$

This is a linear equation. Applying Cramer's rule, we solve for the impact of a change m w on a as follows

$$\frac{da}{dw_m} = \frac{U_{xx}W - \lambda(\mathrm{U_{xx} w_m\ wS + U_n - U_{al}})}{\det D} \quad (9)$$

Where

$$W \equiv [w_0(a_1 U_n + \lambda \mathrm{w_m}) - a_1 w_m U_{al}]U_{xx.}$$

The effect of an increase in manufacturing wages on part-time in the a-sector is ambiguous. In line with the Slutsky equation, the effect can be broken down into an income effect and a substitution effect. An increase in manufacturing wages implies that the income level has increased. As the amenity sector work is a normal good, there is a tendency to spend a larger share of total work in the a-sector. On the other hand, as the manufacturing wage m w rises, consumption of amenity becomes more costly since the price of amenity sector work, m w − wS, rises. Since the two effects are opposite, the net effect is ambiguous.

Similarly, the effects of an increase in m won consumption of X are given by,

$$\frac{dX}{dw_m} \frac{\lambda_0 + U_{aa}\,(a_1 U_u + \lambda \mathrm{w_m}) - a_1 U_{al}^2 - \lambda \mathrm{w_0 U_{al}}}{detD} > 0 \quad (10)$$

In which

$$\lambda_0 \equiv \lambda(\mathrm{w_0 U_n - U_{al} w_m}).$$

As the manufacturing wage is increased, consumption of the commodity is increased. There is
A positive income effect on X as m w rises. As consumption of amenity work becomes more
Costly as the price of a-sector work rises, the individual substitute consumption of the a-sector
Work with consumption of X.

The effects on leisure of increases in m_w are

$$\frac{dl}{dW_m} = \frac{U_{xx}[a_1 w_m U_{aa} - w_0^2(1+w_m)\lambda - a_1 w_0 U_{al}]}{\det D} \quad (11)$$

The effects are ambiguous. The income effect is positive, but the substitution one is negative.

Conclusion

We can similarly carry out the comparative statics analysis with regard to the a-sector wage and the subsidy rate. This example shows the ‘standard procedure’ for analyzing rational behavior by optimization theory. First we describe mathematically rational behavior of individuals as an optimization problem. Then, we find the first-order conditions and solve the associated equations. Finally we check the second-order conditions and examine the impact of changes in some parameters.

References

1. Becker GS. The Economic Approach to Human Behavior. Chicago: The University of Chicago Press. [A collection of important essays on the economic approach to human behavior written by the winner of the Nobel Prize in economics in 1992, Gary S. Becker]. 1976.
2. Diamand MA, Dimand RW. The Foundations of Game Theory, in three volumes. Cheltenham: Edward Elgar. [The collection gathers together primary sources on the development of game theory from its beginnings until 1960-97.
3. Chiang AC. Fundamental Methods of Mathematical Economics, 3rd edition. London: McGraw- Hill. [A user-friendly introductory book on mathematical economics. It includes elementary concepts and techniques of mathematics for social and behavioral sciences as well.]. 1984.
4. Filar JA, Gaitsgory V, Mizukami K. Edited Advances in Dynamic Games and pplications. Boston: Birkhäuser. [The book presents a comprehensive survey of recent developments and advances in dynamic games and their applications]. 2000.
5. Rosser JB Jr. From Catastrophe to Chaos: A General Theory of Economic Discontinuities, Boston: Kluwer Academic Publishers. [The book provides some illustrative examples of nonlinear economic phenomena such as catastrophe, bifurcations, and chaos.]. 1991.
6. Lundborg P. Job amenity and the incidence of double work. Journal of Economic Behavior and Organization 26, 273-287. [This paper explains why moonlighting exists in labor markets by taking account of job amenities in behavior modeling]. 1995.
7. Taha HA. Operations Research – An Introduction. London: Prentice-Hall International. [A preliminary introduction to operations research]. 1997.