



ISSN Print: 2394-7500  
 ISSN Online: 2394-5869  
 Impact Factor: 5.2  
 IJAR 2019; 5(6): 24-26  
 www.allresearchjournal.com  
 Received: 14-04-2019  
 Accepted: 18-05-2019

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## Ubat graphs

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### Abstract

Koshy (2010) introduced and investigated the concept Complementary Distance Pattern Uniform (CDPU) sets in a connected graph. In this paper, introduces one variety of Complementary Distance Pattern Uniform (CDPU) graphs which is ubat graphs. A graph  $G$  is an ubat graph if there exists  $M \subset V(G)$  such that for any vertex  $u \in V(G) \setminus M$ , there exists a vertex  $v \in M$  with  $d(u, v) = e(u)$ , where  $e(u)$  denotes eccentricity of  $u$ . The set  $M$  is called an ubat\* set of  $G$ . An ubat\* set of  $G$  of minimum cardinality is called an ubat set of  $G$ . The cardinality of an ubat set of  $G$  is called the ubat number of  $G$ , denoted by  $\alpha_u(G)$ . A couple of results are generated in this study. Some of which are the following:  $\alpha_u(G + H) \leq \min \{|V(G)|, |V(H)|\}$  where  $G$  and  $H$  be connected graphs. Let  $G$  be a self-centered graph and  $H$  be any graph. Then  $\alpha_u(G+H) = 2$ . Let  $K_{1,n}$ , and  $K_{m,n}$  be a star of order  $n + 1$  and a complete bipartite graph of order  $n + m$ , respectively.

**Keywords:** ubat graphs, distance, disjoint, join, corona, eccentricity

### Introduction

Graph theory deals with the study of networks. It is studied in different directions and is applied in diverse applications. Some would study domination in graphs, others would study dimension in graphs and others would study geodetics covers. Koshy *et al.* (2010) [6, 7, 8] introduced and investigated the concept Complementary Distance Pattern Uniform (CDPU) sets in a connected graph. Koshy stressed that there are strong indications in the literature of Natarajan (2009) [9] that the notion of Complementary Distance Pattern Uniform (CDPU) sets could be used to design a class of Topological indexes that represent certain stereo chemical properties of the molecule. This paper introduces one varieties of Complementary Distance Pattern Uniform (CDPU) graphs which is called an ubat graphs.

A graph  $G$  is an ubat graph if there exists  $M \subset V(G)$  such that for any vertex  $u \in V(G) \setminus M$ , there exists a vertex  $v \in M$  with  $d(u, v) = e(u)$ . The set  $M$  is called an ubat\*-set of  $G$ . An ubat\*-set of  $G$  of minimum cardinality is called ubat-set of  $G$ . The cardinality of an ubat-set of  $G$  is called the ubat number of  $G$ , denoted by  $\alpha_u(G)$ . This paper introduces and investigated ubat graphs. In particular, the researcher determined the ubat number of the join of graphs. Furthermore, we characterized all graphs with  $\alpha_{cdpu}$  number equal to 1. The concept ubat graphs may be used to find a point in a place or network that is farthest from most of the other points. This point maybe utilized by putting on its undesirable things. For example, a nuclear plant may be situated on such place.

### Objectives of the study

This study has the following objectives:

1. To introduced ubat graphs.
2. To obtain bounds or exact values of the ubat number of the join of graphs.
3. To determine relationships, if there are any, between the ubat number of a connected graph and its order, and other graph invariants.

### Research methodology

The following steps were followed in conducting this study. First, the concepts ubat set, ubat number and ubat graph were introduced. Second, these concepts were discussed thoroughly by giving examples. Third, conjectures regarding the properties of concepts were presented. Finally, these conjectures were shown whether they are true or not.

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**Basic concepts**

**Graph/network**

A graph or a network  $G$  is an ordered pair  $G = (V(G); E(G))$ , where  $V(G)$  is a nonempty set whose elements are called vertices, and  $E(G)$  is a set of 2-element subsets of  $V(G)$  called edges. The order of  $G$  is the number of vertices, that is, the cardinality of  $V(G)$ . The size of  $G$ , denoted by  $|E(G)|$ , is the number of edges of  $G$ . Let  $e = uv$  be an edge joining the vertices  $u$  and  $v$  of a graph  $G$ . Then  $u$  and  $v$  are said to be adjacent, or  $u$  and  $v$  are incident with  $e$ , or  $u$  and  $v$  are the ends of  $e$ .

**Distance in graphs**

**Distance**

For any two vertices  $u$  and  $v$  in a graph  $G$ , the distance  $d(u, v)$  from  $u$  to  $v$  is the number of edges of a shortest  $u - v$  path in  $G$ .

**Eccentricity**

For a vertex  $v$  in a connected graph  $G$ , the eccentricity  $e(v)$  of  $v$  is the distance from  $v$  to a vertex farthest from  $v$ . That is,  $e(v) = \max \{d(v, x)\}$ .

**Disjoint union**

Let  $X$  and  $Y$  be sets. The disjoint union of  $X$  and  $Y$ , denoted by  $X \dot{\cup} Y$ , is found by combining the elements of  $X$  and  $Y$ , treating all elements to be distinct. Thus,  $|X \dot{\cup} Y| = |X| + |Y|$ .

**Join and corona in graphs**

**Join**

The join of two graphs  $G$  and  $H$ , denoted by  $G + H$ , is the graph with vertex-set  $V(G + H) = V(G) \dot{\cup} V(H)$  and edge-set  $E(G + H) = E(G) \dot{\cup} E(H) \dot{\cup} \{uv : u \in V(G); v \in V(H)\}$ .

**Corona**

Let  $G$  and  $H$  be a graphs. The corona  $G \circ H$  of two graphs  $G$  and  $H$  is the graph obtained by taking one copy of  $G$  and  $n$  copies of  $H$ , and then joining the  $i$ th vertex of  $G$  to every vertex of the  $i$ th copy of  $H$ .

**Ubat graphs**

A graph  $G$  is an *ubat graph* if there exists  $M \subseteq V(G)$  such that for any vertex  $u \in V(G) \setminus M$ , there exists a vertex  $v \in M$  with  $d(u, v) = e(u)$ . The set  $M$  is called an *ubat\*-set* of  $G$ . An *ubat\*-set* of  $G$  of minimum cardinality is called *ubat-set* of  $G$ . The cardinality of an *ubat-set* of  $G$  is called the *ubat number* of  $G$ , denoted by  $\alpha_u(G)$ .

**Example of ubat graphs**

Consider graph  $G'$  in Figure 2. Then  $M = \{a, d\}$  is an *ubat set*, whence  $G$  is an *ubat-graph*. To see this, let  $M = \{a, d\}$ . Then  $d(b; d) = 2 = e(b)$ ,  $d(c, a) = 2 = e(c)$ ,  $d(e, a) = 2 = e(e)$ , and  $d(f, d) = 2 = e(f)$ . Hence, for every vertex  $u \in V(G) \setminus M$ , there exists a vertex  $v \in M$  with  $d(u, v) = e(u)$ . Thus,  $M$  is an *ubat\** set. Note that  $\{a\}, \{b\}, \{c\}, \{d\}, \{e\}$  and  $\{f\}$  are not *ubat\*-sets*. Therefore,  $M$  is a minimum *ubat\** set, i.e.  $M$  is an *ubat set*. Accordingly,  $G'$  is an *ubat graph*.

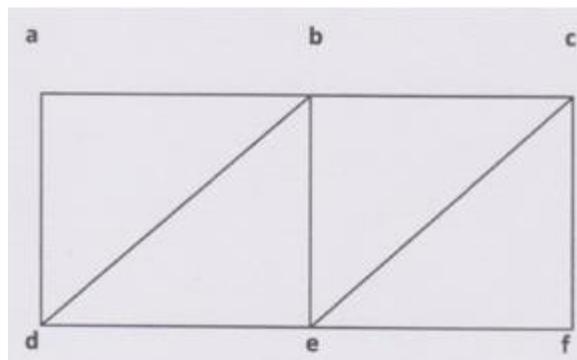


Fig 1: A Graph  $G'$

**Preliminary results**

The following present the results of this study:

**1. Ubat Graphs**

Theorem 3.4.1 Let  $G$  and  $H$  be graphs with isolated vertices  $u \in V(G)$  and  $v \in V(H)$ . Then  $\alpha_u(G + H) = 2$ .

Proof: Let  $G$  and  $H$  be graphs with isolated vertices  $u \in V(G)$  and  $v \in V(H)$ . Consider  $M = \{u, v\}$  and let  $w \in V(G + H) \setminus M$ . Consider the following cases:

Case 1.  $x \in V(G) \setminus M$

If  $x \in V(G) \setminus M$ , then  $d(x, u) = e(x)$ .

Case 2.  $x \in V(H) \setminus M$  If  $x \in V(H) \setminus M$ , then  $d(x, v) = e(x)$ . Hence,  $M$  is an *ubat\*-set*. Therefore,  $\alpha_u(G + H) \leq 2$ . Suppose  $\alpha_u(G + H) = 1$ . Let  $M$  be an *ubat-set* with  $|M| = \alpha_u(G + H) = 1$ . Without loss of generality, let  $M = \{v\}$  with say  $v \in V(G)$  and let  $w \in V(H)$ . Then  $d(v, w) = 1 \neq 2 = e(w)$ . This is a contradiction since  $M$  is an *ubat-set*. Therefore,  $\alpha_u(G + H) = 2$ .

Corollary 3.4.2 Let  $K_{1,n}$ , and  $K_{m,n}$  be a star of order  $n + 1$  and a complete bipartite graph of order  $n + m$ , respectively. Then

1.  $\alpha(K_{1,n}) = 2$ ,
2.  $\alpha(K_{m,n}) = 2$ .

**Summary and Recommendations**

This study encapsulates all the results of this study including particular cases. Moreover, since among results point to very interesting conjectures and very natural generalizations, we also express some recommendations.

**Summary**

1. Ubat-Graphs

1. Let  $G$  and  $H$  be graphs with isolated vertices  $u \in V(G)$  and  $v \in V(H)$ . Then  $\alpha_u(G + H) = 2$ .

**Theorem 3.4.1**

2. Let  $K_{1,n}$ , and  $K_{m,n}$  be a star of order  $n + 1$  and a complete bipartite graph of order  $n + m$ , respectively. Then

- $\alpha(K_{1,n}) = 2$ ;
- $\alpha(K_{m,n}) = 2$ ;

Corollary 3.4.2

### Recommendations

It is recommended that the results in this study be used in endeavors that require the concepts ubat-sets. Moreover, since the concepts ubat-sets are a broad topics in graph theory, there are still a lot of open problems to research on. Some of these includes, finding some bounds or the exact value of the ubat-number of the following:

1. Cartesian product of two arbitrary graphs.
2. Kronecker product of two arbitrary graphs.
3. Composition of two arbitrary graphs.

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