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Unsteady multiple boundary layers viscous flow through MHD porous medium past a porous plate in a rotating system

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Abstract
The aim of the present paper is to study the motion of the three-dimensional unsteady flow of an incompressible, homogeneous, viscous fluid through porous medium bounded by a porous plate with a given velocity to uniform suction or injection in a rotating system. Both the plate and the fluid are in the state of solid body rotation with constant angular velocity Ω about z-axis normal to the plate. A non-torsional oscillation of a given frequency w is super- imposed on the plate for the generation of an unsteady flow in the rotating system. An exact solution of the Navier-Stoke's equations for the unsteady flow is obtained by using the Laplace-transform technique. The effects of uniform suction, injection, permeability and magnetic field on the flow phenomenon in rotating system have also been investigated.

Keywords: Unsteady flow/porous medium/viscous flow

Introduction
The hydromagnetic flow through porous media in a rotating system is of practical interest in many engineering and technological fields. Such type of flow are of great importance to a petroleum engineer concerned with the movement of oil and gas through the reservoir to the hydrologist in his study of migration of underground water.
It is observed that, if the density of the fluid is homogeneous, then the vertical variation of the velocity is completely prohibited and if the density of the fluid is non-homogeneous, then the vector product of the rotation and the vertical gradient of the horizontal velocity must be same as the horizontal gradient of the differential gravitational force. Thus, the fluid in motion satisfies only some restricted boundary conditions, Robinson [9] studied the motion of a rotating stratified inviscid fluid which is bounded above a horizontal plate and below by finite amplitude ridges and presented important characteristics of the flow. Greenspan and Howard [5] have also studied the motion of rotating fluid.
Benney [1], Greenspan [5] and Thornley [8] have studied the flow in a rotating system. Debnath [2] obtained the motion on an unsteady hydromagnetic boundary layer in a rotating fluid. Gupta [4] obtained a solution of the steady state three-dimensional Navier-Stoke's equations for the flow passing through a plate with uniform suction or injection in a rotating system. Debnath and Mukherjee [3] have discussed the motion of an incompressible, homogeneous, viscous fluid bounded by porous plate with uniform suction or injection. Kishore *et al.* [5] have obtained a solution describing the hydrodynamic boundary layer flow of an incompressible, homogeneous, viscous fluid over a porous plate in a rotating system with uniform suction or injection. Kumar and Varshney [7] studied the three-dimensional unsteady flow of an incompressible, viscous fluid through a porous medium past an oscillating porous plate with uniform suction or injection in a rotating system. Reddy [10] studied MHD convective flow of a viscous incompressible and electrically conducting fluid through porous medium bounded by an infinite vertical porous plate in a rotating system. Jat *et al.* [11] discussed three dimensional unsteady flow of an incompressible viscous porous medium past an oscillating porous plate subject to uniform system.
In this paper, we have studied the three-dimensional unsteady flow of a viscous incompressible fluid through a porous medium under the influence of a uniform transverse magnetic field past a porous plate with uniform suction or injection in a rotating system.

Both the plate and the fluid are in the state of solid body rotation with constant angular velocity Ω about z-axis normal to the plate. The unsteady flow is inclined in the fluid by non-torsional oscillations of the plate in its own plate with a given frequency w. The initial value problem is solved in order to determine the significant effects of suction or injection. Special attention has been given to the physical interpretation of mathematical results obtained for the steady state solution with the help of various graphs, in one special case and the effects of suction, injection, permeability and magnetic field on the flow in rotating system are investigated in this case.

Mathematical formulation of the problem

Let us consider the unsteady three-dimensional flow of an incompressible viscous fluid through a porous medium under the influence of a uniform transverse magnetic field of constant strength past an infinite porous plate at z = 0, subject to uniform suction or injection in a rotating system. The uniform magnetic field is applied normal to the plate. Both the fluid and the plate are in a state of solid body rotation with constant angular velocity Ω about z-axis normal to the plate and in addition to it a non-torsional oscillation of a given frequency w is imposed on the plate in its own plane at time t >0 for generation of unsteady flow in rotating system.

Let us take rectangular cartesian system (x, y, z) such that the z-axis be taken normal to the plate which is the axis of rotation and x-axis be taken along the plate. To make the analysis simple following assumptions have been taken.

i. The electrical conductivity 'σ' of the field is sufficiently large so that the displacement current is neglected.
ii. The secondary effect of magnetic induction are neglected.
iii. The induced magnetic field has been neglected.
iv. Only the electromagnetic body force (Lorentz Force) is present.
v. The buoyancy force has been neglected.
vi. Fluid properties (density, viscosity) are constant.
vii. The porosity of the medium tends to one.

The unsteady motion of an incompressible viscous fluid through porous medium under the influence of a uniform magnetic field in a rotating co-ordinate system is governed by the following Navier-Stoke's equations and the continuity equations in the usual notations:

$$\frac{\partial \vec{v}}{\partial t}+\left(\vec{v}.\nabla\right)\vec{v}+2\Omega\vec{K}\times\vec{v}=-\frac{1}{\rho}\nabla p+\upsilon\nabla^2\vec{v}-\frac{\upsilon}{K}\vec{v}-\frac{\sigma B_0^2\vec{v}}{\rho} \quad \text{... (1)}$$

and div $\vec{v}$ = 0. ... (2)

Where $\vec{v}$ (u, v, w) is the velocity vector, $\vec{k}$ is the unit vector along z-axis, p is the pressure including the centrifugal term, ρ is the density of the fluid, K is the permeability of the porous medium, σ is the electrical conductivity and B_o is the constant magnitude of the magnetic induction $\vec{B}$.

We assume that the velocity field depends on z and time t only. It follows from equation (2), together with uniform suction or injection that w = $-w_0$ is constant where w_0 is the suction or injection velocity according as $w_0 > 0$ or $w_0 < 0$.

Now in the absence of any pressure gradient ∇p, the equation (1) can be written as.

$$\frac{\partial u}{\partial t}-w_0\frac{\partial u}{\partial z}-2\Omega v=\upsilon\frac{\partial^2 u}{\partial z^2}-\frac{\upsilon}{K}u-\frac{\sigma B_0^2 u}{\rho}, \quad \text{... (3)}$$

and $\frac{\partial v}{\partial t}-w_0\frac{\partial v}{\partial z}+2\Omega u=\upsilon\frac{\partial^2 v}{\partial z^2}-\frac{\upsilon}{K}v-\frac{\sigma B_0^2 v}{\rho}$. ... (4)

Introducing the notation q = u + iv the equations (3) and (4) reduce into a single equation.

$$\frac{\partial q}{\partial t}-w_0\frac{\partial q}{\partial z}+2i\Omega q=\upsilon\frac{\partial^2 q}{\partial z^2}-\frac{\upsilon}{K}q-\frac{\sigma B_0^2 q}{\rho}. \quad \text{... (5)}$$

For the imposed oscillation on the plate, the equation (5) can be solved subject to a no-slip condition at the plate and no disturbance at infinity. We get $q(z, t) = -U+U_0 (a e^{iwt} + b e^{-iwt})$ at z = 0, t >0 and $q(z, t) \to 0$ or finite as $z \to \infty$, t >0.

Where U and U_0 are constants with dimension of velocity and a, b are some constants. Thus, the initial and boundary conditions for the problem are given as:

t < 0 : q(z, t) = 0 for each z,

$$t>0:\left.\begin{cases} q(z,t)=-U+U_0(ae^{iwt}+be^{-iwt}) at z=0 \\ q(z,t)\to 0 or finite as z\to\infty \end{cases}\right\}. \quad \text{... (6)}$$

Solution of the problem

The following non-dimensional quantities are introduced to put the governing equation and the initial and boundary conditions in the dimensionless form:

$z^* = z\,U_0/\upsilon$, $t^* = t\,\Omega$, $q^* = q/U^0$,

$K_0 = \upsilon^2/K\,U_0^2$, $S = w_0/U_0$, $E = 2\upsilon\Omega\,/\,U_0^2$, ... (7)

$\alpha = w/\Omega$ $M = B_0/U_0\sqrt{\left(\frac{\upsilon\sigma}{\rho}\right)}$ and $K_1 = K_0 + M^2$.

Here, M is the dimensionless Hartmann number.
Using the above non-dimensional quantities and after dropping the stars for convenience the equation (5) reduce to:

$$\frac{\partial^2 q}{\partial z^2} - S\frac{\partial q}{\partial z} - (iE \rightleftarrows + K_1)q = \frac{E}{2}\frac{\partial q}{\partial t}. \quad \text{... (8)}$$

The initial and boundary conditions (6) reduces to:
t < 0 : q = 0 for each z

$$t > 0 : \begin{cases} q = ae^{i\alpha t} + be^{-i\alpha t} - U/U_0 atz = 0 \\ q \to 0 or finite as z \to \infty \end{cases}. \quad \text{... (9)}$$

Equation (8) subject to the initial and boundary conditions (9) can be solved by using the usual Laplace - transform technique. The expression for the velocity field is given as follows:

$$q(z,t) = \frac{a}{2}exp\{i\alpha t - ½Sz\}[exp\{-z\left(\frac{E}{2}\right)^{½}(\beta_1 + i\beta_2)\}.erfc$$

$$\left\{\frac{z}{2}\left(\frac{E}{2t}\right)^{½} - (\beta_1 + i\beta_2)t^{½}\right\} + exp\left\{z\left(\frac{E}{2}\right)^{½}(\beta_1 + i\beta_2)\right\}.erfc$$

$$\left\{\frac{z}{2}\left(\frac{E}{2t}\right)^{½} + (\beta_1 + i\beta_2)t^{½}\right\} + \frac{b}{2}exp \quad \{-(i\alpha t + ½Sz)\} \rightleftarrows \left[exp \quad \left\{-z\left(\frac{E}{2}\right)^{½}\right\}[\tau_1 + i\tau_2]\right]$$

$$\text{erfc}\left\{\frac{z}{2}\left(\frac{E}{2t}\right)^{½} - (\tau_1 + i\tau_2)t^{½}\right\} + \text{exp}\left\{z\left(\frac{E}{2}\right)^{½}(\tau_1 + i\tau_2)\right\}$$

$$\text{erfc}\left\{\frac{z}{2}\left(\frac{E}{2t}\right)^{½} + (\tau_1 + i\tau_2)t^{½}\right\} - \frac{U}{2U_0}exp \quad (-½Sz)\left[exp\left\{-z\left(\frac{E}{2}\right)½\right\}\right]$$

$$(\delta_1 + i\delta_2)\}erfc\left\{\frac{z}{2}\left(\frac{E}{2t}\right)^{½} - (\delta_1 + i\delta_2)t^{½}\right\} + exp \quad \left\{z\left(\frac{E}{2}\right)^{½}(\delta_1 + i\delta_2)\right\}$$

$$\text{erfc}\left\{\frac{z}{2}\left(\frac{E}{2t}\right)^{½} + (\delta_1 + i\delta_2)t^{½}\right\}. \quad \text{... (10)}$$

Where,
$\beta_1 = ½E^{-½}[\{(S^2 + 4K_1)^2 + 4E^2(2+\alpha)^2\}^{½} + (S^2 + 4K_1)]^{½}$
$\beta_2 = ½E^{-½}[\{(S^2 + 4K_1)^2 + 4E^2(2+\alpha)^2\}^{½} - (S^2 + 4K_1)]^{½}$
$\tau_1 = ½E^{-½}[\{(S^2 + 4K_1)^2 + 4E^2(2-\alpha)^2\}^{½} + (S^2 + 4K_1)]^{½}$
$\tau_2 = ½E^{-½}[\{(S^2 + 4K_1)^2 + 4E^2(2-\alpha)^2\}^{½} - (S^2 + 4K_1)]^{½}$
$\delta_1 = ½E^{-½}\left[\{(S^2 + 4K_1)^2 + 16E^2\}^{1/2} + (S^2 + 4K_1)\right]^{½}$ and
$\delta_2 = ½E^{-½}[\{(S^2 + 4K_1)^2 + 16E^2\}^{½} - (S^2 + 4K_1)]^{½}$

By asymptotic representation of the complementary error function as t → ∞, the solution (10) reduces to the steady state form:

$$q(z,t) = a\ exp\left[i\alpha t - z\left\{\frac{S}{2} + \left(\frac{E}{2}\right)^{½}(\beta_1 + i\beta_2)\right\}\right]$$

$$+ \quad b\ exp\left[-i\alpha t + z\left\{\frac{S}{2} + \left(\frac{E}{2}\right)^{½}(\tau_1 + i\tau_2)\right\}\right]$$

$$-\frac{U}{U_0} exp \quad \left\{-z\left(\frac{S}{2}+\left(\frac{E}{2}\right)^{1/2}(\delta_1+i\delta_2)\right)\right\}. \qquad \ldots (11)$$

The solution (10) describes a general feature of unsteady hydrodynamic boundary layer flow in a rotating fluid which includes the effect of uniform suction.

Now, we replace S by -S'. It follows that S' >0 and equations (10) and (11) reduces to other forms which describes a general feature of the unsteady hydrodynamic boundary layer flow in a rotating fluid which includes the effect of uniform injection.

Results and Discussion

In the particular when U = 0, b = 0 and a = 1 then we have a pure oscillation of given frequency w imposed on the plate. The effects of suction and injection on the steady state solution has been explained graphically.

Fig. 1 shows the effect of uniform suction on the steady state solution of the flow in rotating system. We observe that the velocity field decreases with an increase in suction. The velocity decreases more rapidly on an increase in suction parameter. Figs. 1 and 2 shows the effect of magnetic field on the velocity distribution. As the Hartmann number M increases, the velocity field decreases. The effect of Permeability of the medium of steady state solution of the flow in rotating system is also shown in these Figs. As the permeability parameter K increases the velocity field also increases. The effect of Uniform injection on the steady state solution of the flow is shown in Fig. 2. It is to be observed that the velocity field decreases with an increase in injection parameter.

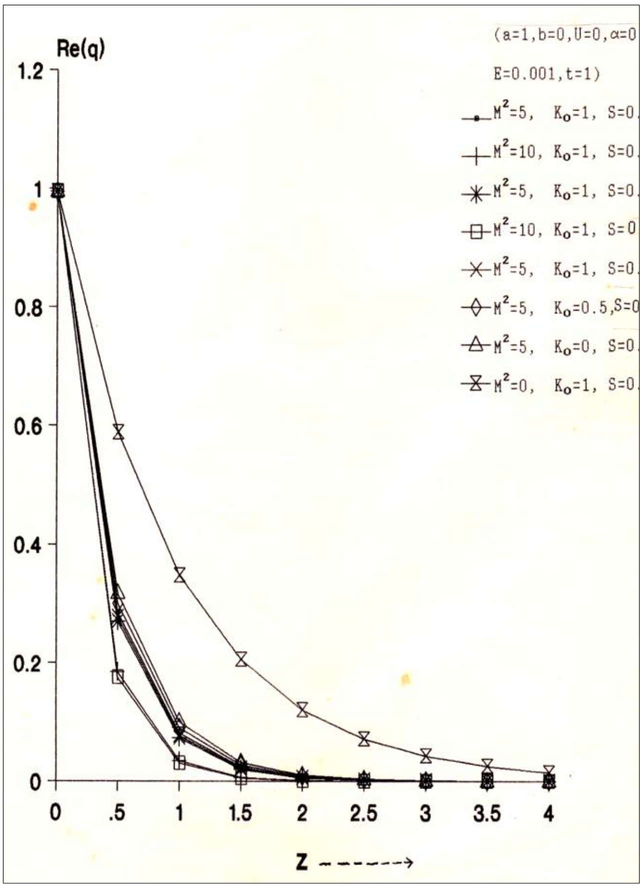

Fig 1: Velocity Profiles

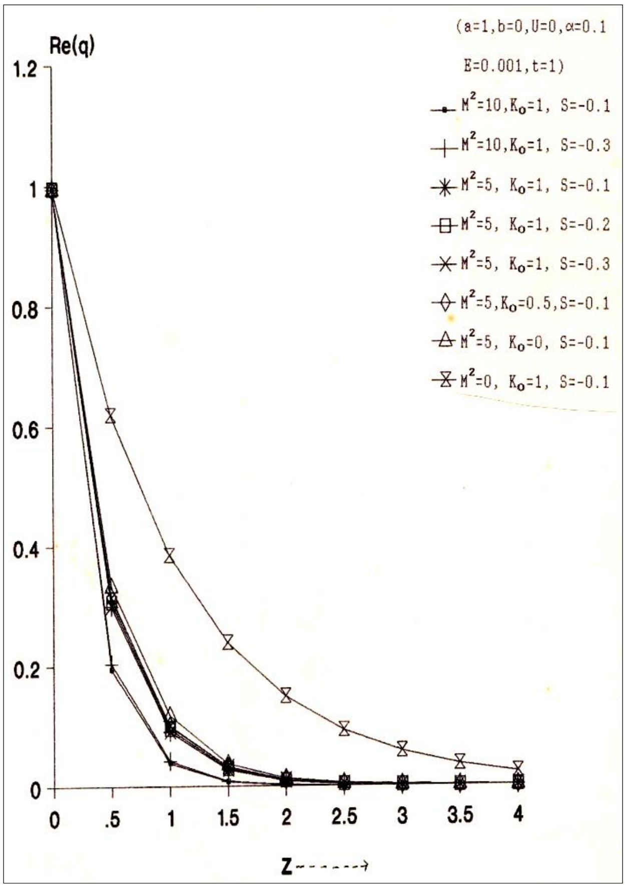

Fig 2: Velocity Profiles

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