International Journal of Applied Research 2019; 5(9): 154-155

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2019; 5(9): 154-155
www.allresearchjournal.com
Received: 08-07-2019
Accepted: 10-08-2019

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On characterisation of p-solvable groups and its invariance

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Abstract
Our main aim is to derive the relationship between representation theory groups with invariants of subgroup of a finite group. We consider p-solvable groups for examining the converse cropped up due to Brauer's hypothesis. The converse of Brauer's hypothesis was proved partially for p-solvable groups by D. Gluck and T. Wolf. We show that for an arbitrary group for p-solvable groups, there exists some characteristics due to the heights of the characters in Irr (B) with respect to the exponents of the character of the group D (Dihedral group).

Keywords: Character, Brauer hypothesis, p-solvable group, irreducible, defect group invariant

Introduction
The most important objective in representation of finite group theory is to establish its connection with theory invariants of a finite group G with some local subgroups of G. Let p be a prime, let G be a finite group and let B a p-block of G with defect group D. If $X \in$ Irr(B) is an irreducible complex character of B, the height h of X is the non-negative integer which satisfy the given relation.

$$\chi(1)_p = P^{a-d+h} \quad \text{.... (1.1)}$$

Where $|G|_p = P^a$ and $|D| = P^d$. If B is a Brauer's p- block of G having defect group D, then complexity of D reflects in the set Irr(B) of complex irreducible characters in B, Which implies that D is a abelian if and only all characters in Irr(B) have height zero. F. Fong proved it for p-solvable groups. We show that the group of outer automorphisms of E that fix all the elements of Z(E), contains as a subgroup, which is either the symplectic group Sp^2_{n-2} (p) or an orthogonal group O_2n (2). Let us choose n to be sufficiently large and assume that out(E) has a subgroup that is a solvable. Frobenius group F= PH whose complement P is a cyclic P-group of order Pm with $m \geq 2$. Let G be this extension of E by F. Then O_p (G)= 1, which means that G has a unique p-block. Now, let $\theta \in$ Irr (E) be non linear and by suitable application of θ.

Theorem
Suppose that A acts coprimely as automorphism on a finite group G. If C_A (G) = 1 then there exists a nilpotent A-invariant subgroup H of G such that $C_A(H)=1$.

Proof: R. Guratinich and R. Robinson proved invariance for groups. We consider here with basis of the classification of finite simple groups. The properties of heights of characters are compared with the order of the when defect group which are comparable if h is the height of an irreducible character of a block B and d is the defect of B, Then $h \leq d$. Also h=d if and only if d=0

If d > 0, the ratio $\frac{h}{d}$ is sufficient condition for G= SL_n (p) with the defining characteristic p, for all characters except the Steinberg character lying in the principal block. Also there exists a unipotent character with height $\frac{(n-1)(n-2)}{2}$. But the defect of the block is n $\frac{(n-1)}{2}$, determined it for p-solvable groups which is contradicts to assumption of the given theorem.

Theorem

Suppose the G is a solvable group. Let $X\in Irr(B)$of height h and B has defect group D. If $|D:Z(D)| = P^n$ then
$h \le 7n/8$.

Proof: We extend the result derived by A. watanbe for p-solvable group. We prove here a generalized version of theorem (1.3) that "almost" yields bound for large primes. We show that it does not hold for an arbitray group as derived by G.R. Robinson concerning Dade's Conjecture.

Let F*(G) denote the generalized fitting subgroup of G. We know that for any group G, C_G (F*(G))≤F*(G). It is applied for proving the three subgroups lemma when N = F*(G).

Theorem

Suppose that A acts faithfully and coprimely on a finite simple group G. Then A acts faithfully on some A- invariant sylow subgroup of G.

Proof: D. Robinson derived that sporadic groups do not admit co–prime automorphism, By an appropriate application the classification of finite simple groups it implies that G is of Lie type.

Let p be the characteristic of G. We show that A acts faithfully on any A-invariant Sylow p-subgroup of G. Let us assume that A = $\langle a\rangle$ is cyclic of prime order. Let U be an A- invariant Sylow p-subgroup of G. We prove its contrary by assuming that [U, A] = 1, which implies that its rank is 1, let B = $N_G(U)$.

Then U = F*(B), So that [B, a] = 1. If G has rank greater then 1, then G is generated by parabolic subgroups of the form $N_G(V)$, where $1\neq V \trianglelefteq U$. We find that N_G (V) is A–invariant as [U, a] =1. But for such Subgroups, F* ($N_G(V)$) = $O_p(N_G(V)) \le U$. So that a centralizes F* ($N_G(V)$) and it centralizes $N_G(V)$. Thus G has a rank 1. We know that G is one of the groups, $L_2(q)$, $U_3(q)$,${}^2G_2(q)$ ${}^2B_2(q)$ where q is a power of p (where q is an odd power of p in the last two cases, and p = 3, 2 respectively). In all cases, G is a doubly transitive group on cosets of B and we obtain|G:B|=1+|U|. But G = $B\bigcup Bw$ B, Where w is an involution. Also w normalizes $B\cap B^w$ = T, and T is a Hall. P'-subgroup of B.

Hence G = B $N_G(T)$B. Hence $|N_G(T) : T|$= 2 in all cases, and as such $[N_G(T), a] \le T$ and $[N_G(T), a, a] \le [T, a] = 1$. Thus $[N_G(T), a]$=1 by co–prime action, So that [G, a] = 1 which is contrary to assumption of the theorem.

Theorem

Suppose that A acts coprimely as automorphism on a finite group G. If $C_A(G)$ = 1, then there exists a nilpotent A-invariant subgroup H of G such that $C_A(H)$ = 1.

Proof: We prove it by induction on |G|, Let Z = Z (G). Let us assume that Z = 1. If not possible, let D = $C_A(G/Z)$. We thus find that A/D acts faithfully on G/Z. By induction there exists an A-invariant nilpotent Subgroup V/Z of G/Z on which A/D acts faithfully, Hence V is A-invariant and nilpotent. Now, C_A (V)D/D) centralizes V/Z, and $C_A(V) \le$ D. Since $C_A(V) \le C_A(Z)$, it implies using co-prime action that $C_A(V) \le C_A(G)$=1. We first show that G is not expressible as the product of two proper A-invariant normal subgroups G=MN with [M,N]=1. Let B = $C_A(M)$, C= $C_A(N)$ then $B\cap C$ = 1. Now A/B acts faithfully on M, hence by induction there is nilpotent A-invariant subgroup H of M with $C_A(H)$ = B. Similarly, A/C acts faithfully on N. By induction there is a nilpotent A-invariant subgroup K of M with $C_A(K)$ = C, then HK is nilpotent, A-invariant and $C_A(HK)$=1 we assume that G=F* (G) such that F*(G)=E(G)F(G), Where E(G) is the layer of G and G=E(G). Now, by suitable application of the result due to T.R. wolf, G = $K_1 \times K_2 \times\cdots \times K_r$, where K_i is the Simple non- abelian.

Now, A acts on $\{K_1, K_2,\cdots, K_r\}$. Let, $\Delta_1,\cdots, \Delta_s$ be the distinct A-orbits and $\wedge_i=\Pi_{T\in\Delta_i}T$. We get $G=\wedge_1\times\cdots \wedge_s$. If S>1, Therefore, A acts transitively on the set $\{K_1,\cdots, Kr\}$. Let us write $K_i = K^{ai}$ for some $a_i \in$ A, Where $K=K_1$ and a_1=1.B= $N_A(K)/C_A(K)$.

We find that B acts faithfully on the non abelian simple group K. There exists a B-invariant Sylow q-subgroup $1 \neq Q$ of K such that B acts faithfully on Q. We show that $C_A(Q) \le C_A(K)$. Let $a\in C_A(Q)$. Let us suppose that $K^a= K_2$, for instance. Then $Q = Q^a \le K_1\cap K_2=1$ and this does not hold. Therefore, $a\in N_A(K)$. Hence, a CA(K) 2B centralize Q and it is proved. Let $Q_i = Q^{ai}$ and let U= $Q_1\cdot\cdot\cdot Q_r$, which is again nilpotent. We thus find that $C_A(Q_i) \le C_A$ (K_i). Now, let a $\in C_A(U)$. Hence, a $\in C_A$ (Q_i). Then a centralizes G and U is A-invariant. Let $X\in A$.

If $K^x{}_i = K_i$, it is sufficient to show that $Q_i{}^a = Q_j$. But $K^2{}_j{}^x = K_j{}^a$. Hence, $a_ixa^{-1}J\in N_A(K)$. Thus $Q^{aixa\text{-}1}{}_j$=Q, and $Q_i{}^x=Q_i$, Hence the theorem is proved.

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