International Journal of Applied Research 2020; 6(10): 354-357

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2020; 6(10): 354-357
www.allresearchjournal.com
Received: 13-08-2020
Accepted: 19-09-2020

Vikash Raj
Assistant Professor, Department of Physics, Lord Krishna College, Samastipur, Bihar, India

Corresponding Author:
Vikash Raj
Assistant Professor, Department of Physics, Lord Krishna College, Samastipur, Bihar, India

Scattering of the transmitted light by randomly rough dielectric surface

Vikash Raj

Abstract
Results can be formulated in terms of the photometric scattering indicatrix, what allows their direct application to the transport theory (TT) problem as the more general boundary conditions. In addition, equations (6) and (9) in combination can be used in TT as an alternative to the well-known Henyey-Greenstein scattering indicatrix. As to the basic question formulated at the beginning of this paper about the scattering of transmitted into a dielectric medium radiation, the calculations have shown that, in the general case of a normal illumination, a light beam having passed through a randomly rough interface retains, to a high degree, its initial direction, while the angular divergence of the transmitted beam is not very high.

Keywords: Transmitted light and dielectric surface

Introduction
At the end of the 20th century the fast development of biomedical tissues optics [1, 2] renews the interest to the light transport and scattering theory. But the general transport equation hasn't today an analytical solution in a common case what makes it difficult to apply the theory to practical tasks. Recently we have reported about one new approach to a multi-dimensional transport task [2]. As it follows from that, among different problems in the classical transport theory (TT) there is one, which is not now widely discussed yet, especially in application to biomedical optics, the effect of boundary scattering by a rough profile of the surface. Today in the TT the boundary conditions of a plane surface are used more often [3]. But the real scattering biological media usually have a non-plane, randomly rough surface, so the randomly rough surface boundary conditions are desirable in the TT in a general case. This effect of boundary scattering can potentially have an influence on a total light distribution into a dielectric or a semi-dielectric scattering media. So, a development of different theoretical approaches to calculate the surface scattering is an important problem of the classical TT. Not long ago a number of authors [4] had attempts to describe some theoretical approaches to the problem of light distribution on a surface of diffusive media. However, the simplified approaches used by them don't allow anyone to obtain acceptable quantitative results and were justifiable on initial stages of study only. One of the more exact and acceptable approaches can be based on the electromagnetic wave and diffraction theory (EWDT). The classical EWDT [5, 6] allows to determine the scattered electromagnetic field in a frontal half space (reflected field) for cases of perfectly conducting, well conducting and, in some cases, dielectric randomly rough surfaces. But for dielectric transparent turbid media and problems of light propagation deep into them a calculation of a transmitted radiation is more interesting as well as for a light waveband and biomedical optics the result must be presented in the photometrical or optic terminology. In this work we attempted to resolve this problem translating the EWDT results into a photometry terminology and deriving the formula for a transmitted throughout a dielectric surface radiation for the practically more important case of a normal incident laser beam and a great rough Gaussian surface.

Theoretical Analysis
In a general case, the overwhelming majority of the modern EWDT results are based on a solution of the Green's integral vector equations, which are also well known as the Stratton-Chu vector equations [7]:

$$\mathbf{E}^S(\mathbf{r}) = rot\int_S [\mathbf{n}' \times \mathbf{E}(\mathbf{r}')]\varphi(\mathbf{r},\mathbf{r}')dS' + \frac{i}{k\varepsilon}rotrot\int_S [\mathbf{n}' \times \mathbf{H}(\mathbf{r}')]\varphi(\mathbf{r},\mathbf{r}')dS'$$

$$\mathbf{H}^S(\mathbf{r}) = rot\int_S [\mathbf{n}' \times \mathbf{H}(\mathbf{r}')]\varphi(\mathbf{r},\mathbf{r}')dS' - \frac{i}{k\mu}rotrot\int_S [\mathbf{n}' \times \mathbf{E}(\mathbf{r}')]\varphi(\mathbf{r},\mathbf{r}')dS' \quad (1)$$

where: $E^S(r)$ and $H^S(r)$ are the complex amplitudes of the scattered by surface electric and magnetic fields at the point of space r, S is the area of the surface, n' is the unit vector of the external normal to S at the surface's point r', k the wave number, ε and μ are the permittivity and permeability of the medium and $\varphi(r, r') = e^{ik|r-r'|}/4\pi\ |r - r'|$ is the Green's function.

Equations (1) are valid for any point of space with no restrictions. They allow calculation of the scattered by the surface S fields through the calculation of the tangential to S components of the field vectors ([n′ × E] ; [n′ ×H]). Usually, the tangential components are determined using various approximate methods. One of the simplest methods employs the Kirchhoff approach and the boundary conditions of a perfectly conducting surface [5]

$n' \times H(r') = J(r'),\ n' \times E(r') = 0$ (2)

$J(r') = 2 \cdot n' \times H^i(r')$ (3)

where $H^i(r')$ is the field of the incident plane wave and J(r') is the surface current on S.

In this case the system (1) becomes much simpler, and for the field scattered by the surface of a perfectly conductor it is sufficient to consider the integral equation [8]:

$$\mathbf{H}^S(\mathbf{r}) = \int_S \nabla_r \cdot \varphi(\mathbf{r},\mathbf{r}') \times [2\mathbf{n}' \times \mathbf{H}^i(\mathbf{r}')]dS'$$

Its solution can be found in the closed form if the geometry of the problem is chosen. The general geometry of problem is shown in the figure 1. It is convenient to present the surface as a square plate with the side length L, illuminated by a unit unbounded linearly polarized plane electromagnetic wave incidents at the angle φ to the normal

$H^i(r) = \eta \cdot e^{i(k,r)}$, $|k| = k$

Where η is the unit basis vector of the plane of polarization of the incident wave.

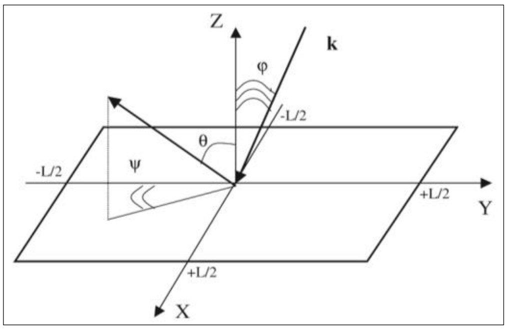

Fig 1: The general geometry of the problem

To describe the randomly roughness it is most convenient to consider the roughness heights as a randomly Gaussian, homogeneous and isotropic field z=ξ(x, y) with zero mean (< ξ(x, y) >= 0), variance $< \xi^2(x, y) >= h^2$, and Gaussian correlation function where $\tau^2=x^2+y^2$ is the distance between the points considered and T is the correlation length.

$$C(r) = \frac{< \xi(x,y)\cdot\xi(0,0) >}{< \xi^2(x,y) >} = e^{-\tau^2/T^2}$$

First of all, let's take into consideration the classical statistical Isakovich-Beckman approach [5] for the field scattered into the outer half-space. Let the direction of scattering will be characterized by the vector ω. Assume also that the central plane of the plate coincides with the XOY plane. Then for a perfectly conducting surface and for the far zone of radiation we can write the well-known solution for a normalized scattered field [8, 9]:

$$H(\phi,\theta,\psi) = \frac{\langle|\mathbf{H^S(r)}|^2\rangle}{|\mathbf{H}^i(\mathbf{r}')|^2} = \frac{1}{R^2}\cdot\frac{S}{\lambda^2}\cdot\mathbf{e}^{-g}\cdot|\omega\times[\mathbf{q}\times\eta]|^2\cdot\frac{1}{q_z^2}$$
$$\times\,(S\cdot(\frac{\sin q_x\cdot\frac{L}{2}}{q_x\cdot\frac{L}{2}})^2\cdot(\frac{\sin q_y\cdot\frac{L}{2}}{q_y\cdot\frac{L}{2}})^2 + \pi T^2\sum_{m=1}^{\infty}\frac{g^m}{m!m}\cdot\mathbf{e}^{\frac{-q_{xy}^2\cdot T^2}{4m}}) \quad (4)$$

Where: q=k-k·ω; R is the distance from the origin of coordinates to the observation point; q_x, q_y, q_z are components of the vector q; q_{xy} =

$$\sqrt{q_x^2+q_y^2};\ S=L^2;\ g=h^2q_z^2;\ \lambda=2\pi/k$$

Wavelength. As it was shown (9, 10) the photometrical reflectance indicatrix ρ_r can be found with the use of (4) as follows:

$\rho_r(\varphi, \theta, \psi) = H(\varphi, \theta, \psi)\cdot \pi \cdot R^2/S$ (5)

It allows us to write a closed-form equation for any reflectance indicatrix as a function of parameters of surface roughness and angels of illumination (observation). For example, in a case of perfectly conducting very rough surface (h @ λ) and a normal illumination [8, 9]:

$$\rho_r(\phi=0,\theta,\psi) = \frac{T^2}{4\cdot h^2\cdot(1+\cos\theta)^2}\cdot\mathbf{e}^{-\frac{\sin^2\theta\cdot T^2}{4h^2(1+\cos\theta)^2}} \quad (6)$$

What is interesting [9], the Eq. (6) describes well the Lambertian scattering when T/h=4. So, the EWDT approach with the use of (5) allows an analytical description of the surface scattering in both the electrodynamics and a photometry terminology.

For the finite conductive media the initial integral equations (1) are to be solved. It can be done most easily for a case of so-called "well-conductive medium", when the impedance boundary conditions (IBC) can be formulated on a rough surface [6, 9]:

$n \times E = \sqrt{\frac{\mu}{\varepsilon}} \cdot n \times n \times H$ (7)

The IBC (7) reflects the fact that the tangential components of the field on the surface of a good conductor are continuously transformed into the transverse components of the field of wave propagating deep into the conductor [6]. When ε → ∞, condition (7), as could be expected, transforms into the (2). In the case of Kirchhoff approximation, for example, for the vertical polarization of the incident beam, the tangential components of the fields on S can be expressed from (7) as follows:

$$\mathbf{n}\times\mathbf{H}=\frac{2}{1+\sqrt{\frac{\mu}{\varepsilon}}\cdot\frac{1}{\cos\phi}}\mathbf{n}\times\mathbf{H}^i;\quad \mathbf{n}\times\mathbf{E}=\frac{2}{1+\sqrt{\frac{\mu}{\varepsilon}}\cdot\cos\phi}\mathbf{n}\times\mathbf{E}^i \qquad (8)$$

Where φ is the local angle of incidence, H^i and E^i are the vectors of the field of the incident wave.

Further simplification can be connected with the replacement of coefficients depending on the local angle of incidence in (8) by their average values. It allows anyone to derive a final analytical solution of (1) in form like equation (4) by changing the vector product $|\omega \times q \times \eta|^2$ only (9) as well as to use (5) to rewrite result into the photometry terminology. However, a lot of kinds of scattering media, biological tissues for example, evidently, are not media with high conductance in the optical waveband. A lot of them are more good dielectrics than conductors, for which the conductance in the optical waveband can be neglected in calculations at all. The solution of (1) for well-dielectric media is much more complex and difficult than one mentioned above. So, if (8) could be valid for dielectric media as minimum in several cases then the solution of the task for such cases can be now quite simple. The applicability of the IBC to dielectric media was detailed discussed by Maradudin and Mendes [11]. It was concluded that (8) are valid not only for media with high conductance but also for perfect dielectrics with a high real part of the refractive index. In this case the angle of refraction of radiation at a medium-air interface can be considered as real and zero, which leads to (8). Moreover, the authors evidently did not notice that, under normal illumination (φ=0), the nonlocal angle of refraction of radiation is also real and nonzero, and the wave penetrating deep into the medium is homogeneous and transverse, so (8) are valid for dielectric media in this case as well.

To obtain a solution for a transmitted radiation it is necessary to direct the propagation vector ω inside the medium in the existing coordinate scheme. It should also be taken into account that the medium under the surface has a refractive index different from 1. As for the rest, the procedure to obtain the final result is quite similar to those described above: the scattering is described by (3), but with modified components of vectors q and k. The vector product $|\omega \times q \times \eta|^2$ transforms in this case (φ=0) into the conventional Fresnel transmission coefficient [12]. This simple reasons allows us to write an analytical equation for the transmitting indicatrix of very rough (h @ λ) surface in the closed form like (6): where ρ_τ is the transmission indicatrix and n is the real part of the refractive index of the medium.

$$\rho_r(\phi=0,\theta,\psi)=\frac{n^3\cdot(n-1)^2\cdot T^2}{h^2\cdot(n+1)^2\cdot(n\cdot\cos\theta-1)^4}\cdot\mathbf{e}^{-\dfrac{n^2\sin^2\theta\cdot T^2}{4h^2(n\cdot\cos\theta-1)^2}} \qquad (9)$$

Numerical results

Some doubts may arise about the fulfillment of the conditions of the far zone just above the rough surface. These doubts can be removed by a more detailed consideration of the procedures of integration of the equations (1) when finding the statistical parameters of the scattered fields, as well as the issue concerning the location of the far zone in the presence of coarse roughness on the surface. As it was indicated (5), in integration (1) over the surface in order to find the incoherent component of the scattered field, the important domain of integration lies within the correlation length T. It is the so-called dominant area in formation of the diffracted field from the coarsely rough surface. Moreover, the distance to the far zone decreases drastically with the appearance of the surface's roughness [13]. Thus, as the radiation passes through the coarsely rough interface, the field is formed within the range of a few T, and all the above equations remain valid for analysis of purely surface effects (neglecting radiation scattering and absorption in the volume of the medium, which are already subjects of the TT investigation).

Table: 1 Illustrated function of randomly rough surface of a dielectric medium

Angle	Surface's parameters						
θ	n=1.4			T/h=8.0			
(deg.)	T/h=4.0	T/h=8.0	T/h=16.0	n=1.2	n=1.6	n=1.8	n=2.0
0.0	47.64	190.6	762.2	571.2	107.7	74.39	56.89
5.0	34.29	43.57	1.773	6.411	46.44	41.17	35.80
10.0	11.41	0.325	0.000	0.001	3.060	6.234	8.263
15.0	1.150	0.001	0.000	0.000	0.016	0.176	0.541
20.0	0.012	0.000	0.000	0.000	0.001	0.001	0.006
25.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Semispherical transmission coefficient						
	0.982	0.975	0.975	0.993	0.951	0.924	0.896

In Table 1, the degree of the beam broadening upon passage through a randomly rough surface of a dielectric medium is illustrated as a function (9). The semispherical transmission coefficient is also calculated there for every case through integration of the indicatrix over the solid angle within the lower hemisphere.
The data of the Table clearly illustrate the beam broadening with increasing surface roughness and refractive index of the medium. True, in all the presented cases, which are closest to the real cases of biological tissues, the transmitted beam retains, to a high degree, its initial direction, and the angular divergence of the transmitted radiation proves not to be very high. Nevertheless, it can influence on the general solution in TT for the radiation field inside the medium as compared to the model of a plane interface. The use of the boundary indicatrix (9) as a boundary condition in TT can allow everyone to consider and study this effect.

Conclusion

In this paper we can state that the solution of the problem of diffraction of electromagnetic waves on a randomly rough surface both with the boundary conditions of an ideal conductor and an impedance interface allows an analytical and closed-form description of the surface scattering. In particular, under a normal incident beam a closed-form equation can be obtained for a light propagating deep into the perfectly dielectric medium as well. In all cases, results can be formulated in terms of the photometric scattering indicatrix, what allows their direct application to the transport theory (TT) problem as the more general boundary conditions. In addition, equations (6) and (9) in combination can be used in TT as an alternative to the well-known Henyey- Greenstein scattering indicatrix. As to the basic question formulated at the beginning of this paper about the scattering of transmitted into a dielectric medium radiation, the calculations have shown that, in the general case of a normal illumination, a light beam having passed through a randomly rough interface retains, to a high degree, its initial direction, while the angular divergence of the transmitted beam is not very high.

References

1. Tuchin VV. “Laser and Fiber Optics in Biomedical Research,” SGU, Saratov 1998.
2. Rogatkin DA. Kvantovaya Elektron. (Rus J Quant Electron) 2001;31(3):279-281.
3. Ishimaru A. “Wave Propagation and Scattering in Random Media,” Academic Press New-York 1978, 1.
4. Ripoll J *et al*. J Opt Soc Am A 2000;17:1671-1681.
5. Beckman P, Spizzino A. “The Scattering of ElectromagneticWaves from Rough Surfaces,” Pergamon Press, Milan-London-New-York 1963.
6. Nikol’skii VV, Nikol’skaya TI. “Electrodynamics and the Radiowave Propagation,” Nauka, Moscow 1989.
7. Colton D, Kress R. “Integral Equation Methods in Scattering Theory,” Wiley, New York 1984.
8. Rogatkin DA, Bulavskii Yu V, Konyakhin VV. Preprint No. 896, VTs SO AN USSR (Computer Center, Siberian Division, USSR Academy of Sciences), Novosibirsk 1990.
9. Rogatkin DA, Tchernyi VV. Proc. SPIE 2002;4617:257-266.
10. Rogatkin DA. Opt. Zh., No. 9, Sov. J Opt. Technol 1992, 72-74.
11. Maradudin AA, Mendes ER. Opt. i Spektrosk 1996;80(3) Opt. Spectrosc 1996;80:459-470.
12. Rogatkin DA. Opt Spectrosc 2004;97:484-493.
13. Rogatkin DA, Konyakhin VV. Pribory i Tekhnika Eksp 1992, 200.