Capillary rise

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Abstract

Results for simulations on single- and two-phase fluids is analysed in this paper. The purpose of these simulations was to better understand the interactions of solid-liquid (wetting) and gas-liquid (surface tension). It will also be important to see how well the LB system would replicate the well-known phenomenon of capillary rise, and the method has therefore been tested in a small capillary pipe on an uprising fluid.

Keywords: Single- and two-phase fluids, surface tension and solid-liquid

Introduction

In recent years, the capillary rise phenomenon has been of broad theoretical and practical importance, with applications ranging from basic capillary rise and imbibition (liquid (droplet) penetration into a porous material) to droplet spreading and other wetting-related phenomena. A long time ago, the basic analytical theories for capillary rise were established, but recent experimental and computational techniques brought new insight into this issue: first the LGA models on the numerical side and then the more recent LB process [1-5]. The movement of an incompressible fluid is viewed as a Poiseuille flow in the classical study by Washburn. When we consider the increase of an incompressible liquid in a capillary pipe, we can therefore proceed from the Hagen-Poiseuille equation under a pressure drop DP for a fully formed pipe flow.

\[ \frac{dQ}{dt} = \frac{\pi \Delta Pr^4}{8\mu (h + h')} \]

where \( dQ \) \( dt \) = \( \pi r^2 dh \) \( dt \) is the volumetric flow rate, \( h \) is the height of the rising liquid column from the level of the liquid surface outside the pipe, \( h' \) is the length of the capillary pipe immersed in the liquid, \( r \) is the radius of the pipe, and \( \mu \) is the viscosity of the liquid. The total pressure drop may be expressed as a sum of capillary pressure and the static pressure exerted by gravity. The contact angle is fixed in the classical capillary rise theory, but the contact angle varies with velocity in the phenomenologically corrected version of this theory, which is thus called the dynamic contact angle.

We first apply the one-dimensional Reynolds transport theorem in a control volume to approximate the rate of change of the system's momentum for an indirect determination of \( qd \). For the meniscus of the liquid column, the upper control surface swings (no outflow from the control volume) and the lower control surface is set at the lower end of the shaft. Then within the control volume, the rate of change of the overall momentum is

\[ \frac{d}{dt} (mv) = \frac{d}{dt} \left[ \int_v \rho \mathbf{dV} \right] + \int_{\Gamma_t} \rho \mathbf{v} (\mathbf{n}) dA. \]

Where \( v \) and \( n \) are liquid velocity and a unit vector normal to the inlet control surface, respectively. Assuming a constant velocity through the control surface, we obtain an expression for \( qd \) by substituting

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\[
\cos \theta = \frac{1}{2\pi r^2} \frac{d(mv)}{dt} - \frac{2}{4} \rho \pi r^2 v^2 + 8\pi \mu (h + h) v + \rho \pi r^2 gh.
\]

**Results and Discussion**

By using the LB method [II] with periodic boundaries in the lateral directions and without any tubing, we first simulated a three-dimensional two-phase fluid structure. The simulation code for single-phase fluid and flow across porous media has already been checked and several benchmark findings have been published \[6\]. After a certain amount of iterations, the pipe was inserted as the machine entered the equilibrium condition, and the simulation proceeded until the capillary increase was saturated (c.f. the snapshots in the insets of Figure 1 and 2).

The simulations reported below were performed for pipe radii \( r = 2, 5, 10 \), while the full domain size was \( 50 \times 50 \times 200 \), and \( 100 \times 100 \times 300 \) for \( r = 20 \). In all capillary rise simulations the relaxation parameter was \( \tau = 1 \), and unless stated otherwise, the adhesive parameters were \( W = -0.1 \) and \( G = -0.15 \). This resulted in \( \rho_f \sim 20 \), with the bulk density of the liquid phase \( \rho_f = 2.25 \), viscosity \( \mu_f = 0.375 \), and the surface tension \( \gamma = 0.085 \).

**Fig 1:** The height of the column and the dynamic contact angle as functions of simulation time for the pipe radius \( r = 5 \). The inset shows three snapshots of the capillary rise: initial, intermediate, and steady state (side view).

**Fig 2:** The height of the column and the dynamic contact angle as functions of simulation time for the pipe radius \( r = 20 \). The inset shows three snapshots of the capillary rise: initial, intermediate, and steady state (side view).

Whether difference in the capillary radius affects the dynamics of the flow in the two-phase flow model is an important question. The velocity profile for pipes with various radii was determined to answer this issue and contrasted with the empirical findings (fig 3). At its equilibrium density between two infinite (periodic boundary conditions in lateral directions) parallel plates, this velocity profile could be established from the steady flow of a single- or two-phase fluid \[7\]. At each lattice point, the flow without any phase separation in the two-phase model was created by a constant body power. The well-known theoretical results of Poiseuille flow are also seen by the full lines in fig 3.
A comparison with analytical observations revealed that the velocity profile had the right (Poiseuille) parabolic shape in our simulations, but it was moved forward. As predicted from the boundary results, this relative variance increased to minimise $r$. A heavy boundary-layer influence was the explanation for this deviation. Due to very low resolution, we did not expect sensible results for $r = 2$, and for strong adhesion also $r = 5$ and $r = 10$ were too small. As seen in the density profiles, the narrow pipe with radius $r = 2$ did not have the bulk liquid density anywhere in the pipe. The decay of density close to the inside walls of the pipe was however similar for all $r = 5,10,20$, at least for weak adhesion forces, e.g. $W = -0.1$. The proportion of the liquid with bulk density decreased with decreasing radius. In Figures 1 and 2 the column height and the dynamic contact angle are shown for $r = 5$ and 20 as functions of simulation time. For comparison we also show the results with and without the presence of gravity, while $W = -0.1$ was kept constant.

At the beginning of the simulations we had $\cos \theta d = 0$, or $\theta d = \pi/2$, as can be seen from Figures 1 and 2. After this the contact angle $\theta d$ decreased with increasing time or decreasing Ca. The steady state contact angle was quickly reached, especially in the presence of gravity, in a few hundred time steps of the total simulation time of about 50,000 time steps. The increase of $\cos \theta d$ from the liquid reaching the end of the pipe. $qd$ reached a constant as expected for limited capillary numbers and long periods, i.e. for slow interface speeds. There was an improvement in the steady state touch angle to increase the pipe radius, which is unphysical and is attributed to the difficulties in calculating the exact angle from the effects of the simulation $\sim t^{1/3}$. As seen in fig 4, as a function of time for both zero and non-zero gravity, we have investigated the velocity of capillary rise. First the column accelerated sharply and then decayed approximately as if toward the stationary state.

**Conclusion**

In conclusion, our test findings are promising: LBM may also research the capillary phenomena at the hydrodynamic level. However for practical kinetics, one requires very thick pipes (in lattice units) because of discrete results. The diameter of the pipe should be at least 30 lattice units for the parametrization used here. In order to verify or change the theoretical velocity profile of a small capillary with heavy adhesive forces on walls, future work needs to be performed. Slightly thinner pipes are ideal for static properties in the presence of gravity, whereas thicker pipes are needed in the case of strong adhesion.

**References**