
**ISSN Print:** 2394-7500
**ISSN Online:** 2394-5869
**Impact Factor:** 5.2
IJAR 2020; 6(5): 157-161
www.allresearchjournal.com
Received: 16-03-2020
Accepted: 18-04-2020

**A Vijayasankar**
Assistant Professor, Department of Mathematics, National College, Affiliated to Bharathidasan University, Trichy, Tamil Nadu, India

**Sharadha Kumar**
Research Scholar, Department of Mathematics, National College, Affiliated to Bharathidasan University, Trichy, Tamil Nadu, India

**MA Gopalan**
Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy, Tamil Nadu, India

**Correspondence Author:**
**Sharadha Kumar**
Research Scholar, Department of Mathematics, National College, Affiliated to Bharathidasan University, Trichy, Tamil Nadu, India

# On the characterizations of integer solutions to the positive pellian equation $y^2 = 11x^2 + 14$

**A Vijayasankar, Sharadha Kumar and MA Gopalan**

**Abstract**
The hyperbola represented by the binary quadratic equation $y^2 = 11x^2 + 14$ (known as positive pellian equation) is analyzed for finding its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed. The formulation of second order Ramanujan Numbers with base numbers as real integers and Gaussian integers is illustrated.

**Keywords:** Binary quadratic, hyperbola, parabola, stright line, integral solutions, pell equation

## 1. Introduction
The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a, b, N \neq 0)$ are rich in variety and have been analyzed by many Mathematicians for their respective integer solutions for particular values $a, b$ and N. In this context, one may refer [1-13].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by $y^2 = 11x^2 + 14$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

## 2. Method of analysis
The positive Pell equation representing hyperbola under consideration is

$$y^2 = 11x^2 + 14 \qquad (1)$$

which is satisfied by

$$x_0 = 5,\ y_0 = 17$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 11x^2 + 1 \qquad (2)$$

whose general solution $(\tilde{x}_n, \tilde{y}_n)$ is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{11}} g_n,\ \ \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(10+3\sqrt{11}\right)^{n+1} + \left(10-3\sqrt{11}\right)^{n+1}$$

$$g_n = \left(10+3\sqrt{11}\right)^{n+1} - \left(10-3\sqrt{11}\right)^{n+1}, \ n = -1,0,1,2......$$

Applying Brahmagupta lemma between $(x_0, y_0)$ and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solutions of (1) are given by

$$x_{n+1} = \frac{5}{2} f_n + \frac{17}{2\sqrt{11}} g_n$$
$$y_{n+1} = \frac{17}{2} f_n + \frac{55}{2\sqrt{11}} g_n$$

The recurrence relations satisfied by the values of $x_{n+1}$ and $y_{n+1}$ are respectively,

$$x_{n+1} - 20x_{n+2} + x_{n+3} = 0, n = -1,0,1......$$
$$y_{n+1} - 20y_{n+2} + y_{n+3} = 0, n = -1,0,1.....$$

A few numerical examples are given in the following table 1 below:

**Table 1:** Numerical examples

| n | $x_{n+1}$ | $y_{n+1}$ |
|---|---|---|
| -1 | 5 | 17 |
| 0 | 101 | 335 |
| 1 | 2015 | 6683 |
| 2 | 40199 | 133325 |
| 3 | 801965 | 2659817 |

From the above table, we observe some interesting relations among the solutions which are presented below:

1. Both $x_{n+1}$ and $y_{n+1}$ are odd.
2. $y_{n+1} - x_{n+1} \equiv 0 \pmod 2$
3. $y_{2n-1} \equiv 0 (\mathrm{mod}\, 5), n = 1,2,3,....$
4. $x_{2n} = 0 (\mathrm{mod}\, 5), n = 0,1,2,.....$

**Observation 1**

Let $\{a_n\}$ and $\{b_n\}$ be two sequences of positive integers

where $a_n = \frac{x_{n+1}+1}{2}$, $b_n = \frac{y_{n+1}-1}{2}$, $n = -1,0,1,2.....$

It is observed that

i) $t_{3,b_n} - 11t_{3,a_n} = 3 - 11a_n$

ii) $12t_{3,b_n} = 11S_n + 25$

iii) $8t_{3,b_n} = t_{90,a_n} - a_n + 24$

**Observation 2**

Let $\{a_{2n}\}$ and $\{b_{2n}\}$ be two sequences of positive integers

where $a_{2n} = \frac{x_{2n}}{5}$, $b_n = \frac{y_{2n}-1}{2}$, $n = 0,1,2.....$

It is observed that

i) $66\left(8t_{3,b_{2n}} - 13\right)$ is a nasty number.

ii) $8t_{3,b_{2n}} - 11t_{52,a_{2n}} - 264a_{2n} = 13$

iii) $8t_{3,b_{2n}} - 11t_{28,a_{2n}} - 11t_{26,a_{2n}} - 253a_{2n} = 13$

**Observation 3**

From the values of $x_{n+1}\ (n > 0)$ and $y_{n+1}\ (n \geq 0)$, one may generate second order Ramanujan Numbers with base numbers as real integers and Gaussian integers. A few illustrations are given below:

**Illustration 1**

$$x_2 = 2015 = 2015*1 = 5*403 = 13*155 = 31*65$$
$$= 1008^2 - 1007^2 = 204^2 - 199^2 = 84^2 - 71^2 = 48^2 - 17^2$$

**Now,**

$$1008^2 - 1007^2 = 204^2 - 199^2 \Rightarrow 1008^2 + 199^2 = 1007^2 + 204^2 = 1055665$$
$$1008^2 - 1007^2 = 84^2 - 71^2 \Rightarrow 1008^2 + 71^2 = 1007^2 + 84^2 = 1021105$$
$$1008^2 - 1007^2 = 48^2 - 17^2 \Rightarrow 1008^2 + 17^2 = 1007^2 + 48^2 = 1016353$$
$$204^2 - 199^2 = 84^2 - 71^2 \Rightarrow 204^2 + 71^2 = 199^2 + 84^2 = 46657$$
$$204^2 - 199^2 = 48^2 - 17^2 \Rightarrow 204^2 + 71^2 = 199^2 + 48^2 = 41905$$
$$84^2 - 71^2 = 48^2 - 17^2 \Rightarrow 84^2 + 17^2 = 71^2 + 48^2 = 7345$$

**Also,**

$$2015*1 = 5*403 \Rightarrow 2016^2 + 398^2 = 2014^2 + 408^2 = 4222660$$
$$2015*1 = 3*155 \Rightarrow 2016^2 + 142^2 = 2014^2 + 168^2 = 4084420$$
$$2015*1 = 31*65 \Rightarrow 2016^2 + 34^2 = 2014^2 + 96^2 = 4065412$$
$$5*403 = 31*155 \Rightarrow 408^2 + 142^2 = 398^2 + 168^2 = 186628$$
$$5*403 = 31*65 \Rightarrow 408^2 + 34^2 = 398^2 + 96^2 = 167620$$
$$13*155 = 31*65 \Rightarrow 168^2 + 34^2 = 142^2 + 96^2 = 29380$$

Note that 1055665, 1021105, 1016353, 46657, 41905, 7345, 4222660, 4084420, 4065412, 186628, 167620, 29380.

**Illustration 2**

$$y_1 = 335 = 335*1 = 5*67 \qquad (3)$$
$$= 168^2 - 167^2 = 36^2 - 31^2$$
$$\Rightarrow 168^2 + 31^2 = 167^2 + 36^2 = 29185$$

Also, notice that

$$336^2 + 62^2 = 334^2 + 72^2 = 116740$$

Further, from (3), we write

$$(335+i)^2 + (67-5i)^2 = (335-i)^2 + (67+5i)^2 = 116688$$

Thus, 29185 and 116740 are second order Ramanujan numbers with base numbers as real integers whereas 116688 is second order Ramanujan numbers with Gaussian integers as base numbers.

**1. A few interesting relations among the solutions are given below**

- $x_{n+2}-10x_{n+1}-3y_{n+1}=0$
- $x_{n+3}-60y_{n+1}-199x_{n+1}=0$
- $y_{n+2}-10y_{n+1}-33x_{n+1}=0$
- $y_{n+3}-199y_{n+1}-660x_{n+1}=0$
- $10x_{n+2}-3y_{n+2}-x_{n+1}=0$
- $x_{n+3}-6y_{n+2}-x_{n+1}=0$
- $10y_{n+3}-199y_{n+2}-33x_{n+1}=0$
- $3y_{n+1}-10x_{n+3}+199x_{n+2}=0$
- $3y_{n+3}-10x_{n+3}+x_{n+2}=0$
- $14y_{n+2}-10667y_{n+1}+1749x_{n+2}=0$
- $y_{n+3}-y_{n+1}-66x_{n+2}=0$
- $x_{n+3}-3y_{n+2}-10x_{n+2}=0$
- $10x_{n+1}-199x_{n+2}+3y_{n+3}=0$
- $y_{n+3}-10y_{n+2}-33x_{n+2}=0$
- $y_{n+1}-10y_{n+2}+33x_{n+2}=0$

**2. Each of the following expressions represents a nasty number**

- $\frac{1}{70}(17x_{2n+4}-6683x_{2n+2}+840)$
- $\frac{6}{14}(34y_{2n+2}-110x_{2n+2}+28)$
- $\frac{6}{70}(17y_{2n+3}-1111x_{2n+2}+140)$
- $\frac{6}{1393}(17y_{2n+4}-22165x_{2n+2}+2786)$
- $\frac{6}{42}(670x_{2n+4}-13366x_{2n+3}+84)$
- $\frac{6}{70}(335y_{2n+2}-55x_{2n+3}+140)$
- $\frac{6}{7}(335y_{2n+3}-1111x_{2n+3}+14)$
- $\frac{6}{70}(335y_{2n+4}-22165x_{2n+3}+140)$
- $\frac{6}{1393}(6683y_{2n+2}-55x_{2n+4}+2786)$
- $\frac{6}{140}(13366y_{2n+3}-2222x_{2n+4}+280)$
- $\frac{6}{7}(6683y_{2n+4}-22165x_{2n+4}+14)$
- $\frac{6}{462}(2222y_{2n+2}-110y_{2n+3}+924)$
- $\frac{6}{4620}(22165y_{2n+2}-55y_{2n+4}+9240)$
- $\frac{6}{231}(22165y_{2n+3}-1111y_{2n+4}+462)$

**3. Each of the following expressions represents a cubical integer**

- $\frac{1}{420}[17x_{3n+5}-6683x_{3n+3}+51x_{n+3}-20049x_{n+1}]$
- $\frac{1}{7}[17y_{3n+3}-55x_{3n+3}+51y_{n+1}-165x_{n+1}]$
- $\frac{1}{70}[17y_{3n+4}-1111x_{3n+3}+51y_{n+2}-3333x_{n+1}]$
- $\frac{1}{2786}[34y_{3n+5}-44330x_{3n+3}+102y_{n+3}-132990x_{n+1}]$
- $\frac{1}{42}[670x_{3n+5}-13366x_{3n+4}+2010x_{n+3}-40098x_{n+2}]$
- $\frac{1}{70}[335y_{3n+3}-55x_{3n+4}+1005y_{n+1}-165x_{n+2}]$
- $\frac{1}{7}[335y_{3n+4}-1111x_{3n+4}+1005y_{n+2}-3333x_{n+2}]$
- $\frac{1}{14}[67y_{3n+5}-4433x_{3n+4}+201y_{n+3}-13299x_{n+2}]$
- $\frac{1}{1393}[6683y_{3n+3}-55x_{3n+5}+20049y_{n+1}-165x_{n+3}]$
- $\frac{1}{140}[13366y_{3n+4}-2222x_{3n+5}+40098y_{n+2}-6666x_{n+3}]$
- $\frac{1}{7}[6683y_{3n+5}-22165x_{3n+5}+20049y_{n+3}-66495x_{n+3}]$
- $\frac{1}{4620}[22165y_{3n+3}-55y_{3n+5}+66495y_{n+1}-165y_{n+3}]$
- $\frac{1}{231}[22165y_{3n+4}-1111y_{3n+5}+66495y_{n+2}-3333y_{n+3}]$

**4. Each of the following expressions represents a bi-quadratic integer**

- $\frac{1}{420}[17x_{4n+6}-6683x_{4n+4}+68x_{2n+4}-26732x_{2n+2}+2520]$
- $\frac{1}{14}[34y_{4n+4}-110x_{4n+4}+136y_{2n+2}-440x_{2n+2}+84]$
- $\frac{1}{70}[17y_{4n+5}-1111x_{4n+4}+68y_{2n+3}-4444x_{2n+2}+420]$
- $\frac{1}{2786}[34y_{4n+6}-44330x_{4n+4}+136y_{2n+4}-177320x_{2n+2}+16716]$
- $\frac{1}{42}[670x_{4n+6}-13366x_{4n+5}+2680x_{2n+4}-53464x_{2n+3}+252]$
- $\frac{1}{70}[335y_{4n+4}-55x_{4n+5}+1340y_{2n+2}-220x_{2n+3}+420]$
- $\frac{1}{7}[335y_{4n+5}-1111x_{4n+5}+1340y_{2n+3}-4444x_{2n+3}+42]$
- $\frac{1}{70}[335y_{4n+6}-22165x_{4n+5}+1340y_{2n+4}-88660x_{2n+3}+420]$
- $\frac{1}{1393}[6683y_{4n+4}-55x_{4n+6}+26732y_{2n+2}-220x_{2n+4}+8358]$
- $\frac{1}{140}[13366y_{4n+5}-2222x_{4n+6}+53464y_{2n+3}-8888x_{2n+4}+840]$
- $\frac{1}{7}[6683y_{4n+6}-22165x_{4n+6}+26732y_{2n+4}-88660x_{2n+4}+42]$
- $\frac{1}{462}[2222y_{4n+4}-110y_{4n+5}+8888y_{2n+2}-440y_{2n+3}+2772]$
- $\frac{1}{4620}[22165y_{4n+4}-55y_{4n+6}+88660y_{2n+2}-220y_{2n+4}+27720]$
- $\frac{1}{231}[22165y_{4n+5}-1111y_{4n+6}+88660y_{2n+3}-4444y_{2n+4}+1386]$

**5. Each of the following expressions represents a quintic integer**

- $\frac{1}{420}(17x_{5n+7}-6683x_{5n+5}+85x_{3n+5}-33415x_{3n+3}+170x_{n+3}-66830x_{n+1})$
- $\frac{1}{14}(34y_{5n+5}-110x_{5n+5}+170y_{3n+3}-550x_{3n+3}+340y_{n+1}-1100x_{n+1})$
- $\frac{1}{70}(17y_{5n+6}-1111x_{5n+5}+85y_{3n+4}-5555x_{3n+3}+170y_{n+2}-11110x_{n+1})$
- $\frac{1}{70}(335y_{5n+5}-55x_{5n+6}+1675y_{3n+3}-275x_{3n+4}+3350y_{n+1}-550x_{n+2})$
- $\frac{1}{7}(335y_{5n+6}-1111x_{5n+6}+1675y_{3n+4}-5555x_{3n+4}+3350y_{n+2}-11110x_{n+2})$
- $\frac{1}{4620}(22165y_{5n+5}-55y_{5n+7}+110825y_{3n+3}-275y_{3n+5}-550y_{n+3}+221650y_{n+1})$
- $\frac{1}{231}(22165y_{5n+6}-1111y_{5n+7}+110825y_{3n+4}-5555y_{3n+5}+221650y_{n+2}-11110y_{n+3})$

**Remarkable observations**

**I. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbola which are presented in the Table 2 below**

**Table 2:** Hyperbolas

| S. No | Hyperbola | $(X,Y)$ |
|---|---|---|
| 1 | $X^2-11Y^2=705600$ | $(17x_{n+3}-6683x_{n+1},\ 2015x_{n+1}-5x_{n+3})$ |
| 2 | $X^2-11Y^2=784$ | $(34y_{n+1}-110x_{n+1},34x_{n+1}-10y_{n+1})$ |
| 3 | $X^2-55Y^2=19600$ | $(17y_{n+2}-1111x_{n+1},67x_{n+1}-y_{n+2})$ |
| 4 | $X^2-11Y^2=31047184$ | $(34y_{n+3}-44330x_{n+1},13366x_{n+1}-10y_{n+3})$ |
| 5 | $X^2-11Y^2=7056$ | $(670x_{n+3}-13366x_{n+2},4030x_{n+2}-202x_{n+3})$ |
| 6 | $X^2-11Y^2=19600$ | $(335y_{n+1}-55x_{n+2},17x_{n+2}-101y_{n+1})$ |
| 7 | $X^2-11Y^2=196$ | $(335y_{n+2}-1111x_{n+2},335x_{n+2}-101y_{n+2})$ |
| 8 | $X^2-11Y^2=7761796$ | $(6683y_{n+1}-55x_{n+3},17x_{n+3}-2015y_{n+1})$ |
| 9 | $X^2-11Y^2=78400$ | $(13366y_{n+2}-2222x_{n+3},670x_{n+3}-4030y_{n+2})$ |
| 10 | $X^2-11Y^2=196$ | $(6683y_{n+3}-22165x_{n+3},6683x_{n+3}-2015y_{n+3})$ |
| 11 | $X^2-11Y^2=853776$ | $(2222y_{n+1}-110y_{n+2},34y_{n+2}-670y_{n+1})$ |
| 12 | $X^2-11Y^2=85377600$ | $(22165y_{n+1}-55y_{n+3},17y_{n+3}-6683y_{n+1})$ |
| 13 | $X^2-11Y^2=213444$ | $(22165y_{n+2}-1111y_{n+3},335y_{n+3}-6683y_{n+2})$ |

**II. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of parabola which are presented in the Table 3 below**

**Table 3:** Parabolas

| S. No. | Parabola | $(X,Y)$ |
|---|---|---|
| 1 | $420X-11Y^2=352800$ | $(17x_{2n+4}-6683x_{2n+2},\ 2015x_{n+1}-5x_{n+3})$ |
| 2 | $14X-11Y^2=392$ | $(34y_{2n+2}-110x_{2n+2},34x_{n+1}-10y_{n+1})$ |
| 3 | $70X-55Y^2=9800$ | $(17y_{2n+3}-1111x_{2n+2},67x_{n+1}-y_{n+2})$ |
| 4 | $42X-11Y^2=3528$ | $(670x_{2n+4}-13366x_{2n+3},4030x_{n+2}-202x_{n+3})$ |
| 5 | $70X-11Y^2=9800$ | $(335y_{2n+2}-55x_{2n+3},17x_{n+2}-101y_{n+1})$ |
| 6 | $7X-11Y^2=98$ | $(335y_{2n+3}-1111x_{2n+3},335x_{n+2}-101y_{n+2})$ |
| 7 | $350X-11Y^2=9800$ | $(67y_{2n+4}-4433x_{2n+3},6683x_{n+2}-101y_{n+3})$ |
| 8 | $1393X-11Y^2=3880898$ | $(6683y_{2n+2}-55x_{2n+4},17x_{n+3}-2015y_{n+1})$ |
| 9 | $140X-11Y^2=39200$ | $(13366y_{2n+3}-2222x_{2n+4},670x_{n+3}-4030y_{n+2})$ |
| 10 | $7X-11Y^2=98$ | $(6683y_{2n+4}-22165x_{2n+4},6683x_{n+3}-2015y_{n+3})$ |

| | | |
|---|---|---|
| 11 | $462X-11Y^2=426888$ | $(2222y_{2n+2}-110y_{2n+3}, 34y_{n+2}-670y_{n+1})$ |
| 12 | $4620X-11Y^2=42688800$ | $(22165y_{2n+2}-55y_{2n+4}, 17y_{n+3}-6683y_{n+1})$ |
| 13 | $231X-11Y^2=106722$ | $(22165y_{2n+3}-1111y_{2n+4}, 335y_{n+3}-6683y_{n+2})$ |

**III. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of straight lines which are presented in the Table 3 below:**

**Table 3:** Straight lines

| S. No. | Straight lines | $(X,Y)$ |
|---|---|---|
| 1 | $Y=30X$ | $(34y_{n+1}-110x_{n+1}, 17x_{n+3}-6683x_{n+1})$ |
| 2 | $Y=6X$ | $(17y_{n+2}-1111x_{n+1}, 17x_{n+3}-6683x_{n+1})$ |
| 3 | $Y=10X$ | $(670x_{n+3}-13366x_{n+2}, 17x_{n+3}-6683x_{n+1})$ |
| 4 | $Y=6X$ | $(335y_{n+1}-55x_{n+2}, 17x_{n+3}-6683x_{n+1})$ |
| 5 | $Y=5X$ | $(34y_{n+1}-110x_{n+1}, 17y_{n+2}-1111x_{n+1})$ |
| 6 | $Y=1X$ | $(335y_{n+1}-55x_{n+2}, 17y_{n+2}-1111x_{n+1})$ |
| 7 | $Y=10X$ | $(335y_{n+2}-1111x_{n+2}, 17y_{n+2}-1111x_{n+1})$ |
| 8 | $Y=10X$ | $(6683y_{n+3}-22165x_{n+3}, 17y_{n+2}-1111x_{n+1})$ |
| 9 | $Y=20X$ | $(22164y_{n+2}-1111y_{n+3}, 22165y_{n+1}-55y_{n+3})$ |
| 10 | $Y=10X$ | $(2222y_{n+1}-110y_{n+2}, 22165y_{n+1}-55y_{n+3})$ |
| 11 | $Y=660X$ | $(6683y_{n+3}-22165x_{n+3}, 22165y_{n+1}-55y_{n+3})$ |
| 12 | $Y=2X$ | $(335y_{n+3}-22165x_{n+2}, 13366y_{n+2}-2222x_{n+3})$ |
| 13 | $Y=20X$ | $(335y_{n+2}-1111x_{n+2}, 13366y_{n+2}-2222x_{n+3})$ |
| 14 | $Y=2X$ | $(335y_{n+1}-55x_{n+2}, 13366y_{n+2}-2222x_{n+3})$ |

**IV. Consider** $p=x_{n+1}+y_{n+1}$ , $q=x_{n+1}$ **observe that** $p>q>0$

Treat p,q as the generators of the Pythagorean Triangle T( $\alpha,\beta,\gamma$ ) where

$$\alpha=2pq, \beta=p^2-q^2, \gamma=p^2+q^2$$

Then the following interesting relations are observed

- $2\alpha-11\beta+9\gamma=-28$
- $13\alpha-2\gamma+28=\frac{44A}{P}$
- $\frac{2A}{P}=x_{n+1}y_{n+1}$

**References**


1. Disckson LE. History of theory of Numbers, Chelsea Publishing co., New York, 1952, 2.
2. Mordell LJ. Diophantine Equations, Academic Press, London, 1969.
3. Gopalan *et al.*, Integral points on the hyperbola $(a+2)x^2-ay^2=4a(k-1)+2k^2, a,k>0$, Indian journal of science. 2012; 1(2):125-126.
4. Gopalan MA, Devibala S, Vidhyalakshmi R. Integral Points on the Hyperbola $2X^2-3Y^2=5$, American Journal of Applied Mathematics and Mathematical Sciences. 2012; 1(1):1-4.
5. Vidhyalakshmi S *et al.*, Observations on the hyperbola $ax^2-(a+1)y2=3a-1$, Discovery. 2013; 4(10):22-24.
6. Meena K, Vidhyalakshmi S, Nivetha A. On the Binary Quadratic Diophantine Equation $x^2-4xy+y^2+14x=0$, Sch. J. Phys. Math. Stat. 2016; 3(1):15-19.
7. Meena K, Vidhyalakshmi S, Janani R. On the Binary quadratic Diophantine Equation $x^2-9xy+y^2+21x=0$, IJETER. 2016; 4(7):1-4.
8. Meena K, Gopalan MA, Nandhini S. On the binary quadratic Diophantine equation $y^2=68x^2+13$, International Journal of Advanced Education and Research. 2017; 2(1):59-63.
9. Meena K, Vidhyalakshmi S, Sobana Devi R. On the binary quadratic equation $y^2=7x^2+32$, International Journal of Advanced Science and Research. 2017; 2(1):18-22.
10. Meena K, Gopalan MA, Hemalatha S. On the hyperbola $y^2=8x^2+16$, National Journal of Multidisciplinary Research and Development. 2017; 2(1):01-05.
11. Gopalan MA, Viswanathan KK, Ramya G. On the Positibe Pell equation $y^2=12x^2+13$, International Journal of Advanced Education and Research. 2017; 2(1):04-08.
12. Meena K, Gopalan MA, Sivaranjani V. On the Positibe Pell equation $y^2=102x^2+33$, International Journal of Advanced Education and Research. 2017; 2(1):91-96.
13. Meena K, Vidhyalakshmi S, Bhuvaneswari N. On the binary quadratic Diophantine equation $y^2=10x^2+24$, International Journal of Multidisciplinary Education and Research. 2017; 2(1):34-39.