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D Manjula
PhD Research Scholar, Department of Statistics, PSG College of Arts & Science, Coimbatore, Tamil Nadu, India

G Uma
Assistant Professor, Department of Statistics, PSG College of Arts & Science, Coimbatore, Tamil Nadu, India

R Nandhinidevi
Assistant Professor, Department of Mathematics, Dr.M.G.R Educational and Research Institute, Chennai, Tamil Nadu, India

Correspondence Author:
D Manjula
PhD Research Scholar, Department of Statistics, PSG College of Arts & Science, Coimbatore, Tamil Nadu, India

Derivation of zero - one truncated Poisson distribution

D Manjula, G Uma, and R Nandhinidevi

Abstract
In Industries every producer expects Zero defect in the producing process but rarely there is a chance of getting zero defect, in such situation omitting the zero tends to Zero truncated. In recent trends the production of food and drinks items tends to zero and one truncated acceptance numbers. In this situation better result cannot be concluded with existing distributions and even applied may mislead sometimes. To overcome this situation a new distribution is instigated called Zero-One Truncated Poisson Distribution (ZOTPD). In this paper, the newly initiated probability distribution function and some of its characteristics are determined using certain assumption and conditions.

Keywords: PD, ZTPD, ZOTPD, AM, GM, SD, Variance, MGF, characteristic function, and MLE.

1. Introduction
The Zero-Truncated Poisson distribution is a sample variant of the Poisson distribution that has no zero value. A simple example of this is distribution of items a customer has in their shopping cart before approaching a register where it is common to presume that the customer will not approach the cash register without any items to purchase.

In this study, our problem is to find a minimum sample size 'n' that necessary to assure a certain average life when the life test is terminated at a pre-assigned time 't' and when the observed number of failures does not exceed a given acceptance number 'c'. The decision procedure is to accept a lot only if the specified average life can be established with a pre-assigned high probability, which provides the protection to the consumer. Based on this concept, truncating 0-1 idea can be implemented to Poisson which as a base line distribution. In this paper, the probability distribution function and their characteristics are derived.

2. Poisson Distribution
Poisson distribution is an ancient probability distribution for modelling rare events (Simeon Denis Poisson, (1978-1840). Several authors have used it for the construction of various sampling plans. The probability mass function (pmf) of the Poisson random variable X can be written as

$$P(X = x) = \sum_{i=0}^{n} \frac{e^{-\lambda}\lambda^{x}}{x!}, \; x = 0,1,2,..... \qquad (1)$$

With mean and variance λ

3. Zero Truncated Poisson Distribution
Under the truncated distributions Zero Truncated Poisson distribution (ZTPD) is used for modelling a chance mechanism whose observational apparatus becomes active only when at least one event occurs. The pmf of ZTPD random variable X can be written as

$$P(X = x) = \sum_{i=1}^{n} \frac{(e^{\lambda}-1)^{-1}\,\lambda^{x}}{x!}, \; x = 1,2,..... \qquad (2)$$

With Mean $\frac{\lambda\, e^{\lambda}}{(e^{\lambda}-1)}$ and Variance $\frac{\lambda+\lambda^{2}}{1-e^{-\lambda}}$

4. Zero - one truncated Poisson distribution

Under the truncated distribution, a Zero - One truncated Poisson distribution is used for modelling a chance mechanism whose observational apparatus becomes active only when at least two events occur. The pmf of ZOTPD random variable X can be written as

$$P(X = x) = \sum_{i=2}^{n} \frac{\lambda^x}{(e^{\lambda} - (1+\lambda))x!},\ x = 2,3,,..... \quad (3)$$

5. Characteristic of ZOTPD

5.1. Performance measures of ZOTPD

a) MGF and Characteristic function

Moment Generating Function of ZOTPD

$$M_x(t) = E\,(e^{tx}) = \frac{1}{x!} \sum_{i=2}^{\infty} \frac{e^{-(\lambda - tx)\,\lambda}}{1-e^{-\,\lambda}\,(1+\lambda)} \quad (4)$$

Characteristic function of ZOTPD

$$\Phi_x(t) = E\,(e^{itx}) = \frac{1}{x!} \sum_{i=2}^{\infty} \frac{e^{-(\lambda - itx)\,(\lambda)}}{1-e^{-\,\lambda}\,(1+\lambda)} \quad (5)$$

b) Moments and recurrence relations

$$\mu_r = \left(\frac{\lambda^x}{(e^{\lambda}-(1+\lambda))x!}\right)^{r} \quad (6)$$

5.2. Estimation of the Parameters of ZOTPD

Maximum Likelihood Estimator (MLE)

$$p(x) = \sum_{i=2}^{n} \frac{\lambda^x}{(e^{\lambda} - (1+\lambda))x!},\ x = 2,3..... \quad (7)$$

$$L = \frac{\prod_{i=1}^{n} \lambda^x}{\sum\left(e^{\lambda}-(1+\lambda)\right)x!} \quad (8)$$

$$\log L = \log\left(\frac{\prod_{i=1}^{n} \lambda^x}{\sum\left(e^{\lambda}-(1+\lambda)\right)x!}\right) \quad (9)$$

Sufficient Estimator

If $x_2, x_3, ..., x_n$ are independent and have a Poisson distribution with parameter λ of Mean $\frac{\lambda(n+n\lambda+1)}{n+n\,\lambda}$, then the sum $T(X) = x_2+x_3+......+x_n$ is a sufficient statistic for λ.

$P(X=x) = P(X_2 = x_2,\ X_3=x_3, ..., X_n = x_n)$. The observations are independent, then it can be written as,

$$\frac{e^{-\lambda}\,\frac{\lambda(n+n\lambda+1)}{n+n\,\lambda}^{x_2}}{x_2!} \cdot \frac{e^{-\lambda}\,\frac{\lambda(n+n\lambda+1)}{n+n\,\lambda}^{x_3}}{x_3!} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \frac{e^{-\lambda}\,\frac{\lambda(n+n\lambda+1)}{n+n\,\lambda}^{x_n}}{x_n!}$$
(10)

Which may be written as

$$e^{-n\lambda}\left(\frac{\lambda}{n+n\lambda}\right)\left[(n(1+\lambda)+1)^{x_2} + (n(1+\lambda)+1)^{x_3} + \ \ldots\right.$$

$$\left. + (n(1+\lambda)+1)^{x_n}\right] \frac{1}{x_1!x_2!..............x_n!} \quad (11)$$

$$e^{-n\lambda}\lambda \cdot \frac{(n+1)^{x_2+x_3+\ ....+x_n}}{x_2!x_3!.......x_n!} \quad (12)$$

This shows that the Zero – One Truncated Poisson Distribution attains Sufficiency condition. $T = \sum_{i=2}^{n} x_i$ is a sufficient statistic for ZOTPD.

5.3. Other Measures

5.3.1. Arithmetic Mean

The most commonly used average is the expected value of X, viz., E(X). This is also called the Arithmetic mean (AM) of X and is denoted by λ

$$E(X) = \frac{\lambda(n+n\lambda+1)}{n+n\,\lambda} \quad (13)$$

5.3.2 Geometric mean

If G_x is the Geometric Mean (GM) of the random variable X, then

$G_x = \exp\{E(\ln X)\}$

$$G_x = \lambda^2 - x!\ \lambda - x!\ (e^{\lambda} - 1) \quad (14)$$

G_x is defined only when $P\{X>0\} = 1$

5.3.3 Standard deviation

The most commonly used measure of variation is the SD. It is defined by

$V(X) = \sigma_x^2 = E(X-E(X))^2 = E(X^2)-\{E(X)\}^2$

$$V(X) = \frac{(n+n\,\lambda+1)\,(1+\lambda)+\lambda}{n(1+\lambda)^2} = \frac{n(1+\lambda)^2 + 2\lambda + 1}{n(1+\lambda)^2} \quad (15)$$

$$SD(X) = \sqrt{\frac{(n+n\,\lambda+1)\,(1+\lambda)+\lambda}{n(1+\lambda)^2}} \quad (16)$$

6. Conditions of ZOTPD

Zero-One Truncated Poisson Distribution is a limiting case of Binomial Distribution under the following assumption

1. The number of trials n should be indefinitely large i.e., $n \to \infty$
2. The probability of success p for each trial is indefinitely small.
3. $np = \lambda_x$, should be finite where λ_x is constant.

7. Properties of ZOTPD

1. Zero-One Truncated Poisson distribution is defined by single Parameter λ_x
2. Mean = $\frac{\lambda(n+n\lambda+1)}{n+n\,\lambda}$
3. Variance = $\frac{(n+n\,\lambda+1)\,(1+\lambda)+\lambda}{n(1+\lambda)^2}$

8. Application of ZOTPD

1. It is used in quality control statistics to count the number of defects of an item except zero & one defect.
2. In a Milk Production the acceptable number of day's production is zero and one; after one day there is a possibility of spoilt the milk, hence determine the probability of acceptance after one day.
3. In Genetic field, the probability of acceptance of fertility after failure of nature form and first attempt by fertility centre.
4. In a Natural drink the quality of product was prepared day and preservative of one another day is possible to drink but after that day there is a chance of risk to drink that Natural drink.

Conclusion

In this paper the probability distribution function, characteristics, properties, assumption and application of Zero-One Truncated Poisson Distribution are derived.

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