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Study of rough surface scattering by single integral equation

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Abstract
An efficient algorithm for electromagnetic wave scattering from rough dielectric surfaces is developed. The algorithm is based on the single magnetic field integral equation (SMFIE) and the surface is discretized using Rao-Wilton-Glisson (RWG-) triangular basis functions. The new feature of the algorithm is the application of the adaptive integral method (AIM) with SMFIE for speeding up the calculation. The developed new method utilizes the flexibility of RWG functions to model arbitrary rough surface enabling accurate simulations over large surfaces.

Keywords: Adaptive integral method (AIM), Rao-Wilton-Glisson (RWG) triangular basis, rough surface scattering, single magnetic field integral equation (SMFIE).

Introduction
Iteration is a well-known method for solving Fred holm integral equations of the second kind arising from the interaction of fields and bodies [1]. The domain decomposition method (DDM) [2, 3] has emerged as a powerful and attractive technique for the numerically rigorous solution of Maxwell's equations. The simulation of electromagnetic scattering from dielectric rough surfaces is very tempting for applications in remote sensing of ocean, soil and ice.
There has been a single dominating method for solving a full vector wave scattering from a two-dimensional (2–D) rough dielectric surface. The method is based on the sparse matrix canonical grid method (SMCG) [4-5].
In this paper we present the formulation of SMFIE and its discetization.

Formulation and discretization
Single Integral Equation Formulation
In this paper a solution to a 3-0 time -harmonic ($e^{-i\omega t}$) scattering problem from a 2-D dielectric random rough surface is presented. Fig.-1 illustrates the problem and Fig.-2 presents the numerical integration performed over the surface of the triangles along the edge. The dielectric medium over (permittivity $\in_1$, permeability μ_1 and under ($\in_2$, μ_2) the interface is assumed to be homogeneous.
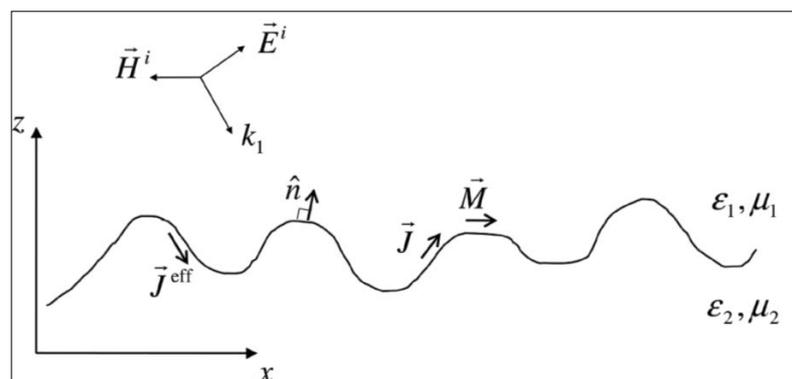

Fig 1: Schematic diagram of the 3–D scattering problem hum a 2–D random rough surface.

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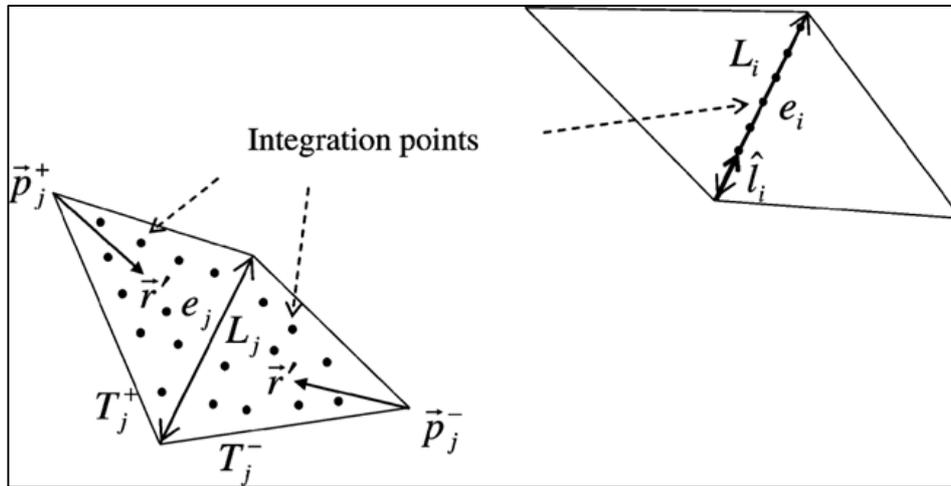Krishna College, Samastipur,
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Fig 2: Triangular RWG function is used as the basis function and the common edge between triangles is used as the testing function.

The single integral equation formulation for a closed dielectric object can be described as follows. The fields outside the object are expressed using the usual surface current densities $\vec{J} = \hat{n} \times \vec{H}_1$ and $\vec{M} = -\hat{n} \times \vec{E}_1$ where $\hat{n}$ is the outer unit normal of the object, yielding

$$\vec{E}_1(\vec{r}) = \vec{E}_1^{\text{inc}}(\vec{r}) - \frac{1}{i\omega\epsilon_1}\mathbf{D}_1(\vec{J})(\vec{r}) - \mathbf{K}_1(\vec{M})(\vec{r}) \qquad (1)$$

$$\vec{H}_1(\vec{r}) = \vec{H}_1^{\text{inc}}(\vec{r}) - \frac{1}{i\omega\mu_1}\mathbf{D}_1(\vec{M})(\vec{r}) + \mathbf{K}_1(\vec{J})(\vec{r}) \qquad (2)$$

Where ϵ_1 is the permittivity of the upper medium with $\epsilon_1 = \epsilon_1 + i\sigma_1/\omega_1\sigma_1$, being the conductivity of the medium: ω is the angular frequency.

Taking the cross product of (2) with the normal vector on the surface, gives the magnetic field integral equation

$$-\frac{1}{i\omega\mu_1}\hat{n} \times \mathbf{D}_1(\vec{M}) + \hat{n} \times \mathbf{K}_1(\vec{J}) - \frac{1}{2}\vec{J} = -\hat{n} \times \vec{H}_1^{\text{inc}}. \qquad (3)$$

Inside the object the fields are expressed in terms of an efficient electric current $\vec{J}^{\text{eff}}$ as

$$\vec{E}_2(\vec{r}) = -\frac{1}{i\omega\epsilon_2}\mathbf{D}_2(\vec{J}^{\text{eff}})(\vec{r}) \qquad (4)$$

$$\vec{H}_2(\vec{r}) = \mathbf{K}_2(\vec{J}^{\text{eff}})(\vec{r}). \qquad (5)$$

The boundary conditions at the interface between the two media

$$\hat{n} \times \vec{E}_1 = \hat{n} \times \vec{E}_2 \qquad (6)$$

$$\hat{n} \times \vec{H}_1 = \hat{n} \times \vec{H}_2 \qquad (7)$$

give a relation between the currents $\vec{J}, \vec{M}$ and $\vec{J}^{\text{eff}}$ [9]

$$\vec{M} = \frac{1}{i\omega\epsilon_2}\hat{n} \times \mathbf{D}_2(\vec{J}^{\text{eff}}) \qquad (8)$$

$$\vec{J} = \hat{n} \times \mathbf{K}_2(\vec{J}^{\text{eff}}) - \frac{1}{2}\vec{J}^{\text{eff}} \qquad (9)$$

on the surface S.

Substituting (8) and (9) into (3) gives SMFIE for $\vec{J}^{\text{eff}}$

$$\hat{n} \times \vec{H}_1^{\text{inc}} = \left(\frac{1}{i\omega\mu_1}\frac{1}{i\omega\epsilon_2}\hat{n} \times \mathbf{D}_1(\hat{n} \times \mathbf{D}_2) \right.$$
$$\left. - \left(\hat{n} \times \mathbf{K}_1 - \frac{1}{2}I\right)\left(\hat{n} \times \mathbf{K}_2 - \frac{1}{2}I\right) \right)(\vec{J}^{\text{eff}}) \qquad (10)$$

Where I is the identity operator.
We note that SMFIE provides unambiguous solution only outside the resonant frequency of the object and for conducting objects. These requirements are met in the application planned in this study, which is ocean surface.
The above treatment. is made for a closed object. The simulation of surface scattering has the problem that the surface is very large and it is more practical to model it as an open surface than a closed object. Hence, the effect of the borders of the surface need to be minimized. Fig.-3 shows a calculation where the scattering from a closed object (box of size $n\lambda \times \lambda/10$, Where?. is the free space wavelength) is compared to the scattering from a plate (of size $n\lambda \times n\lambda$) in order to evaluate the effect of the truncation of the surface to a finite size. The surfaces are fiat and the properties of the surfaces are the same as in the examples in Figs,-3, 4 and 5. The results show that the truncation has no critical effect on the result. This is related to the relative large conductivity of the surface which attenuates the incident field toward the borders of the surface area. The large conductivity originates from the fact that the aim of the study is to simulate scattering from ocean surface.
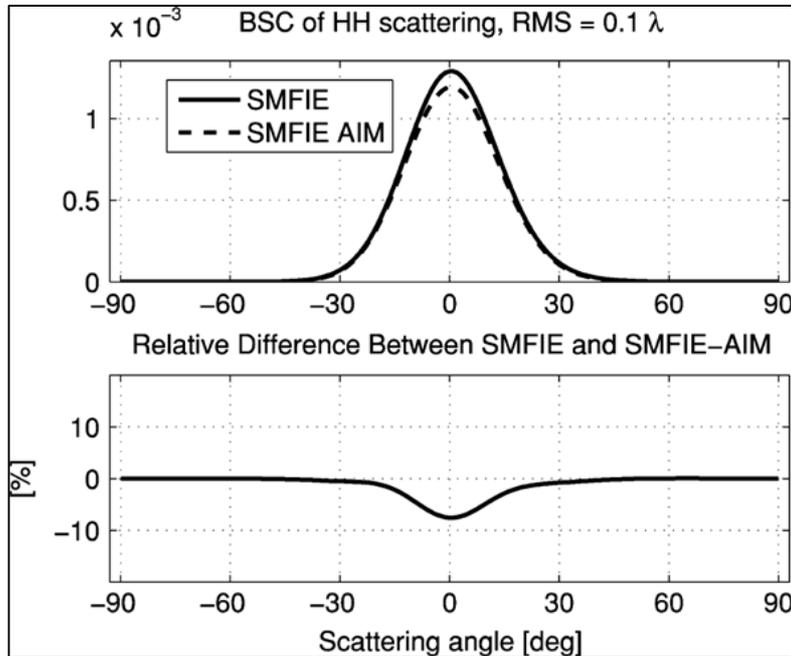
Fig 3: BSC of averaged HH scattering from ten realizations of surfaces with 0.1λ. RMS roughness and 1λ correlation length. The size of the surface is 16λ² and the surface is illuminated from the nadir direction.
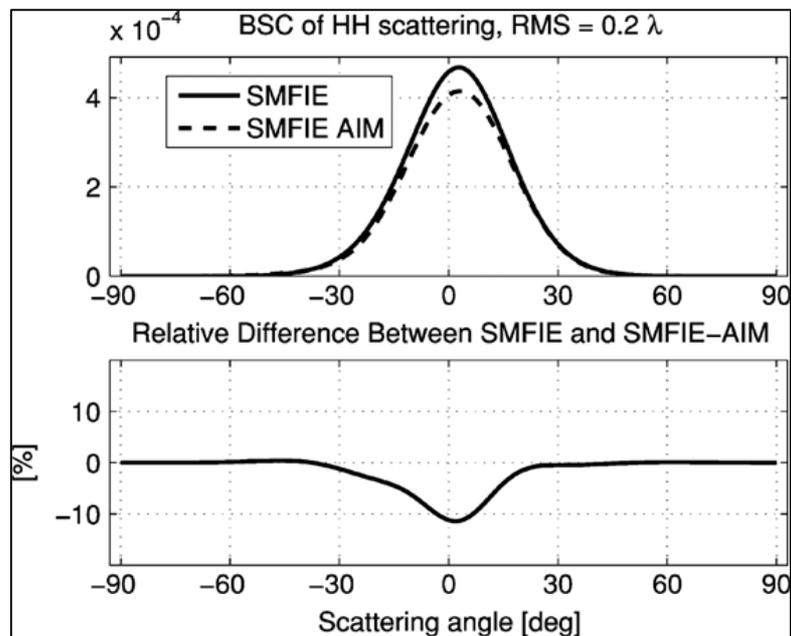

Fig 4: BSC of averaged HH scattering hum ten realizations of surfaces with 0.2λ. RMS roughness and 1λ. Correlation length. The size of the surface is 16λ² and the surface is illuminated from the nadir direction.
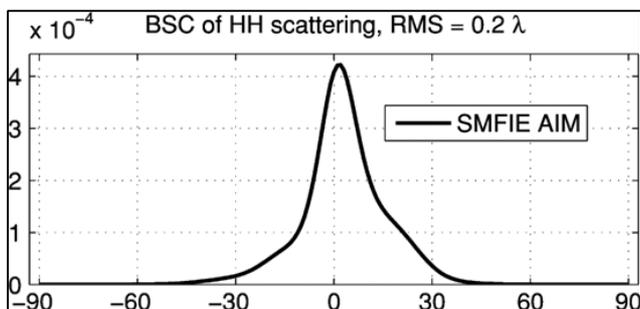

Fig 5: BSC of averaged HH scattering from ten realizations of surfaces with 0.2λ. RMS roughness and 1λ correlation length. The size of the surface is 100λ² and the surface is illuminated from the nadir direction.

Furthermore, a common technique is applied in which an incident field with amplitude tapering toward the edges of the surface is used. In the simulations of this study the field presented in [16] has been used. The x-component of magnetic field is Gaussian tapered and the y– component is zero,

$$H_x^{\text{inc}} = -\frac{1}{\eta_0}e^{-(x^2+y^2)/g^2}$$

(11)

Where g is the parameter that defines the tapering off the field amplitude on the surface.

Conclusion: The SMFIE formulation using AIM for solving the scattering problem of a rough dielectric surface with

MoM was presented. The SMFIE implementation was verified by calculating the bistatic RCS of a sphere. The rough surface scattering was simulated by calculating the BSC for two surfaces with different roughnesses with and without application of AIM. Also, the result for a very large surface was demonstrated.

It is concluded that the combination of SMFIE. RWG functions and AIM is a good basis for flexible and accurate moduling of the surface, imposes fast solving with FFT and less unknowns (compared to coupled integral equations), and enables efficient simulation of large surfaces. However, as the roughness the surface increases the accuracy of the solution using AIM decreases, which has the consequence that in order to use this approach to the simulation of scattering from extremely rough surface an improvement is needed to take the height variations of the surface into account.

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