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Robert J Buenker
Fachbereich C-Mathematik und Naturwissenschaften, Bergische Universität Wuppertal, Gaussstr. 20, D-42097 Wuppertal, Germany

Corresponding Author:
Robert J Buenker
Fachbereich C-Mathematik und Naturwissenschaften, Bergische Universität Wuppertal, Gaussstr. 20, D-42097 Wuppertal, Germany

Time dilation and lightning flashes on a train

Robert J Buenker

Abstract
The Special Theory of Relativity (STR) is used to predict the slowing down of clocks (time dilation) as a consequence of their motion relative to an observer. The starting point is the space-time mixing equation of the Lorentz transformation (LT) which is responsible for the longstanding belief in the remote non-simultaneity of events. Since time dilation is based on a strict proportionality between the measured elapsed times of two observers in relative motion, it is argued that this LT prediction is inconsistent with remote non-simultaneity. This is because the latter requires that there can be a null time difference for one of the observers but not for the other, the occurrence of which is rendered impossible in view of the above proportional relationship. The well-known example of two lightning flashes on a moving train is used to illustrate the discrepancy. A straightforward means of eliminating this contradictory feature of Einstein's theory is presented which makes use of a different space-time transformation than the LT.

Keywords: Lorentz transformation (LT), alternative global positioning system-lorentz transformation (GPS-LT), absolute remote simultaneity, clock-rate proportionality

1. Introduction
Time dilation (TD) is one of the most prominent features of Einstein's Special Theory of Relativity (STR) [1]. It refers to the slowing down of clocks in motion relative to an observer that is predicted by the theory. The first experimental evidence of this phenomenon was obtained in 1938 by Ives and Stilwell [2] based on their measurements of the wavelength of light emitted from an accelerated source. A more direct confirmation came 33 years later with observations of the behavior of atomic clocks carried onboard circumnavigating airplanes [3, 4]. The authors stated in their introductory remarks, however, that at that time there was still a lively debate among physicists as to whether time dilation was a real effect. In the meantime such skepticism has been largely overcome as a result of the everyday experience with the Global Positioning System (GPS). In order to ensure maximum accuracy for this navigation technique, it is necessary to adjust the clocks on GPS satellites so that they run at the same rate as their counterparts on the earth's surface.
Time dilation foresees a strictly proportional relationship between the respective elapsed times Δt and $\Delta t'$ measured for a given event by two different observers who are in constant motion relative to one another. This relationship is derived from the Lorentz transformation (LT) [1], in particular from its space-time mixing equation. It is interesting to note, however, that the same equation has been used to predict that it is possible for two events to occur simultaneously for one observer ($\Delta t=0$) without doing so for another ($\Delta t'\neq 0$). This possibility was first discussed by Poincaré [5] at the end of the 19th century. It stands in stark contrast to the beliefs of Newton and his followers which were held strongly over a period of more than two centuries prior to the introduction of the LT.
The question that will be considered in the present work is how the strict proportionality implied by TD can be reconciled with remote non-simultaneity. After all, if two elapsed times are equal for one observer, the above proportional relationship seemingly demands that they will also be equal for the other observer as well. To investigate this question, a well-known illustrative example for remote non-simultaneity will be considered below.

2. Theoretical Test of Time Dilation
The STR conclusion of remote non-simultaneity is based directly on one of the four equations of the LT (see the discussion in Sect. III) in which space Δx, Δy, Δz and time Δt coordinates are mixed:

$$\Delta t' = \gamma\left(\Delta t - v\Delta x c^{-2}\right) = \gamma\,\eta^{-1}\Delta t \quad (1)$$

with $\gamma = \left(1-v^2c^{-2}\right)^{-0.5}$ and $\eta = \left(\frac{1-vc^{-2}\Delta x}{\Delta t}\right)^{-1}$,

where c is the speed of light in free space (2.99792458×10^8 ms^{-1}). From this equation it is clear that when both v and Δx are non-zero, it is impossible for both Δt and Δt' to have null values (remote non-simultaneity). A popular illustration of this effect involves a moving train that has been struck at opposite ends by two lightning flashes. Application of the LT, specifically using its space-time mixing characteristic, indicates that it is impossible for the light pulses caused by the lightning strikes to reach the midpoint of the train simultaneously for both an observer standing on the station platform and his counterpart riding with the train [6].

There is another way to analyze this problem using the LT which leads to the opposite conclusion, however. The LT predicts (see Sect. III) that the rates of proper clocks in the two rest frames are related by a constant time-dilation factor which depends only on the speed v of the train relative to the platform [1, 7]. Accordingly, each observer will find that the other's clock runs slower by a factor of $\gamma = \left(1-v^2c^{-2}\right)^{-0.5}$. In the following, the use of the above TD relationship will be considered in detail for the train application.

At the beginning of this exercise, it will be assumed that the train is stationary with respect to the station platform. The first theoretical assumption is that all identical clocks located in a rest position anywhere on the train or on the platform are perfectly synchronized. For example, let each clock have a value of T_0 at the start. Since the relative speed of each of these clocks to any other is v=0, it can also be concluded on the basis of the above TD relationship that they are all running at the same constant rate at time T_0.

The train now accelerates and reaches a constant speed v relative to the platform. The LT formula for time dilation is assumed to hold, i.e. $\Delta t=\gamma\Delta t'$, for the two sets of clocks. Because each of the clocks on the train has moved with exactly the same speed at all times during the acceleration process, it follows from the TD that each of them shows the same value at this stage. Let this value be T_1 (train) $=T_0+\tau$. At time T_1 two lightning bolts strike opposite ends of the train, i.e. they do so simultaneously for all stationary clocks on the train.

Light flashes then proceed from each end to the midpoint between the locations of the light flashes. Assume this distance measured on the train is L. Note that the same value L is measured for both distances by the train observer at all stages of the acceleration process up to and including the time when the two light flashes reach the midpoint of the train. It is unaffected by any motion relative to the platform or any other rest frame (recall that the observer on the train feels that he is standing still throughout, while the platform appears to be moving with respect to him). It should also be clear that the same value of L is measured by the platform observer before the train begins to move. The time of the arrival of the light flashes as measured on all stationary clocks on the train is therefore equal to S (train)= T_1 (train) + L/c. It is assumed thereby that the speed of light measured on the train is equal to c in both directions, in accord with Einstein's light-speed postulate [1] which is assumed in deriving the LT.

Next consider the times read from the stationary platform clocks. We can assume for simplicity that the acceleration of the train to speed v relative to the platform occurs instantaneously (the start-up phase has no effect on the simultaneity argument). The TD formula indicates that the platform clocks now all run γ(v) times faster than each of the stationary clocks on the train. This means that the corresponding time on the platform's stationary clocks is T_1 (platform) $=T_0+\gamma(v)\,\tau$ when the two lightning flashes first hit the train. This is the key point in the TD argument. The value of T_1(platform) can be calculated using the TD without any reference to events occurring on the train such as the motion of light flashes toward the midpoint. One simply relies on the constant proportionality relationship between the rates of the two types of clocks that is indicated by the TD formula. Later when the two light flashes reach the midpoint of the train, the stationary clocks on the platform read S (platform) = T_1 (platform)+ γ (v) L/c for both light flashes. Again, the TD formula allows one to make this calculation without regard to any other details about the events under consideration. All one needs to know is the initial time of T_0 along with the subsequent elapsed times on the stationary train clocks.

The above discussion shows that if the two light flashes arrive at the midpoint of the train simultaneously for observers that are at rest anywhere in the train's rest frame, they also must arrive simultaneously for all stationary observers in the platform rest frame provided one agrees to use the TD formula exclusively in the calculations. At the same time, it is well known that if we use the LT in the manner first reported by Einstein, we come to the opposite conclusion. There is no reason to prefer one method of calculation over the other since both are based directly on eq. (1) of the LT without a single error in any of the logical deductions used to arrive at the different results.

3. Derivation of the LT Time-dilation Formula

Time dilation is central to the above proof of the simultaneity of the arrival of the two light flashes from the lightning strikes. It is important to see how TD is derived from STR in order to understand how it should be applied in practice.

The starting point is the Lorentz invariance condition of the LT given below:

$$\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2\Delta t'^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2 \quad (2)$$

An example is considered in which a stationary clock in one of the rest frames (S') described via the LT is used to measure a time difference Δt' for a given pair of events. There is no restriction on the types of events to be considered. The time difference could be a lifetime, a period of a clock or an elapsed time of any kind. It is stipulated that the clock remain in the same position during the entire course of the measurement, which means that $\Delta r'^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2 = 0$ in eq. (2).

The corresponding measurement is also made in the other rest frame (S), and this time difference is designated as Δt. The stationary clock in S' is moving relative to S with speed v in the x direction, however. Consequently,

$\Delta r = \Delta x = v\Delta t$. The right-hand side of eq. (2) is equal to $\Delta r^2 - c^2\Delta t^2$ so upon substitution one therefore has the following result for this application:

$$-c^2\Delta t'^2 = v^2\Delta t^2 - c^2\Delta t^2 . \quad (3)$$

Rearrangement thus leads to the TD formula:

$$\Delta t' = \left(1 - v^2c^{-2}\right)^{0.5} = \gamma^{-1}\Delta t , \quad (4)$$

whereby the key proportionality constant is seen to be γ^{-1}.

There is an interesting twist to this derivation, however. If the tables are turned and the measured times of the stationary clock in S are studied by the observer in S', one must change the derivation so that now $\Delta r = 0$ and $\Delta r' = -v\Delta t'$ are to be substituted in eq. (2). The result is:

$$\Delta t = \left(1 - v^2c^{-2}\right)^{0.5} = \gamma^{-1}\Delta t' . \quad (5)$$

As has been often discussed in the literature, eq. (5) cannot be obtained from eq. (4) by a simple inversion. Instead, there is a symmetry between the two equations that can be conveniently summarized by noting that it is always the moving clock relative to the observer that runs slower. This is a completely subjective relationship, since it indicates that it is a matter of perspective which of two moving clocks has the slower rate.

The point which has been emphasized in Sect. II is that the relationships of eqs. [4, 5] are not consistent with remote non-simultaneity otherwise derived from the LT, specifically from eq. (1). The latter indicates that remote non-simultaneity must occur when v and Δx both have non-zero values, whereas the TD relation states unequivocally that events which are simultaneous for one observer will be simultaneous for all others as well.

It has been shown elsewhere [8-11], however, that there is another space-time transformation which also satisfies both of Einstein's postulates of relativity. The result given below has been referred to as the GPS-LT because of its clear association with the Global Positioning System and its pre-correction procedure [12, 13] for atomic clocks carried onboard its satellites:

$$\Delta t' = \frac{\Delta t}{Q} \quad (6a)$$

$$\Delta x' = \eta\left(\Delta x - v\Delta t\right) \quad (6b)$$

$$\Delta y' = \frac{\eta\Delta y}{\gamma Q} \quad (6c)$$

$$\Delta z' = \frac{\eta\Delta z}{\gamma Q} \quad (6d)$$

with η defined above as $\left(\frac{1 - vc^{-2}\Delta x}{\Delta t}\right)^{-1}$ and Q is a proportionality constant specific for each pair of inertial rest frames described by the transformation. Note that a TD-like clock-rate proportionality relation appears explicitly in eq. (6a).

This raises the question as to whether the same lack of consistency with regard to simultaneity occurs for the GPS-LT. To check this possibility, we can again consider the above example of two clocks in motion. One difference relative to the LT derivation is that the condition of invariance in eq. (2) is not the same for the GPS-LT. Instead, one obtains the following result by squaring each of its eqs. (6a-d) and combining:

$$\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2\Delta t'^2 = \eta^2\gamma^{-2}Q^{-2}\left(\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2\right) . \quad (7)$$

If one makes the same substitutions as previously ($\Delta r' = 0$ and $\Delta r = v\Delta t$), the result is:

$$-c^2\Delta t'^2 = \eta^2\gamma^{-2}Q^{-2}\left(v^2\Delta t^2 - c^2\Delta t^2\right). \quad (8)$$

The constant η has a value of $\left(1 - v^2c^{-2}\right)^{-1} = \gamma^2$ in this case, so evaluation of eq. (8) gives:

$$\Delta t' = \left[\gamma^4\gamma^{-2}Q^{-2}\left(1 - v^2c^{-2}\right)\right]^{0.5}\Delta t = \frac{\Delta t}{Q} , \quad (9)$$

which is seen to be identical to eq. (6a) of the GPS-LT, proving that there is no inconsistency in this case.

Reversing the roles of the two stationary clocks, i.e. substituting $\Delta r = 0$ and $\Delta r' = -v\Delta t'$ in eq. (7), one obtains (note that $\eta = 1$ in this case since $\Delta x = 0$):

$$\left(v^2\Delta t'^2 - c^2\Delta t'^2\right) = \gamma^{-2}Q^{-2}\left(-c^2\Delta t^2\right). \quad (10)$$

Rearrangement then gives the same result as in eqs. (6a, 9), which shows that the GPS-LT is self-consistent in this application, unlike the LT.

4. Conclusion

The famous example of light flashes striking a moving train has been used for many years to make the phenomenon of remote non-simultaneity plausible to the general public. The analysis is based directly on the space-time mixing relation [see eq. (1)] of the Lorentz transformation (LT). One can use the same equation to prove that clocks in motion run slower than their counterparts at rest (time dilation, TD), however. If the train example is analyzed exclusively on the basis of the clock-rate proportionality expressed in the TD relationship, a completely different conclusion is reached about simultaneity. Ultimately, it is impossible for two clocks with rates that are constantly in the same proportion to disagree whether a time difference is zero or not. Clock-rate proportionality is the antithesis of remote non-simultaneity. Physicists need to come to grips with this ambiguity of Einstein's relativity theory (STR) and come to a clear recognition of its consequences for the validity of the LT from which it is clearly derived. It is unacceptable that a physical theory give opposite answers to the same question depending on how it is applied.

Examination of the way in which the TD relation is derived from the LT is instructive in this discussion. It also opens up a straightforward solution for removing the ambiguity. One simply considers the situation when two clocks are used to measure the time difference between the same two events. The two times on one clock are obtained while it remains stationary in a given rest frame (S'). From the point of view of another observer at rest in S, the clock in question moves away from him at speed v in the x direction, consistent with the assumptions used to determine the LT connecting their respective rest frames. Thus the position of the S' clock changes by $\Delta x = v\Delta t$ for the stationary observer in S during the course of the two measurements. Substitution of the latter relation in eq. (1) eliminates any trace of space-time mixing and leads directly to the clock-rate proportionality of the TD relation in eq. (4). The result is therefore clearly inconsistent with eq. (1), the starting point of the derivation.
Consistency is restored by employing the GPS-LT space-time transformation of eqs. (6a-d), particularly the clock-rate proportionality relation of eq. (6a). Substitution of the same $\Delta x = v\Delta t$ condtion as above in the modified Lorentz invariance relation of eq. (7) of the GPS-LT simply leads back to eq. (6a), proving that no comparable lack of consistency exists for this transformation, despite the fact that it also satisfies both of Einstein's postulates of relativity.

5. References

1. Einstein A. Ann. Physik. 1905; 322(10):891
2. Ives WHE, Stilwell GR. J Opt. Soc. Am. 1938; 28:215; 1941; 31:369
3. Hafele JC, Keating RE. Science. 1972; 177:166
4. Hafele JC, Keating RE. Science. 1972; 177:168
5. Poincaré H, Rev. Métaphys. Morale, 1898; 6(1)
6. Phipps TE. Jr., Old Physics for New, Apeiron, Montreal, 2006, 127.
7. Jackson JD. Classical Electrodynamics, John Wiley and Sons, New York, 1962, 357-360.
8. Buenker RJ. Apeiron 2008; 15:254
9. Buenker RJ. Apeiron 2009; 16:96
10. Buenker RJ. Phys. Essays 2013; 26:494
11. Buenker RJ. Relativity Contradictions Unveiled: Kinematics, Gravity and Light Refraction, Apeiron, Montreal, 2014, 55-58.
12. Will CM. Was Einstein Right?: Putting General Relativity To The Test. 2nd ed. New York: Basic Books Inc; 1993, 272.
13. Van Flandern T. in: Open Questions in Relativistic Physics, ed. F. Selleri. Apeiron, Montreal, 1998, 81.