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## The spin-flip mechanism for neutron decay and its relation to the nuclear shell model

**Robert J Buenker**

### Abstract

The exponentially-damped Breit-Pauli Schrödinger (XBPS) model of nuclear physics is reviewed. A key assumption is that the neutron is a compound of three elementary particles: the proton, the electron and the antineutrino. Binding within the neutron is achieved by assuming that the antineutrino plays an essential role in keeping the particles together. An important aspect of this approach is the choice of its charge-to-mass ratio to lie in the neighborhood of 0.5-0.6 bohr magneton, whereby arguments have been presented to show that such a large value is perfectly consistent with the experimentally known extreme penetrability of neutrinos. Previous calculations with the XBPS model have found that the triplet multiplicity of the deuteron can be explained by assuming, in agreement with the nuclear shell model, that spin-dependent forces are involved in addition to those which are purely electromagnetic in nature. Based on both experimental and theoretical inferences, a “spin-flip” mechanism is proposed to account for the instability of the neutron. The essential role of  $e^{-}\bar{\nu}$  complexes of  $0^{-}$  symmetry in producing nuclear binding is emphasized, particularly their attraction for constituent protons in a given nucleus.

**Keywords:** Nuclear structure, nuclear shell model, internal composition of the neutron, spin-flip mechanism for neutron decay

### Introduction

In recent work <sup>[1]</sup>, calculations have been presented which are consistent with the known properties of both the <sup>2</sup>H and <sup>3</sup>He nuclei. A key assumption in this approach is that the neutron has a three-particle composition consisting of a proton, electron and antineutrino. The above calculations have been carried out with the *exponentially damped* Breit-Pauli Hamiltonian which contains a spin-spin  $\delta$  operator with a charge-to mass ratio of 1.0 bohr magneton for the proton. The purpose of the momentum-dependent damping factors is to prevent variational collapse of the associated wavefunctions that are otherwise obtained by solving a Schrödinger equation using the unmodified Breit-Pauli Hamiltonian.

A key aspect of the present (XBPS) model is that there is an  $e^{-}\bar{\nu}$  binary complex with  $0^{-}$  symmetry for every neutron in the composition of each nucleus. This assumption is seen to lead to binding energies of the proper magnitude for the various light nuclei when a value of the antineutrino charge-to mass ratio is assumed to be equal to approximately 0.6 bohr magneton. The latter value has been determined by adjusting it so that the known total energy of the neutron results from the calculations. Investigation of the interactions of protons with  $e^{-}\bar{\nu}$  complexes then gives insight into the composition of heavier nuclei. It will be seen that this information is consistent in many ways with the nuclear shell model of Goeppert-Mayer and Jensen <sup>[2, 3]</sup>.

### Nuclei heavier than the alpha particle: comparison with the nuclear shell model

The discussion of the internal composition of the deuteron and <sup>3</sup>He <sup>[1]</sup> has been shown to be largely consistent with the nuclear shell model of Goeppert-Mayer and Jensen <sup>[2, 3]</sup>, differing from it mainly in the concept of protons interacting with  $e^{-}\bar{\nu}$  units rather than neutrons as elemental particles. The lowest-energy proton  $s_{1/2}$  orbitals  $\phi_a$  and  $\phi_b$  simply replace the corresponding neutron and proton species employed in the shell model. A key element in the exponentially damped Breit-Pauli-Schrödinger (XBPS) calculations which suggests this

adjustment is the finding that the most stable state of the  $e^- \bar{\nu}$  system is a singlet. Since neither the electron nor the antineutrino can exist inside the nucleus without the other, it follows that pairs of these fermions can be added indefinitely without directly affecting the magnitude of the angular momentum of the constituent protons of a given system.

The possibility exists that the hierarchy of spin-orbitals for proton-neutron shells in the original model can be obtained from calculations based on an augmented XBPS Hamiltonian similar to that employed above in Sects. III and IV of Ref. [1], in which case the resulting proton shells are expected to occur in pairs of the same symmetry, in analogy to the  $\phi_a$  and  $\phi_b$   $s_{1/2}$  species already considered. It is likely that the  $p_{1/2} s_{1/2}$  configuration for  $e^- \bar{\nu}$  is not the only possibility, however. Any configuration of two orbitals of the same  $j$  but differing by one unit in  $l$  contains a single  $0^-$  multiplet, and thus different  $e^- \bar{\nu}$  units could vary widely with respect to their  $l$ -type orbital composition while still retaining their singlet characteristic. One can conceive of a series of concentric spherical units, each differing in volume from their neighbor of smaller radius by a constant amount, consistent with the arguments given in Sect. III of Ref. [1] based on the Darwin term. The exact nature of these relationships is clearly a matter to be settled by explicit calculations for larger nuclei.

The negative parity of the  $e^- \bar{\nu}$  units must have an effect on the overall nuclear symmetry, but it is generally accepted that there is no way to determine absolute parities of nuclear states experimentally [4]. In the case of nuclei it should be noted that parity is always inferred indirectly on the basis of angular momentum changes which occur upon scattering relatively light systems such as deuterons off heavier nuclei. Analysis of the resulting proton or neutron capture reactions [5] allows a determination of the  $l$  value of the incoming nucleon from which the corresponding parity change is deduced. Since the  $e^- \bar{\nu}$  complex has  $J = 0$ , it is impossible to learn anything about its parity from such investigations, so that existing assignments of nucleon quantum numbers necessarily only refer to the proton orbitals which are occupied in the calculations of the present (XBPS) model.

By assuming that the  $e^- \bar{\nu}$  units always have  $0^-$  symmetry, it becomes a simple matter to convert the total  $J$  values obtained on the basis of the present model to those deduced from the nucleon capture angular momentum distributions. For example, since the deuteron has a single such unit, it follows that the  $1^-$  symmetry computed for its ground state is tantamount to predicting that the results of such experiments will lead to a  $1^+$  assignment, as actually observed. Nonetheless, although the intrinsic negative parity of the  $e^- \bar{\nu}$  unit defies detection in the traditional nuclear physics experiments described above, it will be seen to have interesting consequences regarding the interpretation of other pivotal observations in elementary particles physics, as will be discussed in succeeding sections.

It is well to emphasize that the XBPS treatment does not assume fixed positions for the various nucleons, in contrast to the practice in the Born-Oppenheimer approximation commonly used in molecular calculations. As a result, the symmetry of the corresponding Hamiltonian is that of the full rotation group with inversion, exactly as assumed in the nuclear shell model. This feature does not really distinguish

molecules from nuclei, however, because the rotational states of the former also transform rigorously as irreducible representations of the same point group [6]. This seems surprising in view of the fact that molecules are said to have electric dipole moments, but a closer analysis shows that such properties only result as a consequence of the symmetry reduction produced by the introduction of an external electric field [7]. The electric dipole moments of molecules are thus seen to result because perturbations of this nature cause rotational states of different parity to mix with one another. In the last analysis it is the relatively small energy separation of rotational states which distinguishes molecules from nuclei in this respect. In summary, the rotational symmetry of the XBPS wavefunctions results because no comparable "clamped-nuclei" assumption to that of the Born-Oppenheimer approximation is made, rather than because a convenient simplification is introduced *ad hoc* in order to maintain identification with the principles of the nuclear shell model. The calculations discussed above have the disadvantage of simultaneously including translational effects as well, but the difficulties brought about by this characteristic become somewhat less critical for systems containing relatively heavy particles such as protons.

After the  $1s_{1/2}$  shells have been filled, the nuclear shell model leads us to expect that the next most stable orbital available to succeeding protons is  $1p_{3/2}$ , closely followed by  $1p_{1/2}$ . Since the XBPS model foresees pairs of such proton orbitals, it might be expected that the same symmetry is not always involved for both partners. Because the two orbitals are assumed to occupy essentially equi-potential locations (see Sect. III and Fig. 2 of Ref. [8]) relative to a given  $e^- \bar{\nu}$  unit, however, it seems at least plausible that they differ only in their respective radial distributions. The operators involved in the XBPS model are of the spin-orbit and related type, and since this choice has been suggested in part by the results of the nuclear shell model, it is thus clearly consistent with this approach. If the traditional  $q/m_0$  factors of the Breit-Pauli Hamiltonian [9] (see Table 1 of Ref. [10]) are employed exclusively, it is seen that the only interactions of this nature which are of great importance for protons are the spin-same-orbit and Darwin terms, whereas the  $e^- \bar{\nu}$  units participate in all such interactions (except the Darwin term for the most part). A spin-orbit term which can account for the large  $p_{3/2}$ - $p_{1/2}$  and related splittings deduced from the locations of nuclear energy levels [2, 3, 11, 12] is also required, however, which has far larger coupling constants than those found in the conventional Breit-Pauli interactions [9] between protons.

A spin-spin  $\delta$ -function operator employing a unit (electronic)  $q/m_0$  value for the proton-proton interactions has been found to be effective in explaining the triplet multiplicity of the deuteron's ground state (Sect. III of Ref. [1]), but it is clear that such a term cannot account for the observed splittings of  $j$  levels belonging to the same  $l$  quantum number. It needs to be recognized that such spin-orbit interactions cannot be assumed to occur between a proton and an antiproton without upsetting the arguments of Sect. 2.4 of Ref. [8]. They predict that the  $e^+e^-$  and  $p^+p^-$  systems have wavefunctions which are strictly related by a scaling operation. It is also clear that some type of long-range damping is required, similar to that originally suggested by Yukawa [13, 14], to account for the fact that the

influence of the strong interaction does not appear to extend beyond nuclear dimensions.

The search for such spin-dependent proton-proton interactions is clearly far easier when an accurate description of the central field attracting these particles is available. It is in this context that the present XBPS Hamiltonian can play a crucial role. In this model the  $e^- \bar{\nu}$  units provide a strong attraction for protons while having no direct effect on the angular momentum of the nucleus as a whole by virtue of their  $0^-$  symmetry. In this view the bond between a proton and a neutron comes about to a large extent because two protons are strongly attracted to the same electron and antineutrino. Since the protons in question occupy different orbitals (Sect. III of Ref. [1]), their individual relationships to the lighter particles can still be quite different from one another, thereby making it appear that they are associated with two fundamentally different particles, i.e. a neutron on the one hand and a proton on the other, exactly as foreseen in the nuclear shell model.

### The isospin property and its relation to the XBPS model

Before concluding this comparison between the results of the preceding calculations and the concepts which form the basis of the theory of nuclear physics, it is important to give close consideration to another fundamental quantity, the isospin property [15, 16]. The XBPS model employs a Hamiltonian which depends only on the spatial and spin coordinates of the constituent particles of a given system, and on this basis the corresponding energies and other expectation values are computed without making any prior assumptions whatsoever regarding the isospin property. It is thus important to see how the fundamental relationships underlying the isospin theory might be inferred from the results of such calculations. The idea which led to the introduction of isospin can be traced back to the suggestion of Heisenberg [17] in 1932 that a neutron and a proton can be looked upon as two different states of the same system (nucleon). As such, these fundamental particles are referred to as an isospin doublet, differing only in the z component of the vectorial quantity I. The latter's transformation properties are assumed to be identical to those of orbital and spin angular momentum, and so a well-defined mathematical framework for the theoretical treatment of isospin is immediately at hand.

The most important physical conclusion regarding such isospin multiplets is that the various component systems would be perfectly degenerate if only the hadronic force were active, and not the electromagnetic (gravitational forces can be safely ignored because of the nature of the problem). The theory asserts that under these circumstances the rest masses of the proton and neutron would be of identical magnitude. In the present model emphasis is placed instead on the possibility of converting a proton into a neutron by the addition of an electron and an antineutrino in the form of an  $e^- \bar{\nu}$  binary unit. In this sense, elimination of the electromagnetic force can be equated with placing bare protons in an environment which is absolutely free of electrons and antineutrinos. Because of the high density of massless  $e^+e^-$  and  $\nu\bar{\nu}$  binary systems, so the XBPS model goes, this state of affairs is impossible to achieve by any experimental means, similarly as it is not really feasible to turn the electromagnetic force off and on at one's volition.

Explicit calculations with the XBPS Hamiltonian show that a system of two protons forms neither bound states nor

resonances, and hence in this sense the pp interaction is purely repulsive. The reason that roughly one hundred years of scattering experiments have been interpreted differently is understandable in terms of the premise that whenever two protons approach each other at very close range, they have a sufficiently strong attraction for an electron and an antineutrino to overcome the dissociation barriers of the ever-present  $e^+e^-$  and  $\nu\bar{\nu}$  binary systems, thus making these particles available, i.e. "creating" them in the conventional terminology. The fact that the partner positrons and neutrinos are always set free in such nuclear formation processes is consistent with this interpretation, although the evidence is clearly less than totally unambiguous. To settle the matter definitively would require experimental proof that particles can be created or destroyed with appropriate application of energy, which amounts to observing the unobservable. Suffice it to say that the charge independence of nuclear reactions, which is at the root of the isospin concept, is accounted for in the XBPS model. Thus, to the extent that a suitable Hamiltonian can be found and the associated Schrödinger equation solved to a satisfactory degree of accuracy, one can expect the key results of isospin theory to emerge from such treatments without having to make additional assumptions of an *ad hoc* nature.

One of the greatest successes of the isospin theory in the realm of nuclear physics is the elucidation of trends in the rest masses of isobaric nuclides [18], i.e. groups of systems with the same mass number. One of the clearest examples of this type is the isospin triplet  $^{14}\text{C}$ ,  $^{14}\text{N}$  and  $^{14}\text{O}$  in their respective  $0^+$  states. In such cases it is found that the differences in the rest masses of these systems can be estimated to a good approximation by comparing the respective magnitudes of the Coulomb repulsion of their constituent protons and correcting for the promotion energy required to convert protons into neutrons. In the present model isobaric nuclides differ from one another mainly in the number of their constituent  $e^- \bar{\nu}$  units, each of which changes the atomic number of a given nucleus without changing its mass number. Assuming that the various proton shells correspond to pairs of orbitals  $\phi_a$ ,  $\phi_b$  which can be associated on a one-to-one basis with the neutron and proton one-particle functions of the nuclear shell model, it is only necessary in the present view that the occupation of the outermost shells (in the above case of  $p_{1/2}$  symmetry) change in concert with the addition or loss of the  $e^- \bar{\nu}$  units.

The relatively good agreement achieved with experimental data through the above approximation suggests that proton orbitals can be distinguished in a reasonably unambiguous manner on the basis of their spatial relationship to the  $e^- \bar{\nu}$  unit lying closest to it. Again the existence of pairs of equipotential points in the exponential damped Breit-Pauli terms, as indicated in the schematic diagram of Fig. 2 of Ref. [8], make such an identification at least plausible. More significant in this respect is perhaps the spin-dependent force which exists between the protons, however. The (partial) concretization of this interaction discussed in Sect. III of Ref. [1] prevents the development of atomic-like trends in which shells tend to be completely filled with electrons before the next most stable species is to be occupied. By providing a strong repulsive interaction between protons of opposite spin, this term minimizes the importance of stability differences between two neighboring shells and causes them to be occupied alternately in a manner akin to

the way houses and hotels are added to properties of the same color in the parlor game Monopoly. Moreover, the  $p^+e^-\bar{\nu}$  calculations of Sect. IV of Ref. [1] indicate that the gain in stability possible by further occupying one of the "neutron" orbitals is very much dependent on the number of neighboring  $e^-\bar{\nu}$  units in the system.

Another key aspect of the isospin formulation is its assumption of nuclear wavefunctions which are anti-symmetric with respect to any exchange of a proton and a neutron [15]. This specification is referred to as the generalized Pauli principle [19] to call attention to the fact that in the original work [20], an anti-symmetric form was only required for indistinguishable fermions. In the XBPS model no such generalization is necessary because protons are already assumed to satisfy the Pauli anti-symmetry principle, and accordingly it is actually two of these particles which are being permuted when one speaks in the conventional model of an exchange of a proton and a neutron. The corresponding electron and antineutrino of the "neutronic" proton are totally separate particles in this view, and thus are not affected by such a permutation. In retrospect, it would be more difficult to bring the XBPS model in line with the isospin formulation if the latter insisted that its nuclear wavefunction *did not have to satisfy* a particular permutation symmetry for the exchange of a proton and a neutron. This eventuality would force an exception to the original Pauli principle [20] to be allowed in the present model according to which the exchange of two such indistinguishable protons would not lead to a sign change in the total wavefunctions under these circumstances. Instead the realization of the consequences of  $\beta$  decay of neutrons left no other choice in the formulation of the isospin theory [15, 16] to broaden the definition of "indistinguishable" particles in the Pauli principle to include the proton and the neutron in the same class despite their apparently distinctive characteristics.

In this discussion it is also important to recognize that the predictions of the isospin theory in nuclear physics are not of a generally quantitative nature. The comparisons of the energies of isobaric nuclides mentioned at the beginning of this section do often work remarkably well for ground states, but the experience with excited states is less sanguine. In their book Goeppert-Mayer and Jensen [21] point out, for example, that although excited levels of a nucleus with  $I_z = 1$  should be closely related to some of those of the corresponding isobar with  $I_z = 0$ , "seldom has it been possible to make an unambiguous assignment of this kind [12, 22]." As a result these authors prefer to distinguish between the "charge symmetry" of nuclear forces, involving either two protons or two neutrons ( $I_z = \pm 1$ ), and "charge independence" when a neutron-proton interaction is present, concluding that the experimental evidence for the latter property is not nearly as strong as for the former. Furthermore, it is well known that correspondingly simple relationships in either isotopic or isotonic series are all but non-existent, as illustrated by the deuteron- $^3\text{He}$ - $\alpha$  particle series [23] discussed in Sect. IV of Ref. [1]. There is thus a danger of oversimplifying the theory of nuclear binding by rigidly assuming that all nucleons in equivalent shells have the same properties except for electromagnetic effects.

As an additional remark on the relation of the isospin concept to the XBPS model, it seems inescapable that at some point the number of  $e^-\bar{\nu}$  units reaches a critical value, at which point it is no longer possible to form even a meta-

stable nucleus with the original number of nucleons. In isospin theory such theoretical isobaric members are simply assumed to be non-existent, whereas in an *ab initio* approach they correspond to virtual states whose mass and other properties can be computed in principle, but which in all likelihood are quite unstable to either  $\beta$  decay or electron-capture processes. A similar situation is not uncommon in atomic and molecular physics, whereby states which correspond to bound species for other (isoelectronic) systems are found to lie in some continuum region for the system of immediate interest. Sometimes a resonance of the expected character can be located, but more often than not, such corresponding (adiabatic) states are totally absent in the computed spectrum, or more precisely, they are apportioned among the wavefunctions of continuum states lying in the appropriate energy range.

In summary, the present model can be expected to be at least indirectly relevant to the isospin concept by virtue of its (hoped-for) capacity to successfully predict the masses and properties of a variety of related nuclear systems. To the extent that the solutions of the corresponding Schrödinger equations can be obtained quantitatively, and in the process be shown to reproduce the pertinent experimental data to a satisfactory degree of approximation, such results will inevitably be found to be in agreement with the conclusions otherwise reached independently through application of the theory of isospin. Such an attitude toward a concept which has become so well established over the years as to have been referred to by one author [24] as an "industry" might well be perceived by some as unacceptably *leger*. Opinions have varied widely on this issue, however, as illustrated by a passing reference made in 1950 by Goldstein [25] in his survey of the principles of classical mechanics in which he wrote: "indeed for nuclear forces we don't have any theory worth speaking about." We shall return to the subject of isospin in subsequent work when the relationship of the XBPS model to the quark theory of elementary particles is taken up.

### Nuclear reactions

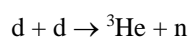
The discussion of nuclear forces in terms of the XBPS model has thus far being confined to the study of structure alone. The possibility of carrying out calculations for such systems on an *ab initio* basis also carries with it the prospect of studying nuclear reactions in a new light. One can order these processes most conveniently in terms of the types of particles which are produced by them. These include  $\alpha$ ,  $\beta$  and  $\gamma$  rays in the original language used to describe this field [26]. Later on, neutrons were also discovered as decay products [27], and the general possibilities of nuclear fusion and fission became known. When studying reactions it is necessary to know something about energy barriers separating products from reactants, and not just relative energies in their respective equilibrium states.

Since the Born-Oppenheimer approximation is not used in the XBPS treatment, there are no potential curves as such from which to deduce the magnitudes of such barrier heights. For many purposes the variation of the total energy with scale factor  $\eta$  for the associated one-particle basis set provides equivalent information, however, as demonstrated in the  $e^+e^-$  and  $\nu\bar{\nu}$  treatments (see Fig. 3 of Ref. [8] and Fig. 2 of Ref. [10]). As  $\eta$  is increased from small values, the energy always rises until a certain point when the short-range interactions begin to overtake the effects of kinetic

energy (Fig. 1 of Ref. [8]). The heights of such computed barriers are necessarily basis-set dependent, since for a complete set the corresponding results must be independent of scale factor. For relatively large basis sets this degree of saturation still remains an unattainable ideal, however, and one can hope that data such as shown in the above figures can be successfully analyzed in the suggested manner.

Because of the extensive computations required in the present model for the treatment of systems consisting of a few particles, it is likely that such investigations will not be feasible for heavy nuclei, thus making the study of nuclear fission and alpha-particle decays accessible only if suitable approximations can be found which allow inert shells to be treated in a simplified manner. The subject of  $\beta$  decay will be taken up in detail in later work, however, and the treatment of photon emissions from light nuclei also appears to be a practical goal of *ab initio* calculations of this type. In the latter case it is necessary to develop methods which clearly distinguish pairs of states differing mainly in translational energy from those corresponding to distinct internal states between which spectroscopically observable transitions are expected to occur.

The most attractive reaction type that might be accessible to treatment by such computational methods is nuclear fusion, since only relatively small nuclei in their respective ground states are involved in the most interesting cases. There has recently been renewed interest in this subject because of reports on electrochemical studies on heavy water with platinum and palladium electrodes [28, 29]. The results of these experiments have led to speculation regarding the identity of the reactions which might be involved, although there is mounting evidence that some of the original observations [30] can be explained to at least a large extent on the basis of purely chemical transformations. Traditional high-temperature fusion experiments have been based largely on the reaction of tritium and deuterium to give  $^4\text{He}$  plus a neutron [31]. The exothermicity of the above tritium reaction is 646458.62 hartree (17.6 MeV), which compares favorably with that of the analogous fusion process



whose exothermicity is only 120238.74 hartree (3.3 MeV). Despite the fact that no radioactive substances are required to drive the latter reaction, the higher heat output of the tritium process makes it technically more appealing. The reaction of  $^3\text{He}$  plus a neutron is itself quite exothermic, however, since it leads to the formation of the highly stable alpha particle. If this product were formed directly in the  $d + d$  reaction, the corresponding exothermicity would increase to 876582.49 hartree (23.8 MeV), 6.2 MeV more than in the  $t + d$  counterpart preferred in the high-temperature fusion investigations [31]. Moreover, both reactants and product of the former process are thermodynamically stable.

The reason that  $^4\text{He}$  is not formed with sufficient regularity from  $d + d$  collisions is understandable in terms of the laws of conservation of energy and momentum. A second product is indispensable for sharing the energy potentially released in this reaction, and decomposition into  $^3\text{He}$  plus a neutron achieves this purpose with relatively high probability under typical reaction conditions. A photon could carry away most of the available 23.8 MeV energy if the intermediate  $^4\text{He}$  state reached were of non-zero multiplicity, however, so as to make a subsequent emission process to the  $J=0$  ground

state allowed. Since deuterons have zero isospin such an excited  $^4\text{He}$  state must also be expected to be an isospin singlet, but this in itself would not seem to rule out the formation of an intermediate with angular momentum  $J = 1$  or 2 because of the triplet multiplicity of each deuteron. Once the pertinent  $^4\text{He}$  wavefunctions became available, it would be possible to compute the rate for radiative decay to the ground state as well as for decomposition into the  $^3\text{He}$ -plus-neutron products. The practicality of such calculations is something which remains open to question at the present stage of development, but this example at least illustrates how the above quantum mechanical approach might be used to shed light on the mechanisms governing the reactions of small nuclei.

### Weak interaction in the XBPS model

In the preceding sections, the strong interaction responsible for nuclear binding has been represented in large part by means of an exponentially-damped, momentum-dependent potential which has as its low-velocity limit the spin-orbit and related operators of the Breit-Pauli Hamiltonian [9]. The key assumption in the accompanying theoretical model is that the antineutrino is capable of undergoing an attractive short-range interaction with other particles because it possesses a non-zero charge-to-rest-mass ratio (its charge and rest mass are each assumed to be zero). Under these conditions it is shown that the two-body potential defined above (XBPS Hamiltonian) is capable of binding protons to  $e^- \bar{\nu}$  binary systems with sufficient force to allow identification of the stable products with experimentally observed nuclei. Instead of assuming that the electron and antineutrino decay particles of a neutron are created at the time of its decomposition, it is demonstrated that, contrary to past argumentation, a potential does exist which is sufficiently attractive to confine such light particles within a volume of typical nuclear dimensions [32]. In this view, the electron and antineutrino retain their existence within the bound nucleus, and they are simply set free in  $\beta$  decay processes.

The question that obviously arises as a result is whether such a model can be used in a consistent manner to describe the weak interaction itself, i.e. the force which is believed to be responsible for the  $\beta$  decay of nuclides. The accepted view in theoretical physics today is that the strong and weak interactions are basically different in nature, although the possibility has long been sought of unifying them in some way with each other as well as with at least the electromagnetic interaction. In more recent times a theory which succeeds in combining the electromagnetic and the weak interactions has been formulated [33], but the hoped-for union with the strong nuclear force has yet to be achieved. Once it has been assumed that the binding of nuclei can be described in terms of interactions involving three different constituents [32], the proton, electron and antineutrino, however, it is straightforward to assume that the same types of effects must be involved when such systems undergo spontaneous decomposition. This line of approach is investigated below.

### Quantum mechanical theory of radiationless transitions

It has been mentioned in Sect. II of Ref. [32] that the neutron can be looked upon as an excited state of a system whose ground state is unbound, a state of affairs which is analogous to that existing for excimers in molecular physics.

The  $\beta$  decay of a neutron involves a radiation-less transition in the language of that field, since photons are not emitted in the process. In order to compute the lifetime of any radiation-less decay the most direct way is to solve the corresponding Schrödinger equation for the excited state in question (more commonly referred to as a resonance). The eigenvalues associated with such meta-stable states are complex ( $E_r - iE_i$ ) and the line width  $\Gamma$  associated with its decay is equal to twice the imaginary component  $E_i$  [34]. The same Hamiltonian is employed for this purpose as for the computation of bound states. The distinction between stability and meta-stability of a given state arises naturally out of the solution of the corresponding Schrödinger equation, namely in terms of the magnitude of  $E_i$  (zero for stable, non-zero for unstable or resonance states) [35]. Upon carrying over this formally simple approach to the present discussion, one must expect that the neutron's energy eigenvalue is actually complex and corresponds to a line width  $\Gamma = \hbar/\tau$ , where the lifetime  $\tau$  is observed experimentally to be 918 s.

Because the neutron's lifetime is quite long in comparison with those of typical atomic or molecular radiation-less processes, it is quite impractical to compute its value in the above manner, however. The imaginary part of the corresponding energy eigenvalue is only in the order of  $10^{-18}$  eV, as compared with a total (real) energy of 29000 hartree, so the accuracy required in obtaining a useful result in this way is prohibitively high. Instead, it is far better in such cases to resort to an approximate method closely related to it, namely the Fermi golden rule [36], which can be derived from the time-dependent perturbation formalism introduced by Dirac [37].

In essence, low-order perturbation theory is employed to obtain the desired result, and because the effect of interest is very small, the accuracy expected from this level of approximation is correspondingly high. The usual procedure is to divide the total Hamiltonian into two parts,  $H_0$  and  $H'$ , and first obtain solutions for the former operator's Schrödinger equation which can subsequently be used as a basis in which to apply the golden-rule formalism. In a typical application a spin-free Hamiltonian ( $H_0$ ) is diagonalized, while the spin-orbit operator serves as  $H'$ . In the present example, it is very difficult to know how to divide up the XBPS Hamiltonian, however, especially to identify a small interaction in it which is responsible for inducing the transition from initial to final. Such a division of the total interactions is not essential, however, as can be judged from the complex eigenvalue approach first discussed, in which only the full Hamiltonian is actually needed. Because the golden-rule formalism deals with off-diagonal matrix elements of  $H'$  in a basis of functions which diagonalize  $H_0$ , it follows that the corresponding full Hamiltonian matrix elements have the same magnitude as those for the perturbing term alone.

On this basis, it can be recognized that the main purpose of dividing up the full Hamiltonian into  $H'$  and  $H_0$  components is to provide a suitable definition of the initial and final states of the problem. The difficulty in applying the golden-rule formalism for the XBPS Hamiltonian in the description of neutron decay thus comes down to identifying realistic initial and final states which represent the physical situation satisfactorily. In a spin-orbit pre-dissociation of a molecular energy level [38], for example, one chooses pure spin states of different multiplicity for this purpose. In another typical

example, Born-Oppenheimer solutions are taken as initial and final states and allowed to interact via the nuclear kinetic energy operator (non-adiabatic effects). A completely equivalent way of proceeding in both cases, however, consists of first diagonalizing the full Hamiltonian and identifying solutions which correspond to mixtures of the initial and final states normally employed in the golden-rule approach. A unitary (diabatic) transformation can then be defined which connects the diagonal representation of the full Hamiltonian to one which is non-diagonal, based on the physically appropriate golden-rule states. The off-diagonal matrix elements of the latter Hamiltonian matrix are then recognized as the quantities to be substituted in the golden-rule formula for the desired lifetime computation.

When described in this way the above procedure sounds a bit arbitrary, which in a certain sense it is, but no less so than in the alternative method of dividing up the full Hamiltonian into two parts. The correct lifetime can only be obtained if the choice of initial and final states conforms to the experimental conditions at hand. That is why the complex eigenvalue method is more acceptable in principle because no comparable choice is needed in this case. Instead, it is only necessary to identify the Schrödinger equation solution which corresponds to the physical state of interest. In summary, to describe the weak interaction in terms of the XBPS Hamiltonian it is not necessary to first devise a new interaction, but rather to identify a suitable diabatic transformation which leads to the physically meaningful initial and final states involved in the decay processes of interest.

### Spin-flip mechanism for neutron decay

In Sect. V of Ref. [10] it was noted that the most stable state of the  $p^+e^-\bar{\nu}$  system to be associated with the neutron has  $1/2^-$  symmetry and that the  $M_s = 1/2$  component can be described primarily in terms of the spin configuration  $p\alpha(e^-\alpha\bar{\nu}\beta - e^-\beta\bar{\nu}\alpha)$ , that is, the  $e^-\bar{\nu} 0^-$  complex discussed above is bound meta-stably to the proton in this model. The spin of the neutron is thus determined almost entirely by that of the proton. The spins of the three particles emitted after neutron decay have been studied experimentally by Burgy *et al.* [39]. These authors found that of the three  $M_J = 1/2$  spin combinations possible for the  $p^+e^-\bar{\nu}$  system, one is missing almost entirely, namely  $\alpha\alpha\beta$ . It is thus seen that the antineutrino carries away the original total spin of the neutron to a large extent, and this implies that the proton and electron form a singlet complex once decomposition takes place. Since the  $\beta$  decay process involves a transition between two states of the same system, we can combine the above theoretical and experimental results to arrive at the following interpretation. In the initial (meta-stable) state, the spins of the electron and antineutrino are opposite and constantly alternating. At the time of decay a *spin flip* occurs so that the proton and electron spins become opposite and alternating and the  $\bar{\nu}$  spin consequently must remain fixed ( $\alpha$ ). The system is expected to be quite unstable in the resulting spin configuration, so that decomposition occurs as a consequence.

A key result of the above  $p^+e^-\bar{\nu}$  calculations based on the XBPS Hamiltonian is that the absence of the  $\alpha\alpha\beta$  spin configuration is not complete. The CI coefficients of Table 1 of Ref. [10] show that the ideal ratio of  $\sqrt{3}:1$  of the two doublet spin eigenfunctions (see Sect. V of Ref. [10])

corresponding to the same spatial orbital occupation is never quite attained. Discrepancies of 1- 2 % in the square of this ratio are generally observed.

With reference to the general discussion of lifetime computations given in the preceding section, it seems reasonable to assume that this admixture of another 1/2<sup>-</sup> spin eigenfunction is at least partially responsible for the neutron's instability. Because of this impurity in the p<sup>+</sup>e<sup>-</sup> $\bar{\nu}$  wavefunction ( $\Psi_1$ ) there is a small probability that the system foregoes the 0<sup>-</sup> arrangement preferred by the e<sup>-</sup> $\bar{\nu}$  complex, thereby causing the system as a whole to break apart. A diabatic transformation can then be defined along the lines discussed above which removes this impurity by mixing in a second state ( $\Psi_2$ ) which consists primarily of the other (undesirable) spin configuration. The p<sup>+</sup>e<sup>-</sup> $\bar{\nu}$  diabatic state  $\Psi_1^D$  with the pure e<sup>-</sup> $\bar{\nu}$  0<sup>-</sup> component is then defined as:

$$\Psi_1^D = \cos \Theta \Psi_1 + \sin \Theta \Psi_2$$

A second diabatic state corresponding to the orthonormal 1/2<sup>-</sup> spin function is similarly defined as:

$$\Psi_2^D = -\sin \Theta \Psi_1 + \cos \Theta \Psi_2$$

Normally several such excited states are required to allow for a thorough de-perturbation of the desired initial diabatic state, in which case a unitary transformation of higher dimension is required.

In order to use the results of the p<sup>+</sup>e<sup>-</sup> $\bar{\nu}$  calculations discussed in Sect.V of Ref. [10] to obtain an estimate of the corresponding decay lifetime, it is necessary to compute matrix elements over the full XBPS Hamiltonian between the diabatic states defined in the last section. This is a simple matter if the eigenvalues corresponding to the adiabatic excited states  $\Psi_n$  needed to define the required unitary transformation are known, involving a reverse transformation of the diagonal Hamiltonian matrix in the original eigenvector basis. This brings us to an important qualitative point, however. The decay process must involve a final state of the same total energy as that of  $\Psi_1^D$  which rules out any of the other diabatic species described above. All such states must be far less stable, corresponding to a highly undesirable change in the spin orientations of the three component particles relative to those in the initial state. Instead the true final state  $\Psi_c^D$  is a member of a continuum corresponding to the free proton, electron and antineutrino system of the same energy as  $\Psi_1^D$ . More precisely there are an infinite number of such states which correspond to the continuous range of antineutrino and electron energies whose distribution was successfully predicted by Fermi [40-42] in his original theory of  $\beta$  decay. The other diabatic species mentioned above are thus to be looked upon as virtual states which only act as intermediates in the decay process.

The decay is accordingly expected to be predominantly of second order, and properly described by the golden rule formula [36]:

$$\langle \Psi_1^D | H \Psi_c^D \rangle = \sum_n \langle \Psi_1^D | H \Psi_n^D \rangle \langle \Psi_n^D | H \Psi_c^D \rangle (E_n^D - E_1^D)^{-1}$$

$$\text{and } \tau^{-1} = 2\pi |\langle \Psi_1^D | H \Psi_c^D \rangle|^2 dN/dE,$$

where  $\tau$  is the lifetime in atomic units ( $\hbar a_0/e^2$ ) and  $dN/dE$  is the density of states of the continuum [42]. From these equations it is relatively easy to imagine why the neutron's decay lifetime is so long. First, the energy denominators  $E_n^D - E_1^D$  are expected to be quite large, as already mentioned. Secondly, each of the continuum functions  $\Psi_c^D$  is a free-particle state and as such cannot be expected to have very large Hamiltonian matrix elements with the very compact diabatic functions  $\Psi_n^D$  which are confined for all practical purposes to within nuclear dimensions. In this connection it is well to note that the matrix element  $\langle \Psi_1^D | H \Psi_c^D \rangle$  is not computed *ab initio* in the original theory [42], but rather is inferred from the measured lifetime of the decay process. Thus far we have only emphasized the effect of different spin orientations on the lifetime of the

p<sup>+</sup>e<sup>-</sup> $\bar{\nu}$  system, but it also seems clear that the s<sub>1/2</sub>-p<sub>1/2</sub> (or p<sub>3/2</sub>-d<sub>3/2</sub>) polarization effects discussed in Sect. V of Ref. [10] also should be an important factor. One of the major inducements for the binding of a proton in the XBPS calculations is its attractive spin-same-orbit interaction with the electron, which for an s<sub>1/2</sub> proton orbital requires that the electron occupy a p<sub>1/2</sub> species as far as possible. The antineutrino resists any polarization of the electronic charge distribution from s<sub>1/2</sub> to p<sub>1/2</sub> because such a change decreases its own attraction for the electron. When a relatively small number of protons is present in a nucleus as compared to the number of e<sup>-</sup> $\bar{\nu}$  units, the competition for the electron's favor is expected to be intense, and this in turn increases the possibility of a spin flip of the type discussed above. When more protons are added, however, the electron can more readily adjust its orbital character to satisfy them at the expense of the antineutrino without sacrificing the overall stability of the nucleus. It can thus be anticipated that nuclei of high binding energy are characterized by relatively highly polarized e<sup>-</sup> $\bar{\nu}$  units relative to their respective proton-free states, while at the same time preserving the 0<sup>-</sup> character of the light-particle binaries more thoroughly than has been computed above for the p<sup>+</sup>e<sup>-</sup> $\bar{\nu}$  neutron system.

Nevertheless, in view of the variety of Breit-Pauli terms which contribute to the stability of the e<sup>-</sup> $\bar{\nu}$  units as opposed to those with a major influence on the nuclear protons (Tables 2-4 of Ref. [43]), it still seems highly plausible that the electron and antineutrino remain indispensable to one another for the purposes of keeping them bound within the narrow confines of any given nucleus. This characteristic of the XBPS Hamiltonian (Table 1 of Ref. [10]) thus ensures adherence to the well-known fact that  $\beta$  decay always involves an addition or loss of the same number of electrons as antineutrinos (or positrons and neutrinos). In this way neutrons can be converted into protons and *vice-versa* as a consequence of  $\beta$  decay. When an excess of protons is present, the results of the XBPS calculations indicate that a relatively high binding energy is still possible. Under these circumstances, however, the nucleus becomes unstable to electron capture that would otherwise lead to the conversion of protons into neutrons via this mechanism in order to further enhance stability. Ultimately in the present model it is the high density of the e<sup>+</sup>e<sup>-</sup> and  $\nu\bar{\nu}$  mass-less binaries throughout the universe [8] which keeps systems such as <sup>4</sup>Li from having a stable existence.

## Conclusion

The above description of the deuteron binding process can then be extended at least qualitatively to the structure of heavier nuclei, of which  ${}^3\text{He}$  and the hypothetical  ${}^4\text{Li}$  system have been explicitly treated in Ref. [1]. A key feature of the XBPS model is the identification of a neutron as a proton which is relatively tightly bound to an  $e^- \bar{\nu}$  unit inside the nucleus. There is thus a clear relationship between this proposal and the questioning of 'the creation-annihilation hypothesis, particularly in the form first suggested by Fermi according to which the electron and antineutrino are assumed to not be physically present inside the bound nucleus.

The exponential form of the damped Breit-Pauli interactions allows for two regions of similar potential for protons relative to the tightly bound but meta-stable  $e^- \bar{\nu}$  system, which in turn allows for the existence of a pair of (mutually orthogonal) proton orbitals of similar energies. Hence the conventional interpretation of nuclei as combinations of protons and neutrons is consistent with the XBPS model. The overall spin of the nucleus is determined exclusively by the proton constituents in this view, i. e. the ordinary proton and that of the neutronic  $p^+ e^- \bar{\nu}$  system. The latter conclusion rests primarily on the finding of a singlet structure for the  $e^- \bar{\nu}$  complex, which characteristic explains in a quite simple way why the electron and antineutrino do not appear to affect the magnitude of the nuclear spin despite the fact that they each possess  $s = 1/2$  themselves.

An important consequence of the above identification is that it gives a perfectly straightforward explanation for the observed adherence of both types of nucleons to the (generalized) Pauli principle. In this view no separate postulate is required to explain the relevant experimental data, since all that is ever involved is the exchange of indistinguishable protons. While such an interpretation is easily reconciled with the general procedures employed in isospin theory, it does call into question the latter's underlying precept, as introduced in 1932 by Heisenberg, namely that the proton and neutron are simply two different states of the same system which are characterized by the same spatial and spin wave functions. Specifically, it holds open the possibility that the governing Hamiltonian is completely independent of the hypothetical isospin coordinates. This position clearly does not diminish the practical utility of this theory, as applied for over a half-century in several important fields of physics, but it does raise questions as to whether the isospin quantities should be regarded as true constants of motion in the description of physical interactions.

Once the above interpretation has been made, it is possible to understand the Aufbau principle for nuclei in a straightforward manner in terms of protons and neutrons, consistent with the shell model of nuclear structure. The increased binding energies of successive protons are thereby associated with an overall contraction of the nuclear system which occurs because of the increase in the number of proton-electron and proton-antineutrino attractive interactions resulting therefrom. The well-known volume effect observed for nuclei appears to be closely tied up with the fact that the electrons and antineutrinos must avoid each other to a high degree (in order to have a small Darwin term) in order to maximize their Breit-Pauli attraction for one another. In this view the proportional increase in volume with atomic number is directly related to the

corresponding increase in the number of  $e^- \bar{\nu}$  bosonic complexes needed to form the required number of neutrons to bind the nucleons together.

One of the goals of the XBPS model is to help to understand on an *ab initio* basis the characteristics of nuclear reactions. The most promising reaction type which could be studied on a quantitative basis is that of nuclear fusion. The  $d + d$  reaction has a relatively low exothermicity, but it would be easier to deal with on an *ab initio* basis than the better studied  $t + d$  process. The reason that

${}^4\text{He}$  is not formed from the  $d+d$  reaction is because of problems with the laws of energy and momentum conservation, but the practicality of doing the necessary calculations is very low.

The XBPS model differs in a clear way from previous descriptions of the weak interaction. It leads to the conclusion that the antineutrino is capable of undergoing an attractive short-range interaction. There is a key binary system ( $e^- \bar{\nu}$ ) which becomes quite central to the mechanism of nuclear binding. The key distinction is that the XBPS model assumes that the electron is actually present in the neutron prior to its decay. Consequently, one can look upon the neutron as a resonance with a long lifetime, similar to the case of excimers in molecular studies. The calculated wavefunctions indicate that the decay process results from a "spin-flip" mechanism. Accordingly, the neutron prior to decay has an ( $e^- \bar{\nu}$ ) complex in a  $0^-$  state. In this state the spins of the electron and antineutrino are constantly alternating. After decay, experiments indicate that the antineutrino almost always has  $\alpha$  spin, so that it is now the spins of the proton and electron that are constantly alternating.

As a result the decaying state is quite unstable compared to the previous arrangement. If there is a relatively small number of protons in a nucleus compared to the number of  $e^- \bar{\nu}$  pairs, there is a fierce competition among the protons for the electrons, and this leads to the spin flip that leads to the decay of the nucleus. This competition is relaxed when more protons are added. This characteristic of the XBPS Hamiltonian provides a ready explanation for the well-known experimental fact that  $\beta$  decay always involves the addition or loss of the same number of electrons and antineutrinos (or positrons and neutrinos).

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