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Aspects of black hole physics and formation of super-massive black hole from ultra-light-dark bosons

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Abstract
When the universe was barely billion years old motivates one to consider ultra-light bosons as alternate candidates for dark matter. In this research paper by using the framework of non-relativistic Gross-Pitaevskii equation it is found that SMBHs with mass ~ 10^{10} M⊙ can be created from the collapse of ultra-light dark boson condensates formed around remnants of population III stars on time scales of ~ 10^8 yrs., if the dark boson mass is ≳ 10^{-20} eV. In order that these dark bosons act both as dark matter and dark energy, their rest mass has to be less than ~ 10^{-23} eV, implying that SMBHs with mass ≳ 10^{12} M would be generated from collapse of halo size BEC. In this paper, we studied collapse of such ultra-light bosonic halo DM that are in a Bose-Einstein condensate (BEC) phase to give rise to SMBHs on dynamical time scales. We undertake a simplified analysis to explain the observed alignments from the standpoint of collapsing, cluster size, rotating dark boson BEC. Finally, on concluding remarks pertaining to our proposed mechanism of Super-Massive Black Hole from Ultra-Light-dark Bosons (SMBH) generation.

Keywords: Gross-Pitaevskii, super-massive black hole (SMBH), ultra-light-dark bosons

Introduction
Cosmology, at present, faces three cardinal challenges - explaining the existence of supermassive black holes (SMBHs), dark matter (DM) and dark energy (DE). A very recently discovered quasar J1342+0928located at z = 7:54, surpassing the distance of quasar J1120+0641 (z = 7:09), is estimated to have a SMBH of mass ~ 8 ×10^8M⊙.Bose-Einstein condensate (BEC) phase, can entail creation of a SMBH. In the subsection that follows, to study the quantum evolution of such BECs as well as the conditions under which rotating SMBHs can be produced on dynamical time scales. We conclude the section by commenting on the GWs generated during the dynamical changes in such rotating BECs. Recently observed radio-galaxies in the ELAIS-N1 deep field with aligned jets can also possibly be explained if vortices of a rotating cluster size BEC collapse to form spinning SMBHs with angular momentum J ≲ 3:6 $n_W GM^2/c$, where n_W and M are the winding number and mass of a vortex, respectively.

Gravity of dark matter from recent observations: Gravitational lensing, which is a direct outcome of GR, leads to strong evidence for the existence of DM in clusters of galaxies. This result implies that either gravitational dynamic mimic MOND, deviating considerably from GR predictions, By this authors have also pointed out that many of these problems can be circumvented if DM is made up of ultra-light bosons with mass ~ few×10^{-22} eV.

Black hole Thermodynamics: Black holes (BHs) and gravitational waves (GWs) are two of the signature predictions of general relativity.

The second law of BH thermodynamics (SLBHT)) so that, $\frac{dA}{dt} \geq 0$ ….. (1)

Bose-einstein condensates: T < Tc, bosons have a tendency to occupy a single quantum mechanical state, forming a Bose-Einstein condensate (BEC). This results in quantum effects becoming evident on a macroscopic scale.

For a particle of mass m in an ensemble in thermodynamic equilibrium at temperature T, the thermal wavelength is

$$\lambda_T = (h/\ 2\pi\ k_B m T)^{1/2} \tag{2a}$$

Where k_B is Boltzmann's constant. By the relationship $n\ell^3 \simeq 1$.Therefore, the condition $\lambda_T > l$ can be re-expressed as $n\lambda_T^3 > 1$, yielding the transition temperature

$$T_c = (h^2\ n^{2/3}/2\ \pi\ k_B m) \tag{2b}$$

The gravitational wave luminosity, due to a source with slow internal motion, is given by,

$$L_{GW} = \frac{G}{2c^5}\langle \mathbb{I}_{jk}\mathbb{I}^{jk}\rangle \tag{3a}$$

Where

$$\mathbb{I}^{jk} = I^{ij}(t) - \frac{1}{3}\delta^{ij}I_k^k(t)$$

is the reduced mass quadrupole moment, with mass quadrupole moment $\mathbb{I}^{jk}(t)$defined by,$I^{jk}(t) = \int(\rho(t,\vec{r}))x^i x^j d^3r$ $\rho(t,\vec{r})$being the mass density of the source. Furthermore, if one considers Planck energy

$$L_{Pl} = \frac{E_{Pl}}{t_{Pl}} = \frac{c^5}{G} \tag{3b}$$

$$L_{GW} = \frac{2G}{c^5}\omega^2 E_{nonsph}^2 \approx 2\times 10^{50}\left(\frac{E_{nonsph}}{10^{51}\text{erg s}^{-1}}\right)^2\left(\frac{f}{1\text{kHz}}\right)^2 \text{erg s}^{-1} \tag{3c}$$

By gravitational collapse of a compact cosmic initial size $R = \alpha_1 R_s$,

$$L_{GW} = \frac{\alpha_2^2}{4\alpha_1^2}\frac{c^5}{G} = 9\times 10^{58}\frac{\alpha_2^2}{\alpha_1^2}\text{erg s}^{-1} \tag{3d}$$

While

$$\mathbb{I}_{jk}\sim\frac{E_{nonsph}}{\tau_{dyn}}\sim\frac{\alpha_2}{2\sqrt{2}\alpha_1}\frac{c^5}{G}\text{From eq.} \tag{3d}$$

$$L_{GW}\sim\frac{GE_{nonsph}^2}{c^5\tau_{dyn}^2}\lesssim\frac{GcM^2}{R^2} \tag{3e}$$

Therefore, one arrives at an upper limit for L_{GW} by substituting the smallest possible size

$$R_{min}\sim GM/c^2 \text{ in eq.} \tag{3e}$$

$$L_{GW} < \frac{GcM^2}{R_{min}^2} = \frac{c^5}{G} \tag{3f}$$

This indicates that Dyson luminosity may represent an upper limit for the GW luminosity.

$$\tau_{dyn} = \frac{\alpha_2 GM}{c^3} = 1.5\times 10^{-5}\alpha_2\left(\frac{M}{3M_\odot}\right)s \Rightarrow f \le \frac{c^3}{\alpha_2 GM} \cong 67\alpha_2^{-1}\left(\frac{M}{3M_\odot}\right)^{-1}kHz \tag{3g}$$

For GW150914, if one takes the mass scale to be $\sim 30\text{–}40 M_\odot$, as it radiates away energy with flux $F_H = \sigma T_H^4$, where $T_H = c^3\hbar/8\pi G k_B M$ is the Hawking temperature, is given by,

$$L_H = F_H\times 4\pi R_s^2 = \frac{1}{15360\pi}\left(\frac{m_{Pl}}{M}\right)^2\frac{c^5}{G} \tag{3h}$$

This implies that even for a Planck mass primordial BH, the Hawking luminosity (eq.(3h)) is four orders of magnitude smaller than the Dyson bound.

Bose-Einstein Condensation of Ultra-light Dark Bosons and Formation of Massive Black Holes

A very recent study, discussing SMBHs associated with three most distant quasars, J0100+2802 (z = 6.33), J1120+0641 (z = 7.09) and J1342+0928 (z = 7.54), has arrived.

Dark matter condensates, uncertainty principle and the central region of galaxies

When such light, non-relativistic and weakly interacting bosons of mass m constitute DM halo of size R_h,a fraction of them with very low momenta p can form a condensate provided,

$$\lambda_{DB} \sim \frac{h}{p} \gtrsim \left(\frac{3N}{4\pi R_h^3}\right)^{-1/3} = R_h \left(\frac{3M}{4\pi m}\right)^{-\frac{1}{3}} \quad (4)$$

Where N and M are the number and the total mass of dark bosons making up the BEC.
If the DM halo develops

$$E \sim \frac{p^2}{2m} + \frac{l^2}{2mR_h^2} - \frac{GMm}{R_h} < 0 \quad (5)$$

So that

$$p^2 < \frac{2GMm^2}{R_h} - \frac{n^2\hbar^2}{R_h^2} \quad (6)$$

The total angular momentum of the BEC is, of course

$$,L \sim nN\hbar = n\hbar\frac{M}{m} \quad (7)$$

Heisenberg's uncertainty principle demands that

$$\Delta \mathrm{p} \sim p \gtrsim \frac{\hbar}{2R_h} \quad (8)$$

According to eq. (4) and (8)

$$\frac{h}{4\pi p} \lesssim R_h \lesssim \frac{h}{p}\left(\frac{3N}{4\pi}\right) \quad (9)$$

The above condition, in the case of a BEC, is self-consistent as N ≫1.
From minimum energy consideration, eq. (8) entails $\mathrm{p} \sim \frac{h}{4\pi \mathrm{R_h}}$ be substituted in eq.(5) so that,

$$E \sim \frac{\hbar^2}{8mR_h^2} + \frac{n^2\hbar^2}{2mR_h^2} - \frac{GMm}{R_h} < 0 \quad (10)$$

$$R_h \gtrsim \left[n^2 + \frac{1}{4}\right]\frac{\hbar^2}{GMm^2} = 0.5\left(n^2 + \frac{1}{4}\right)\left(\frac{m_{Pl}^2}{mM}\right)\frac{\hbar}{mc} = 4.3\left(n^2 + \frac{1}{4}\right)\left(\frac{10^7 M_\odot}{M}\right)\left(\frac{10^{-22}eV}{m}\right)^2 kpc \quad (11)$$

For a fixed angular momentum, L = nN(h/2π)

$$\frac{\partial E}{\partial \mathrm{R_h}} = \frac{GMm}{R_h^3}\left[R_H - \left(n^2 + \frac{1}{4}\right)\frac{\hbar^2}{GMm^2}\right] = 0 \quad (12)$$

So that the size R_{h0} that leads to minimum energy configuration for the BEC is given by

$$R_{h0} = \left[n^2 + \frac{1}{4}\right]\frac{\hbar^2}{GMm^2} \cong 86\left(n^2 + \frac{1}{4}\right)\left(\frac{10^9 M_\odot}{M}\right)\left(\frac{10^{-22}}{m}eV\right)^2 pc \quad (13)$$

Corresponding to a single boson energy

$$\mathrm{E_{min}} = -\frac{\mathrm{GMm}}{2\mathrm{R_{h0}}} = -0.5\left(\frac{\mathrm{mc^2}}{\mathrm{n^2} + \frac{1}{4}}\right)\left(\frac{\mathrm{m_{Pl}^2}}{\mathrm{mM}}\right)^{-2} \quad (14)$$

Eq.(13) tells us

$$\mathrm{R_h(t)} = \mathrm{R_{h0}} + \mathrm{x(t)} \quad (15)$$

Where $|x(t)| << R_{h0}$ is the amplitude of oscillation. Substitution of eq. (15) in eq.(10) leads to

$$E = E_{min} + \left[\frac{3\mathrm{h}^2\left(n^2 + \frac{1}{4}\right)}{2mR_{h0}} - GMm\right] = E_{min} + \frac{1}{2}\frac{GMm}{R_{h0}^3}\mathrm{x}^2 \quad (16)$$

$$\omega = \sqrt{\frac{GM}{R_{ho}^3}} \quad (17)$$

Corresponding to a time period, τ=2 π/ ω,

$$\tau = 2\pi \left(n^2 + \frac{1}{4}\right)^{\frac{3}{2}} (\frac{1}{GM})^2 (\frac{\hbar}{m})^3 \cong 2 \times 10^6 (n^2 + \frac{1}{4})^{3/2} (\frac{m}{10^{-22} eV})^{-3} (\frac{M}{10^9 M_{\odot}})^{-2} \text{ yrs} \quad (17)$$

Such shocks could enhance a top heavy SFR on time scales~ 10^6–10^7 yrs, and plausibly. The BEC will implode to form a BH if its size R_{h0} is less than the Kerr EH radius given by,

$$R_{BH} = \frac{R_S}{2} + \sqrt{(\frac{R_S}{2})^2 - (\frac{L}{Mc})^2} \quad (18)$$

By making use of the condition $R_{ho} \le R_{BH}$ along with eq's(7), (13) and (18), we may express the criteria for the BH formation to be,

$$\left(n^2 + \frac{1}{4}\right) \frac{\hbar^2}{GMm^2} \le \frac{GM}{c^2} [1 + \sqrt{1 - (\frac{n\hbar c}{GMm})^2} \quad (19)$$

So as to obtain the inequality

$$\left(n^2 + \frac{1}{4}\right) \left(\frac{m_{Pl}^2}{Mm}\right)^2 \le 1 + \sqrt{1 - (\frac{nm_{Pl}^2}{Mm})^2} \quad (20)$$

In order to avoid appearance of imaginary numbers

$$mM \ge nm_{Pl}^2 \quad (21)$$

From the inequality in eq. (1.32), it follows that

$$mM \ge \frac{n^2 + \frac{1}{4}}{\sqrt{n^2 + 1/2}} m_{Pl}^2 \quad (22)$$

For n=1, the above result implies

$$mM \ge \; 1.02 \, m_{Pl}^2 \quad (21)$$

In what follows, we will derive a more accurate constraint by studying the evolution of DM in the BEC phase using the framework of Gross-Pitaevskii equation.

The Gross-Pitaevskii equation

The wave function of the condensate satisfies the non-linear Schrodinger equation (NLSE), also called the Gross-Pitaevskii equation (GPE) which we normalize such that…

$$\int |\Psi|^2 d^3r = N \ldots \ldots \quad (22)$$

Where N is the total particle number. The coupling constant g is related to the scattering length by the equation

$$g = \frac{4\pi\hbar^2 a}{m} \ldots \ldots \ldots \ldots \ldots. \quad (23)$$

This effective interaction is attractive if g < 0, N-particle Hamiltonian that corresponds to this pseudo potential can be written as

$$\hat{H} = \sum_i^N [-\frac{\hbar^2}{2m} \nabla_i^2 + V(r_i)] + \sum_{i<j} g\delta(r_i - r_j) \ldots \ldots \ldots \ldots .. \quad (24)$$

Where ∇_I and V (R_i) represents external potentials. This leads to the following non-linear Schrodinger type equation (NLSE):

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V - \mu + g|\Psi|^2\right] \Psi \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (25)$$

Which is the Gross-Pitaevskii- equation. Below the critical temperature, T < Tc, the chemical potential is well approximated by $\mu(T) = -\kappa T \ln 1 + 1/N \simeq 0$ and thus it can be ignored for large N. Hence from now on, we assume μ = 0.

Self-gravitating systems

In a self-gravitating system with no external potential, the potential V is the gravitational potential that is determined from Poisson's equation for gravity:

$$\nabla^2 V = 4\pi G m |\Psi|^2 \ldots\ldots\ldots\ldots \quad (26)$$

Where G is Newton's gravitational constant. For a self-gravitating system, the GPE can be derived using the variational principle from the energy functional

$$\varepsilon = \frac{\hbar^2}{2m}|\nabla\Psi|^2 + \frac{1}{2}V|\Psi|^2 + \frac{1}{2}g|\Psi|^4 \quad (27)$$

From (28), the total energy of the system can be calculated as

$$E_{tot} = E_{kin} + E_{pot} + E_{int} \ldots \quad (28)$$

Where the kinetic energy E_{kin}, (gravitational) potential energy E_{pot} and internal energy E_{int} are given by, respectively

$$E_{kin} = \int \frac{\hbar^2}{2m}|\nabla\Psi|^2 d^3 r \ldots \quad (29a)$$

and

$$E_{pot} = \int \frac{V}{2}|\nabla\Psi|^2 d^3 r \ldots\ldots (29b) E_{int} = \int \frac{1}{2} g |\Psi|^4 d^3 r \quad (30)$$

To achieve a stable self-gravitating object, it is necessary to choose a coupling coefficient (g) that is positive. For an initial estimate a_0 for the scattering length a, we use the value

$$a_0 = G m^3 (R/\pi\hbar)^2 \quad (31)$$

Which is derived from the Thomas-Fermi approximation. Since we do not rely on that approximation, other values for a are of course also possible.

Quantization of circulation

In a frame of reference that is rotating in the xy-plane with angular velocity Ω, the GPE is written as

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V + g|\Psi|^2 - \Omega L_Z\right]\Psi \quad (32)$$

Where $L_Z = -i\hbar(x\partial_y - y\partial_x)$.sThe GPE is formally similar to the Ginzburg-Landau equation, which describes a superfluid with Ψ as a complex order parameter field. The phase ϕ of the order parameter around any closed contour K must be

$$2\pi q, \oint_K \nabla\varphi . dI = 2\pi q \quad (33)$$

The gradient of the phase describes the local velocity flow$= \hbar m^{-1}\nabla\varphi$. The superfluid rotation induced by a vortex line can be expressed as the circulation, Γ, about K:

$$\Gamma = \oint_K v . d = 2\pi q \frac{\hbar}{m} \quad (34)$$

Which is an integer multiple of 2πħ/m. Also from Eq. (34) it follows that the velocity around a single circular vortex of radius r is

$$v = \frac{\Gamma}{2\pi r} = \frac{q\hbar}{mr} \quad (35)$$

There is an energy barrier between non-vortex and vortex states. The minimum density grows to the bulk value over a length-scale of order the healing length.

Gross-Pitaevskii Equation and Ultra-light Dark Matter Particles

Evolution of the condensate wavefunction ψ(ŕ,t) can be described by the following Gross-Pitaevskii equation (GPE),

$$i\hbar\frac{\partial\Psi}{\partial t} = [-\frac{\hbar^2}{2m}\nabla^2 + V_{ext} + N\int V(\vec{r}-\vec{u})|\psi(\vec{u},t)|^2\, d^3u]\,\psi(\vec{r},t) \quad (36)$$

Where

$$V(\vec{r}-\vec{u}) = \frac{4\pi\hbar^2 a\delta^3(\vec{r}-\vec{u})}{m} + V_g(\vec{r}-\vec{u}) \quad (37)$$

So

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{ext} + Ng|\psi(\vec{r},t)|^2 + N\int V_g(\vec{r}-\vec{u})|\psi(\vec{u},t)|^2\, d^3u\right]\psi(\vec{r},t) \quad (38)$$

Where m is the dark boson rest mass, $g = \frac{4\pi\hbar^2 a}{m}$ characterizes a short-range contact interaction between the bosons and $V_{ext}(r)$ given by

$$V_{ext}(r) = -\frac{GM_0 m}{r} \ldots. \quad (39)$$

The GPE of eq.(36) can be derived by extremizing the following action

$$S = \int dt \int d^3r\, L \;\ldots.. \quad (40)$$

$$L = \frac{i\hbar}{2}\left\{\Psi\frac{\partial\Psi^*}{\partial t} - \Psi^*\frac{\partial\Psi}{\partial t}\right\} + \frac{\hbar^2}{2m}\nabla\Psi^*.\nabla\Psi + V_{ext}|\psi|^2 + \frac{gN}{2}|\Psi|^4 + \frac{N}{2}\int V_g(\vec{r}-\vec{u})|\psi(\vec{u},t)|^2\, d^3u \quad (41)$$

Of course, the mutual gravitational interaction between any pair of ultra-light dark bosons separated by a distance r is given by

$$V_g(r) = -\frac{Gm^2}{r} \quad (42)$$

Substituting eqs. (37), (39) and (41) in the action given by eq(40) results in and solving,

$$S = \int dt \int d^3r\, L \int dt\, \Psi^*\left\{-i\hbar\frac{\partial\Psi}{\partial t} - \frac{\hbar^2}{2m}\nabla^2\Psi + V_{ext}\Psi + \frac{gN}{2}\int V_g(|\vec{r}-\vec{u}|)\,|\psi(\vec{u},t)|^2 d^3u\right\}\Psi \quad (43)$$

From here onwards, we will set g = 0, Now, to obtain an approximate solution of eq.(38) we make use of the time dependent variational method by employing a trial wave function

$$\psi(\vec{r},t) = A(t)r\exp\left(\frac{-r}{\sigma(t)}\right)\exp(-iB(t)r)\,Y_{lm}(\theta,\varphi) \quad (44)$$

The normalization condition entails A(t) and $\sigma(t)$ to be related by,

$$|A(t)|^2 = \frac{4}{3}\sigma(t)^{-5} \Rightarrow A(t) = \frac{2}{\sqrt{3}}\sigma(t)^{-5/2} \quad (45)$$

Where

A(t), σ(t)andB(t)areamplitude, width, andphaseparameterofwavefunction.

$$\mathcal{L} = \int d^3r\, L = -\frac{5}{2}\hbar\sigma\dot{B} + \frac{\hbar^2}{2m}B^2 + \frac{\hbar^2}{2m\sigma^2} + L_{int} - \frac{GM_0 m}{2\sigma} \quad (46)$$

Where the self-gravity term $\boldsymbol{L_{int}}$ **is given by,**

$$L_{int} = \frac{N}{2}\int d^3r|\psi(\vec{r},t)|^2\int V_g(\vec{r}-\vec{u})|\psi(\vec{u},t)|^2\, d^3u \quad (47)$$

In the scenario according to the GPE of eq.(38) with g = 0.if we choose l=1 and m=1 so that,

$$\psi(\vec{r},t) = A(t)r\exp\left(\frac{-r}{\sigma(t)}\right)\exp(-iB(t)r)\,Y_{11}(\theta,\varphi) \quad (48)$$

Then the self-gravity term of equation (47) is given by

$$L_{int} = -\frac{0.37NGm^2}{2\sigma}\ldots \quad (49)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}\,(EULERLANGRANGiIANEQUATION) \quad (50)$$

$$B(t) = -\frac{5m\dot{\sigma}}{2\hbar} \quad (51)$$

and,

$$\frac{5\hbar\dot{B}}{2} + \frac{\hbar^2}{m\sigma^3} - \frac{G[0.37Nm+M_0]m}{2\sigma^2} = 0 \quad (52)$$

So that the above two equations can be combined to yield

$$m\ddot{\sigma} = -\frac{dV_{eff}}{d\sigma} \quad (53)$$

Where

$$V_{eff} = \frac{2}{25}\left[\frac{\hbar^2}{m\sigma^2} - \frac{G[0.37Nm+M_0]m}{\sigma}\right] \quad (54)$$

$$\frac{1}{2}m\dot{\sigma}^2 + V_{eff} = \text{constant}(K_0) \Rightarrow \dot{\sigma} = \pm\frac{2}{5}\sqrt{\left[\frac{G\bar{M}}{\sigma} - \frac{\hbar^2}{m^2\sigma^2} + \frac{25}{2}K_0\right]} \quad (55)$$

Where $\bar{M} = 0.37Nm + M_0$. The time evolution of the trial wavefunction is completely determined by specifying the initial data $\sigma(t_i)$ and $\dot{\sigma}(t_i)$ at an initial time t_i. If at time t_i, the condensate wavefunction is spread over a galactic scale and is contracting ever so slowly, implying $2\sigma_i \equiv 2\sigma(t_i) \approx 25$ kpc and $\dot{\sigma}(t_i) = \epsilon$ where$\epsilon \approx 0$ then,$K_0 \approx 0$,So that one can easily integrate eq. (55) to

$$t - t_i = \frac{5}{3}\sqrt{\frac{\sigma_i^3}{G\bar{M}}}\left[\left(1 - \frac{\hbar^2}{G\bar{M}m^2\sigma_i}\right)^{3/2} - \left(\frac{\sigma(t)}{\sigma_i} - \frac{\hbar^2}{G\bar{M}m^2\sigma_i}\right)^{3/2}\right] - \frac{2\hbar^2}{G\bar{M}m^2\sigma_i}\left[\left(1 - \frac{\hbar^2}{G\bar{M}m^2\sigma_i}\right)^{1/2} - \left(\frac{\sigma(t)}{\sigma_i} - \frac{\hbar^2}{G\bar{M}m^2\sigma_i}\right)^{1/2}\right] \quad (56)$$

The turning point occurs atσ_{min} corresponding to$\dot{\sigma} \doteq 0$ so that

$$\sigma_{min} = \frac{\hbar^2}{G\bar{M}m^2}\cdots \quad (57)$$

However, BEC size$\approx 2\sigma(t)$ becomes comparable to the associated event horizon radius (eq.(18), In order to find out the conditions under which the BEC implodes into a BH, we adopt a heuristic approach by first considering the total mass enclosed within a sphere of radius $2\sigma(t)$ at time t,

$$M_0 + M_{\boldsymbol{\psi}}(< 2\sigma(t), t) = M_0 + Nm\int_0^{2\sigma(t)}\int_0^{2\pi}\int_0^{\pi}|\psi(\vec{r},t)|^2 d^3r = M_0 + \frac{4Nm}{3\sigma(t)^5}\int_0^{2\sigma(t)} r^4 \exp\left(-\frac{2r}{\sigma(t)}\right)dr = 0.37N\hbar.\ldots \quad (58)$$

The corresponding angular momentum of condensate is given by,

$$\hat{L} = N\hbar\int_0^{2\sigma(t)}\int_0^{2\pi}\int_0^{\pi}|\psi(\vec{r},t)|^2 d^3r = 0.37N\hbar \quad (59)$$

Hence, when$\bar{M}$ and $\hat{L}$ of eq. (58) and (59) are substituted as effective mass and angular momentum, respectively, in the EH radius given by eq. (18), we get

$$R_{BH} = \frac{G\bar{M}}{C^2}\left[1 + \sqrt{\left[1 - \frac{m_{Pl}^4}{\bar{M}^2m^2}\left(1 - {M_0}/{\bar{M}}\right)^2\right]}\right] \quad (60)$$

The dark condensate will certainly collapse to from a BH if

$$\sigma_{min} = \frac{2\hbar^2}{G\bar{M}m^2} < R_{BH}\ \ldots\ldots\ldots. \quad (61)$$

Therefore, from eq.(61),we have the following criteria for the formation of a BH of maas $\bar{M}$:

$$m\bar{M} > \frac{m_{Pl}^2}{\sqrt{\left(1 - \frac{1}{4}\left(1 - {M_0}/{\bar{M}}\right)^2\right)}} \cong 1.15m_{Pl}^2 \quad (62)$$

It is interesting to note that the condition given by eq. (21), although derived using simple physical arguments in the previous subsection, is not much different from the above inequality. So eq.(56) and eq.(62) imply that the dark boson mass must satisfy,

$$m > 1.54 \times 10^{-20} \left(\frac{\bar{M}}{10^{10} M_{\odot}}\right) eV \tag{63}$$

In order that SMBHs heavier than billion solar masses are formed on time scale,

$$\tau_{dyn} = t - t_i \cong \frac{5}{3}\sqrt{\frac{\sigma_i^3}{G\bar{M}}} \approx 10^8 - 10^9 \text{ yrs} \tag{64}$$

Assuming an initial BEC size $\boldsymbol{2\sigma_i} \approx 20 - 30$ kpc.
Assuming an asymmetric variation in size, energy carried away by gravitational radiation is,

$$E_{GW} = \epsilon \frac{G\bar{M}^2}{\boldsymbol{\sigma_{min}}} = \epsilon 2.4 \times 10^{64} \left(\frac{\bar{M}}{10^{10} M_{\odot}}\right)^3 \left(\frac{\text{m}}{1.54\times10^{-20} eV}\right)^2 \text{ erg} \tag{65}$$

Over a time scale given by eg. (64)

$$\tau_{dyn} = 9.4 \times 10^8 \left(\frac{\bar{M}}{10^{10} M_{\odot}}\right)^{-1/2} \left(\frac{\boldsymbol{\sigma_i}}{25\text{kpc}}\right)^{3/2} \text{yrs} \tag{66}$$

Where the parameter ϵ characterises the asymmetric changing size.Hence the gravitational wave luminosity can be estimated to be, L

$$L_{GW} \sim \frac{E_{GW}}{\tau_{dyn}} \approx \epsilon 10^{48} \left(\left(\frac{\bar{M}}{10^{10} M_{\odot}}\right)\right)^{\frac{7}{2}} \left(\frac{\text{m}}{1.54\times10^{-20} eV}\right)^2 \left(\frac{\boldsymbol{\sigma_i}}{25\text{kpc}}\right)^{-\frac{3}{2}} \frac{\text{erg}}{\text{s}} \tag{67}$$

Associated with a very low characteristic frequency,

$$\upsilon_{GW} \sim \tau_{dyn}{}^{-1} = 3 \times 10^{-17} \left(\frac{\bar{M}}{10^{10} M_{\odot}}\right)^{1/2} \left(\frac{\boldsymbol{\sigma_i}}{25\text{kpc}}\right)^{-\frac{3}{2}} \text{Hz} \tag{68}$$

Also, if the asymmetric kinetic energy E_{nhonsp} associated with a collapsing condensate at a distance d from us is ~ G $\bar{M}^2$/σmin then the ensuing GW amplitude on Earth can be estimated to be

$$h_{GW} \sim \frac{4G\text{E}_{\text{nonsph}}}{c^4 d} = \frac{2}{d}\left(\frac{G\bar{M}}{c^2}\right)\left(\frac{m\bar{M}}{m_{Pl}^2}\right)^2 \sim 3 \times 10^{-10} \left(\frac{\bar{M}}{10^{10} M_{\odot}}\right)^3 \left(\frac{\text{m}}{1.54\times10^{-20} eV}\right)^2 \left(\frac{\text{d}}{10\text{Mpc}}\right)^{-1} \tag{69}$$

On the other hand, if the central BH is not formed, the dark bosons can form a stable self-gravitating system with a characteristic size $R_0 = 2\sigma_{nim}$ that minimizes V_{eff} of eq.(54).

$$2\sigma_{min} = 176 \left(\frac{10^9 M_{\odot}}{M}\right)\left(\left(\frac{10^{-22} eV}{m}\right)\right)^2 pc \tag{70}$$

That follows from eq. (57).The normal modes of angular frequency ω of such an undulation is given by,

$$\omega_{osc}{}^2 = \frac{1}{m}\frac{d^2 V_{eff}}{d\sigma^2}\bigg|_{R_0} = \frac{G^4 \bar{M}^4 m^6}{100\hbar^6} \tag{71}$$

$$\upsilon_{osc} = \frac{\omega_{osc}}{2\pi} = 1.3 \times 10^{-15} \left(\left(\frac{\bar{M}}{10^{10} M_{\odot}}\right)^2\right) \left(\frac{m}{10^{-22} eV}\right)^3 Hz \ldots \tag{72}$$

Bose-einstein condensates and gross-Pitaevskii equation

Now, in the T = 0◦K mean field approximation, evolution of the condensate wave function $\psi(\vec{r}, t)$ (normalized to unity) is described by the Gross-Pitaevskii equation (GPE).

$$i\hbar \frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{ext} + \text{N} \int \text{V}_\text{g}(\vec{r} - \vec{u})|\psi(\vec{u}, t)|^2 \, d^3u\right] \psi(\vec{r}, t) \tag{73}$$

Where m, $V_{ext}(\vec{r})$ and V $(\vec{r})$ are the mass of the boson, The boson-boson interaction energy is given by,

$$V(\vec{r}-\vec{u}) = \frac{4\pi\hbar^2\delta^3(\vec{r}-\vec{u})}{m} + V_g(\vec{r}-\vec{u}) \qquad (74)$$

Where $\vec{r}$ and $\vec{u}$ are the position vectors of the two bosons, respectively. In the present study, the dynamical evolution of BEC is based on eq.(73), and by eq.(74), GPE,

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{ext} + Ng|\psi(\vec{r},t)|^2 + N\int V_g(\vec{r}-\vec{u})|\psi(\vec{u},t)|^2\, d^3u\right]\psi(\vec{r},t) \qquad (75)$$

Where

$$g = \frac{4\pi\hbar^2 a_s}{m} \qquad (76)$$

The GPE of eq. (75) can be derived by extremizing the following action,

$$S = \int dt \int d^3r\, L\ L = \frac{i\hbar}{2}\left\{\Psi\frac{\partial\Psi^*}{\partial t} - \Psi^*\frac{\partial\Psi}{\partial t}\right\} + \frac{\hbar^2}{2m}\nabla\Psi^*.\nabla\Psi + V_{ext}|\psi|^2 + \frac{gN}{2}|\Psi|^4 + \frac{N}{2}|\psi|^2\int V_g(\vec{r}-\vec{u})|\psi(\vec{u},t)|^2\, d^3u. \qquad (77)$$

The potential energy of a dark boson due to such a central compact remnant plays the role of external potential energy $V_{ext}(r)$ of eq. (73) and is given by

$$\mathrm{Vext}(r) = -\frac{GM_0 m}{r}\ldots\ldots\ldots \qquad (78)$$

$$V_g(\vec{r}-\vec{u}) = -\frac{Gm^2}{|\vec{r}-\vec{u}|}\ldots\ldots\ldots\ldots\ldots \qquad (79)$$

Thus we determine its parameters by demanding that the trial wave function extremizes the action for which the Lagrangian density is given by eq. (77). We consider the following normalized trial wave function,

$$\psi(\vec{r},t) = A(t)r\exp(-r^2/2\sigma^2(t))\exp(-iB(t)r^2) \qquad (80)$$

Where A(t), σ(t) and B(t) represent the amplitude, width and a phase parameter for the macroscopic wave function, respectively

$$|A(t)|^2 = (\sqrt{\pi}\sigma(t))^{-3} \Rightarrow A(t) = \left(\sqrt{\pi}\sigma(t)\right)^{-3/2}\exp(i\gamma t) \qquad (81)$$

BEC mass enclosed within a sphere of radius R at time t is given by,

$$M_{bec}(<R,t) = \int_0^R |\psi(\vec{r},t)|^2 d^3r = \frac{4\pi Nm}{\pi^{\frac{3}{2}}\sigma^3(t)}\int_0^R r^2\exp(-r^2/2\sigma^2(t))d^3r \qquad (82)$$

So that the BEC mass confined within the Gaussian width σ(t) is,

$$M_{eff} = M_{bec}\left(<\sigma(t)\right) = \frac{4Nm}{\pi^{\frac{1}{2}}}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!(2k+3)} \cong 0.43Nm \qquad (83)$$

Which turns out to be independent of time. Since occupation numbers are high for these ultra-light bosons48, the relevant dynamical equations are,

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\vartheta}\frac{\partial a(x^\vartheta)}{\partial x^\vartheta}\right) + f\left(a(x)\right) = 0\ and\ G_{\mu\vartheta} = \frac{8\pi G}{C^4}T_{\mu\vartheta}(a(x))$$

where f(a(x)), $T_{\mu\nu}(a(x))$ and $G_{\mu\nu}$ are the nonlinear term due to self-interactions, stress-tensor for the bosons and Einstein tensor Assume for simplicity that the dark boson condensate collapses to form a black hole when its width becomes smaller than the Schwarzschild radius so that,

$$\sigma(t) < \frac{2GM_{eff}}{c^2} \qquad (84)$$

Where M_{eff} denotes the total mass of dark bosons within the width σ(t). In the variational method formalism, the time evolution of ψ (t,r) is determined from the stationarity of the action. Substituting the trial wave function of eq. (80) in the Lagrangian density of eq.(77), and then integrating the latter over space, we arrive at the following Lagrangian,

$$\mathcal{L} = \int d^3r\, L = \hbar\dot{\gamma} + L_{int} + \frac{gN}{4\sqrt{2}\pi^{3/2}\sigma^3} - \frac{2GM_0 m}{\sqrt{\pi}\sigma} + \frac{3}{2}\sigma^2\left[\hbar\dot{B} + \frac{2\hbar^2}{m}B^2 + \frac{\hbar^2}{2m\sigma^2}\right] \quad (85)$$

Where the self-gravity term $L_{int}$is given by.

$$L_{int} = \frac{N}{2}\int d^3r|\psi(\vec{r},t)|^2\int V_g(\vec{r}-\vec{u})|\psi(\vec{u},t)|^2\, d^3u \quad (86)$$

The standard Newtonian gravity eq(79), the trial wave function of equation eq.(80) leads to,

$$L_{int} = -\frac{NGm^2}{\sqrt{2\pi}\sigma} \quad (87)$$

Using Euler-Lagrange equations ensuing from eternizing the action

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \quad (88)$$

For i=1, 2, with q_1=B,q_2=$\boldsymbol{\sigma}$and L given by eq.(85) have the following forms,

$$\hbar\dot{B} + \frac{2\hbar^2}{m}B^2 - \frac{\hbar^2}{2m\sigma^4} - \frac{gN}{4\sqrt{2}\pi^{3/2}\sigma^5} = -\frac{G[Nm+2\sqrt{2}M_0]m}{3\sqrt{2\pi}\sigma^3} \quad (89)$$

$$\dot{\sigma} = \frac{2\hbar B}{m}\sigma \quad (90)$$

The variable γ(t) is non-dynamical because its corresponding contribution to eq.(85) is just a total derivative term. Eqs.(89) and (90) can be combined to give,

$$m\ddot{\sigma} = -\frac{dV_{eff}}{d\sigma} \quad (91)$$

Where

$$V_{eff} = \frac{\hbar^2}{2m\sigma^2} - \frac{3}{3\sqrt{2\pi}}\frac{G[Nm+2\sqrt{2}M_0]m}{\sigma} + \frac{gN}{\sqrt{2}6\pi^{3/2}\sigma^3} \quad (92)$$

And

$$B = \frac{m}{2\hbar}\frac{\dot{\sigma}}{\sigma} \quad (93)$$

Formulation and solution of black holes

Time evolution of the trial wave function by solving eqs. (91) and (92) by specifying the initial data $\sigma(t_i)$ and $\sigma(t_i)$ at time t_i which is equivalent to supplying $\psi(t_i)$ by virtue of eqs. (80) and (93). Eq(91) leads readily to the first integral

$$\frac{1}{2}m\dot{\sigma}^2 + V_{eff} = constant(K_0) \Rightarrow \dot{\sigma} \doteq \pm\sqrt{\frac{2}{m}\left[K_0 - V_{eff}\right]} \quad (94)$$

From eq.(94) to determine the turning points by setting $\dot{\boldsymbol{\sigma}} = \mathbf{0}$.

Case I: K_0 = 0 and g = 0

Let us consider a scenario in which a very large number N of dark bosons with ∼ zero momentum and initially spread over a very large scale ∼ 20 - 30 kpc evolve quantum mechanically. Then, if we take the initial conditions to be $\dot{\sigma}^2 \approx 0$ and B $B \lesssim 0$ with an initial σ_i∼ 25 kpc (where V_{eff}≈ 0), the constant K_0 of eq.(21) can be taken to bezero so that,

$$\dot{\boldsymbol{\sigma}} = -\sqrt{\frac{4}{3\sqrt{2\pi}}\frac{G[Nm+2\sqrt{2}M_0]}{\sigma} - \frac{\hbar^2}{m^2\sigma^2} - \frac{gN}{3\sqrt{2}\pi^{\frac{3}{2}}m\sigma^3}} \quad (95)$$

Provided

$$\frac{4G[Nm+2\sqrt{2}M_0]\sigma}{3\sqrt{2\pi}} - \frac{\hbar^2}{m^2}\left[1 + \frac{2\sqrt{2}Na_s}{3\sqrt{\pi}\sigma}\right] > 0 \quad (96)$$

Where one has used eq. (76) for $\boldsymbol{g}$. An attractive contact interaction (as < 0) helps gravity to oppose the repulsive force arising due to quantum theory. Eqs. (95) and (96) lead to,

$$t - t_i = \int_{\sigma(t)}^{\sigma_i} \sigma \left(\frac{4G[Nm+2\sqrt{2}M_0]\sigma}{3\sqrt{2\pi}} - \frac{\hbar^2}{m^2}\left[1 + \frac{2\sqrt{2N a_s}}{3\sqrt{\pi}\sigma}\right] \right)^{-1/2} d\sigma \quad (97)$$

If there is strictly no short range interaction between the bosons so that $a_s = 0$, and if the condition eq.(96) is valid all through,

eq.(97), $t - t_i = \int_{\sigma(t)}^{\sigma_i} \sigma \left(\frac{4G[Nm+2\sqrt{2}M_0]\sigma}{3\sqrt{2\pi}} - \frac{\hbar^2}{m^2} \right)^{-1/2} d\sigma$

$$= \left(\frac{2}{3}\right)\left(\frac{4G[Nm+2\sqrt{2}M_0]}{3\sqrt{2\pi}\sigma_i^3}\right)^{-\frac{1}{2}}\left[\left(1 - \frac{3\sqrt{2\pi}\hbar^2}{4G[Nm+2\sqrt{2}M_0]m^2\sigma_i}\right)^{\frac{3}{2}} - \left(\frac{\sigma(t)}{\sigma_i} - \frac{3\sqrt{2\pi}\hbar^2}{4G[Nm+2\sqrt{2}M_0]m^2\sigma_i}\right)^{\frac{3}{2}} + \frac{9\sqrt{2\pi}\hbar^2}{4G[Nm+2\sqrt{2}M_0]m^2\sigma_i}\left(\sqrt{1 - \frac{3\sqrt{2\pi}\hbar^2}{4G[Nm+2\sqrt{2}M_0]m^2\sigma_i}} - \sqrt{\frac{\sigma(t)}{\sigma_i} - \frac{3\sqrt{2\pi}\hbar^2}{4G[Nm+2\sqrt{2}M_0]m^2\sigma_i}}\right)\right] \quad (98)$$

As long as the inequality in eq. (84) is violated, the above equation entails σ(t) to decrease steadily with time till it reaches the turning point,

$$\sigma_{min} = \frac{3\sqrt{2\pi}\hbar^2}{4G[Nm+2\sqrt{2}M_0]m^2} \quad (99)$$

Where $V_{eff}(\sigma_{min}) = 0$. After the bounce at the turning point, σ(t) starts increasing again. By imposing the criteria

$$\sigma_{min} > \frac{2GM_{eff}}{c^2} \quad (100)$$

That there is no black hole formation, we can constrain the dark boson mass m as shown below. Making use of eqs.(83) and (96) in the condition eq.(100), we find the criteria for no black hole formation to be,

$$m < \frac{0.64 m_{Pl}^2}{M_{eff}}\left(1 + \frac{1.22 M_0}{M_{eff}}\right)^{-\frac{1}{2}} \quad (101)$$

Where $m_{Pl} = \sqrt{\hbar c/G}$ is the Planck mass? In other words, instead collapses to form a black hole because of eq.(84), the dark boson mass has to be larger than

$$\frac{0.64 m_{Pl}^2}{M_{eff}}\left(1 + \frac{1.22 M_0}{M_{eff}}\right)^{-\frac{1}{2}} \quad (102)$$

If one chooses $M_{eff} = 10^{10} M_\odot$ and $M_0 = 150 M_\odot$, one finds from eq.(102), thatblack holes of mass ~ M_{eff} are formed from dark boson BEC provided, $m \gtrsim 10^{-53} = 0.56 \times 10^{-20} eV$ From eq.(98) one can estimate the time taken for the width of the condensate to decrease to the value of Schwarzschild radius.

$$\tau_{dyn} = \left(\frac{2}{3}\right)\left(\frac{4G[Nm+2\sqrt{2}M_0]}{3\sqrt{2\pi}\sigma_i^3}\right)^{-\frac{1}{2}} \approx 10^8 yrs \quad (103)$$

Therefore, formation of SMBHs can happen even when the universe is barely ~ 10^9yrs old.

Case II: $K_0 > 0$ and $g = 0$

If we consider the initial value of B to be negative and large in magnitude, $\dot{\sigma}^2$would be initially large implying $K_0 > 0$. Then, From eqs. (92) and (94), this occurs when,

$$\frac{2K_0\sigma^3}{m} + \frac{4G[Nm+2\sqrt{2}M_0]\sigma^2}{3\sqrt{2\pi}} - \frac{\hbar^2\sigma}{m^2} = \quad (104)$$

So that the turning point occurs at,

$$\sigma_{min} = \frac{G[Nm+2\sqrt{2}M_0]}{3\sqrt{2\pi}K_0}\left[\sqrt{1 + \frac{9\pi\hbar^2 K_0}{G^2[Nm+2\sqrt{2}M_0]^2 m^3}} - 1\right] \quad (105)$$

Since, in the present study, we are considering ultra-cold bosons and K_0 is the classical analogue of energy for a single boson (eq.(94)), we may express it as

$$K_0 = \epsilon mc^2 \ldots \quad (106)$$

With $0 < \epsilon << 1$. Use of eq.(106) makes eq.(105) take the following form,

$$\sigma_{min} = \frac{G[Nm+2\sqrt{2}M_0]}{3\sqrt{2\pi}\epsilon c^2}\left[\sqrt{1+\frac{9\pi m_{Pl}^4\epsilon}{G^2[Nm+2\sqrt{2}M_0]^2m^3}}-1\right] \quad (107)$$

For black holes not to form entails

$$\frac{m_{Pl}^4}{m^2} > 2.47M_{eff}^2\left[1+\frac{1.22M_0}{M_{eff}}+3.23\epsilon\right] \ldots\ldots\ldots \quad (108)$$

Implies that the mass of the dark boson must satisfy

$$\mathrm{m} > \frac{0.64\mathrm{m}_{\mathrm{Pl}}^2}{\mathrm{M}_{\mathrm{eff}}}\left(1+\frac{1.22\mathrm{M}_0}{\mathrm{M}_{\mathrm{eff}}}+3.23\epsilon\right)^{-\frac{1}{2}} \ldots.. \quad (109)$$

Case III: $K_0 < 0$ and $g = 0$

When $K_0 < 0$, we may modify eq.(106) to, $K_0 = -\epsilon mc^2$ (110) where $0 < \epsilon \ll 1$. Therefore, to obtain the turning point where $\dot{\sigma}$ vanishes, we use eqs.(92), (94) and (110) to arrive at the quadratic equation,

$$\sigma_{min}^2 - \frac{2G[Nm+2\sqrt{2}M_0]\sigma_{min}}{3\sqrt{2\pi}\epsilon c^2} + \frac{\hbar^2}{2m^2C^2\epsilon} = 0 \quad (110)$$

$$\sigma_{min} = \frac{2G[Nm+2\sqrt{2}M_0]}{3\sqrt{2\pi}\epsilon c^2}\left[1-\sqrt{1-\frac{9\pi m_{Pl}^4\epsilon}{[Nm+2\sqrt{2}M_0]^2m^2}}\right] \quad (111)$$

Applying the condition given by eq.(84) for black hole formation to eq.(112), one gets the inequality,

$$m > \frac{0.64m_{Pl}^2}{M_{eff}}\left(1+\frac{1.22M_0}{M_{eff}}-3.23\epsilon\right)^{-\frac{1}{2}} \quad (112)$$

It is easy to see that one may combine the conditions for a black hole formation given by eqs.(102), (109) and (112) into a single criteria for all values of K_0,

$$m\left(1+\frac{1.22M_0}{M_{eff}}+3.23\frac{K_0}{mc^2}\right)^{\frac{1}{2}} > \frac{0.64m_{Pl}^2}{M_{eff}} \quad (113)$$

In the scenario under consideration, $M_0 << M_{eff}$ and $|K_0| << mc^2$. Therefore, the above condition can, for all practical purposes, be expressed simply as,

$$mM_{eff} \gtrsim 0.64m_{Pl}^2 \quad (114)$$

An inequality that essentially reflects an interplay of black hole formation and uncertainty principle. Suppose we assume that the results eqs.(83) and (113) derived for a non-rotating BEC with winding number n_W and consisting of N dark bosons of mass m, then it is easy to see that the spin angular momentum parameter

$$a \sim \frac{c\hbar Nn_W}{GM_{eff}^2} = 2.33n_W\frac{m_{Pl}^2}{mM_{eff}} \quad (115)$$

$$a \lesssim 3.63n_W\left(1+\frac{1.22M_0}{M_{eff}}+3.23\frac{K_0}{mc^2}\right)^{\frac{1}{2}} \approx 3.63n_W \quad (116)$$

Using Fe emission lines diagnostics, the spin parameter $a \equiv cJ/GM^2$ has been estimated to be in excess of 0.84 for the SMBH in NGC 1365, a nearby AGN at a redshift of 0.00545, and a > 0.89 for the SMBH in NGC3783, an AGN at z=0.00933,90 both at 90 % confidence level.

Summary and Conclusions

Using the framework of non-relativistic Gross-Pitaevskii equation in our work, we found that SMBHs with mass $\sim 10^{10}$ $M_\odot$ can be created from the collapse of ultra-light dark boson condensates formed around remnants of population III stars on time scales of $\sim 10^8$ yrs, if the dark boson mass is $\gtrsim 10^{-20}$ eV. This implying that SMBHs withmass $\gtrsim 10^{12}$ M would be generated from collapse of halo size BEC. Recent observation of large scale alignment of jets coming out of radio-galaxies in ELAIS-N1 GMRT deep field suggests that spins of a statistically significant number of SMBHs responsible for jet orientation display preferential alignment. Using a simplified analysis, we showed that the observed upper limits ($\lesssim$0.9, in the case of several nearby AGNs) on the SMBH spinparameter a can be explained provided the associated black holes are created from collapse of dark boson BEC vortices with low winding numbers.

References

1. Banados, *et al*. Nature, 2017. http://dx.doi.org/10.1038/nature25180.
2. Hui L, Ostriker JP, Tremaine S, Witten E. Phys. Rev., D 95, 043541; and the references therein, 2017.
3. Tan A, *et al*. Phys. Rev. Lett. 2016;117:121303.
4. Aprile E, *et al*., Phys. Rev., D 94, 122001, 2016.
5. Akerib DS, *et al*. Phys. Rev. Lett., 118, 021303, 2017.
6. Das Gupta P, Thareja E. Class. Quan. Grav. 34; and the references therein, 2017.
7. King A. MNRAS, 456, L109, 2016.
8. Abbott BP, Abbott R, Abbott TD, Abernathy MR, Acernese F, Ackley K, *et al*. Phys. Rev. Lett. 2016;116:241103.
9. Abbott BP, Abbott R, Abbott TD, Acernese F, Ackley K, Adams C, *et al*. arXiv: 1706.01812, 2017.
10. Abbott BP, Abbott R, Abbott TD, Acernese F, Ackley K, Adams C, *et al*. Phys. Rev. Lett. 2017;119:141101.
11. Das Gupta P. Journal of Scientific Review 2016;6(1).
12. Abbott BP, Abbott R, Abbott TD, Abernathy MR, Acernese F, Ackley K, *et al*. Annalen der Physik 2017;529:1.
13. Mingarelli CM, Lazio TJW, Sesana A, Greene JE, Ellis JA, Ma CP, *et al*. Nature Astronomy, 1, 886; and the references therein 2017.