International Journal of Applied Research 2021; 7(4): 162-167

International Journal of Applied Research

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 8.4
IJAR 2021; 7(4): 162-167
www.allresearchjournal.com
Received: 10-02-2021
Accepted: 12-03-2021

Ab. Rafi Rather
Research Scholar, Department of Mathematics, Rabindranath Tagore University, Bhopal, Madhya Pradesh, India

Dr. Chitra Singh
Head of Department
Department of Mathematics, Rabindranath Tagore University, Bhopal, Madhya Pradesh, India

Corresponding Author:
Ab. Rafi Rather
Research Scholar, Department of Mathematics, Rabindranath Tagore University, Bhopal, Madhya Pradesh, India

A study of Anti Q-fuzzy R-closed PS-ideals in PS-algebras

Ab. Rafi Rather and Dr. Chitra Singh

Abstract
In this research study the investigator explore the Notion of Anti Q-fuzzy R-closed PS-ideals in PS-Algebras. The systematic procedure has been adopted by the investigator for exploration of the results.

Keywords: anti Q-fuzzy R-closed PS-ideals in PS-algebras

1. Introduction
Algebraic structures play an important role in mathematics with wide range of applications in many disciplines such as computer sciences, control engineering, theoretical physics, information systems and topological spaces. It gives enthusiasm to the researchers to review various concepts and results from the area of abstract algebra in the broader framework of fuzzy setting. Classical algebra was first developed by the ancient Babylonians, who had a system similar to our algebra. The word "Algebra" literally means the re-union of broken parts based on the origins of Arabic language. It was first used around 800AD by Arabic scholars, and is still in our language today. As a branch of mathematics, algebra emerged at the end of 16th century, with the work of François Viète. Algebra can essentially be considered as doing computations similar to that of arithmetic with non-numerical mathematical objects. However, until the 19th century, algebra consisted essentially of the theory of equations. For example, the fundamental theorem of algebra belongs to the theory of equations and is not, nowadays, considered as belonging to algebra. Modern Algebra has come into existence much more recently, emerging over the past 200 years. This is a very complicated study of abstract ideas that are useful for mathematicians and scientists. It also includes some more basic topics like Boolean algebra and matrix multiplication. Modern day physics and quantum physics rely heavily on the new concepts of modern, or abstract, algebra.

2. Review of the related literature
The investigator has surveyed the related literature as under:
In this Section, we give a comprehensive review of the published work that is necessary for this thesis. K. Iseki and S. Tanaka [26, 27] captured the concept of algebra and introduced the theory of BCK algebras in 1978. Later on K. Iseki [25, 28] introduced the theory of BCI algebras in1980 and it has been shown that the class of BCK algebras is the proper subclass of BCI algebras. These two are the important classes of logical algebras. Q.P.Hu and X. Li [23] introduced the notion of a BCH algebra which generalization of BCK and BCI algebras in 1983.They have shown that the class of BCI algebra is a proper subclass of the class of BCH algebras. Young Bae Jun and E.H. Roh [66] studied G-part of BCI – algebras in 1993 and discussed some of its properties. Jun *et al.* [63] introduced a new notion called BH - algebra in 1998, a generalization o BCH / BCI /BCK- algebra. Ahn and Kim [7] introduced another type of algebra-QS-algebra in 1999, which is also a generalization of BCK/BCI algebras and obtained several results iterms of sub algebras, ideals, implicatives, etc. J. Neggers and H.S. Kim [42] introduced the concept of d algebras in 1999, which is another generalization of BCK algebras and investigated relations between d algebras and BCK algebras. J. Neggers, S.S. Ahn and H.S. Kim [40] introduced the notion of Q algebras in 2001, which is a generalization of BCK / BCI / BCH algebras and some theorems are discussed.

J. Neggers and H.S. Kim [39] also introduced a new notion of B algebras in 2002 and discussed their properties. C.B. Kim and H.S. Kim [31] introduced the concept of BG algebra in 2005,which is a generalization of B algebra. K.H. Dar and M. Akram [16] introduced a new notion of K (G) - algebra, which isa generalized class of B – algebra in 2005. Dar *et al.* [2, 17, 18] also studied the characterization of K (G) algebras through homomorphism and Cartesian product in2007. Walendziak [60] introduced the notion of BF algebra in 2007, as a generalization of B algebra. Meng [35] introduced the new notion of CI algebra in 2009. C. Prabpayak and U. Leerawat [43] introduced the concept of KU algebra in 2009, which is another generalization of BCK / BCI / BCC algebras. They have been studied ideals and congruence of BCC algebras of W.A. Dudek [19, 20, 21] and established results in KU algebra.

3. Rationale of the study: Anti Q-fuzzy R-closed PS-ideals of PS - algebras

In this section, we defined new ideal, namely R-closed PS-ideal, and studied the properties of anti-Q-fuzzy structure of PS-algebra through the newly defined ideal and established numerous results in detail.

We proved that μ is a Q-fuzzy R-closed PS-ideal of a PS-algebra X if and only if μ^c is an anti Q-fuzzy R-closed PS-ideal of X.

Definition 3.1

Let Q and G be any two sets. A mapping β: G x Q → [0, 1] is called a Q–fuzzy set in G.

Definition 3.2

An ideal A of a PS-algebra X is said to be R-closed if x * 0 A for all x A.

Definition 3.3

Let (X, *, 0) be a PS-algebra. A non empty subset I of X is called R-closed PS- ideal of X if it satisfies the following conditions:

1) x * 0 ∈ I
2) y * x ∈ I and y ∈ I ⇒ x ∈ I for all x, y ∈ X.

Example 3.4

Let X = {0, a, b} be the set with the following table.

*	0	a	b
0	0	a	b
a	0	0	b
b	0	b	0

Then (X, *, 0) is a PS – Algebra.

From the above example it is clear that A_1 = {0, a} and A_2 = {0, a, b} are R- closed PS-ideals of X.

Definition 3.5

A Q- fuzzy set μ in X is called a Q-fuzzy PS- ideal of X if

(i) μ (0, q) ≥ μ (x, q)
(ii) μ (x, q) ≥ min {μ (y * x, q), μ (y, q)},for all x, y∈ X and q Q.

Definition 3.6

A Q-fuzzy set μ of a PS-algebra X is called an anti Q-fuzzy PS-ideal of X, if i) μ (0, q) ≤ μ (x, q)

ii) μ (x, q) ≤ max {μ (y * x, q), μ (y, q)}, for all x, y∈ X and q Q.

Definition 3.7

A Q-fuzzy set μ of a PS-algebra X is called an anti Q-fuzzy R-closed PS-ideal of X, if it satisfies the following conditions:

(i) μ (x * 0, q) ≤ μ (x, q)
(ii) μ(x, q) ≤ max {μ (y * x, q), μ(y, q)}, for all x, y∈ X and q Q.

Theorem 3.8

Every Anti Q-fuzzy R-closed PS- ideal μ of a PS-algebra X is order preserving.

Proof

Let μ be an anti Q-fuzzy R-closed PS-ideal of a PS-algebra X

Let x, y ∈ X and q Q be such that x ≤ y, then y * x = 0 Then
μ(x, q) ≤ max {μ ((y * x), q), μ (y, q)}
= max {μ (0, q), μ (y, q)}
= max {μ (y * 0, q), μ (y, q)}
= μ (y, q)
⇒ μ(x, q) ≤ μ (y, q), which completes the proof.

Theorem 3.9

μ is a Q- fuzzy R-closed PS-ideal of a PS-algebra X if and only if μ^c is an anti Q - fuzzy R- closed PS-ideal of X.

Proof:

Let μ be a Q-fuzzy R-closed PS- ideal of X. Let x, y, z ∈ X and q Q.
i) μ(x * 0, q) ≥ μ(x, q)
1 - μ^c (x * 0, q) ≥ 1 - μ^c (x, q)
μ^c (x * 0, q) ≤ μ^c (x, q)
ii) μ^c (x, q) = 1 - μ (x, q)
≤ 1 – min {μ (y * x, q), μ (y, q)}
= 1 – min {1 - μ^c (y * x, q), 1 - μ^c (y, q)}
= max {μ^c (y * x, q), μ^c (y, q)} That is μ^c (x * z, q) ≤ max {μ^c (y * x, q), μ^c (y, q)}.
Thus μ^c is an anti Q-fuzzy R-closed PS-ideal of X.
Conversely let us assume that μ^c is an anti Q-fuzzy R-closed PS-ideal of X. i) μ^c (x * 0, q) ≤ μ^c (x, q)
1 - μ (x * 0, q) ≤ 1 - μ (x, q)
ii) μ (x, q) = 1 - μ^c (x, q)
≥ 1 – max {μ^c (y * x, q), μ^c (y, q)]
= 1 – max {1 - μ (y * x, q), 1 - μ (y, q)]
= min {μ (y * x, q), μ (y, q)} That is μ^c (x * z, q) ≥ min {μ (y * x, q), μ (y, q)]. Thus μ is a Q-fuzzy R-closed PS-ideal of X.

Theorem 3.10

If μ is an anti Q-fuzzy R-closed PS-ideal of PS–algebra X, then for all x, y ∈ X and q Q, μ(x*(x *y), q) ≤ μ (y, q)

Proof

Let x, y X and q Q.
μ (x * (x * y), q) ≤ max {μ (y * (x * (x * y)), q) , μ (y, q)]
= max {μ (0, q), μ(y, q)}
= max {μ (y * 0, q), μ(y, q)}
= μ (y, q)

$\therefore$ μ (x * (x * y), q) ≤ μ (y, q)

Theorem 3.11

Let X be a PS-algebra. For any anti Q- fuzzy R-closed PS-ideal μ of X, X_μ =
{x ∈ X and q Q / μ(x, q) = μ (0, q)} is a PS-ideal of X.

Proof

Let y * x, y ∈ X_μ.
Then μ(y * x, q) = μ(y, q) = μ (0, q)
Since, μ is an anti Q-fuzzy R-closed PS-ideal of X,
μ(x,q) ≤ max {μ (y * x, q), μ (y, q)}
= max {μ (0, q), μ (0, q)]= μ (0, q)
Hence x ∈ X_μ.
Therefore X_μ is a PS-ideal of X.

Theorem 3.12

If λ and μ are anti Q-fuzzy R-closed PS ideals of a PS-algebra X, then λ ∩ μ is also an anti Q-fuzzy R-closed PS-ideal of X.

Proof

Let x, y ∈X and q Q. Then
(λ ∩ μ) (0, q) = min {λ (0, q), μ (0, q)]
≤ min {λ (x, q), μ (x, q)]
= (λ ∩ μ) (x, q)
(λ ∩ μ) (x, q) = min {λ (x, q), μ(x, q)]
≤ min {max {λ(y * x, q), (y, q)}, max {μ(y * x, q), μ(y, q)}}
= min {max {λ(y * x, q), μ(y * x, q)}, max {λ(y, q), μ(y, q)}}
≤ max {min {λ(y * x, q), μ(y * x, q)}, min {λ(y, q), μ(y, q)}}
= max {(λ∩μ) (y *x, q), (λ∩μ) (y, q)}.
⇒ (λ ∩ μ) (x, q) ≤ max {(λ ∩ μ) (y *x, q), (λ ∩ μ) (y, q)}.
Thus (λ ∩ μ) is also an anti Q-fuzzy R-closed PS ideal of X.

Theorem 3.13

The union of any set of anti Q-fuzzy R-closed PS-ideals in PS-algebra X is also an anti Q-fuzzy R-closed PS-ideal.

Proof

Let {μ_i] be a family of anti Q-fuzzy R-closed PS-ideals of PS-algebras X. Then for any x, y ∈ X and q Q.
(∪ μ_i) (0, q) = sup (μ_i (0, q))
≤ sup (μ_i (x, q))
= (∪ μ_i) (x, q) And (∪ μ_i) (x, q) = Sup (μ_i (x, q))
≤ Sup {max {μ_i (y * x, q), μ_i (y, q)}}
= max {Sup (μ_i (y * x, q)), Sup (μ_i (y, q))}
= max {(∪ μ_i) (y * x, q), (∪ μ_i) (y, q)}
⇒ (∪ μ_i) (x, q) max {(∪ μ_i) (y * x, q), (∪ μ_i) (y, q)]
This completes the proof.

4. Lower Level Cuts in Anti Q-Fuzzy R-Closed PS-Ideals Of PS-Algebra

In this section, we discussed about the lower level cuts in anti Q-fuzzy R- closed PS-ideals of PS-algebra. We proved that If μ is an anti Q-fuzzy R-closed PS-ideal of PS - algebra X, then μ_t is a R-closed PS-ideal of X for every t ∈ [0, 1]. Also we showed that if μ is an anti Q-fuzzy R-closed PS-ideal of X then
∀ ⇒ where μ is a Q-fuzzy set in PS-algebra X.

Definition 4.1

Let μ be a Q-fuzzy set of X. For a fixed t ∈ [0, 1], the set μ_t ={x ∈ X | μ (x, q) ≤ t, for all q Q} is called the lower level subset of μ. Clearly μ^t ∪ μ_t = X for t ∈ [0, 1] if $t_1 < t_2$, then $\mu_{t1} \subseteq \mu_{t2}$.

Theorem 4.2

If μ is an anti Q-fuzzy R-closed PS-ideal of PS-algebra X, then μ_t is a R-closed PS-ideal of X for every t ∈ [0, 1].

Proof

Let μ be an anti Q-fuzzy R-closed PS-ideal of PS-algebra X.
(i) Let y μ_t ⇒ μ (y, q) ≤ t.
μ (x * 0, q) ≤ max {μ (y * (x * 0)),q) , μ (y, q)]
= max {μ (y * 0),q) , μ (y, q)]
= μ (y, q) ≤ t.
⇒ x * 0 μ_t.
(ii) Let y * x μ_t and y μ_t, for all x, y ∈ X and q Q.
⇒ μ (y * x, q) ≤ t and μ (y, q) ≤ t.
μ (x, q) ≤ max {μ (y * x, q), μ (y, q)} ≤ max {t, t} = t.
⇒ x μ_t.
Hence μ_t is an R-closed PS- ideal of X for every t∈ [0, 1].

Theorem 4.3

Let μ be a Q-fuzzy set of PS- algebra X. If for each t [0, 1], the lower level cut μ_t is a R - closed PS-ideal of X, then μ is an anti Q- fuzzy R-closed PS-ideal of X.

Proof

Let μ_t be a R-closed PS-ideal of X.
If μ(x* 0,q) > μ(x,q) for some x X and q Q, then μ(x * 0,q) > t_0 > μ(x, q) by taking t_0= {μ(x*0,q) + μ(x,q)}.
Hence x*0 ∉ μ_{t0} and x μ_{t0}, which is a contradiction. Therefore, μ(x*0,q) ≤ μ(x,q).
Let x, y X and q Q be such that μ (x, q) > max {μ (y * x, q), μ(y, q)}. Taking t1 = {μ(x, q) + max {μ (y * x), q), μ(y, q)}}
⇒ μ (x, q) > t_1 > max {μ (y * x, q), μ(y, q)}. It follows that (y * x), y μ_{t1} and x ∉ μ_{t1}.
This is a contradiction.
Hence μ(x, q) ≤ max {μ (y * x, q), μ(y, q)} Therefore μ is an anti Q-fuzzy R-closed PS-ideal of X.

Definition 4.4

Let X be an PS- algebra and a, b X. We can define an set A (a, b) by
A (a, b) = {x X/ a * (b * x) = 0}. It is easy to see that 0, a, b A (a, b) for all a, b X.

Theorem 4.5

Let μ be a Q-fuzzy set in PS-algebra X. Then μ is an anti Q-fuzzy R-closed PS- ideal of X iff μ satisfies the following condition.
∀ ∀ ⇒

Proof

Assume that μ is an anti Q-fuzzy R-closed PS- ideal of X. Let a, b μ_t. Then μ (a, q) ≤ t and μ (b, q) ≤ t.
Let x A (a, b). Then a * (b * x) = 0. Now,
μ (x, q) ≤ max {μ ((b * x), q), μ (b, q)}
≤ max {max {μ (a * (b * x), q), μ (a, q)}, μ(b, q)]
= max {max {μ (0, q), μ (a, q)}, μ (b, q)}

= max {max {μ (a * 0, q), μ (a, q)}, μ(b, q)}
= max {μ (a, q), μ (b, q)}
≤ max {t, t}
= t
⇒ μ (x, q) ≤ t
⇒ x μ_t.

Therefore A (a, b) μ_t.
Conversely suppose that A (a, b) μ_t.
Obviously x * 0 = 0 A (a, b) μ_t for all a, b X. Let x, y X be such that (y * x) μ_t and y μ_t.
Since (y * x) * (y * x) = 0. We have x A (y * x, y) μ_t.
∴ μ_t is a R-closed PS- ideal of X.
Hence, by theorem 6.3.3, μ is an anti Q-fuzzy R-closed PS-ideal of X.

Theorem 4.6
Let μ be a Q-fuzzy set in PS-algebra X. If μ is an anti Q-fuzzy R-closed PS-ideal of X then ∀ ⇒

Proof
Let t [0, 1] be such that.
Since x * 0 = 0 μ_t, we have Now, let x
Then there exists (u, v) such that x A (u, v) by theorem 6.3.5. Thus ∴

5. Homomorphism on Anti Q-fuzzyR-closed PS-ideals of PS-algebras
In this section, we discussed about homomorphism on Anti Q-fuzzy R-closed ideals of PS-algebras and some of its properties in detail.

Definition 5.1
Let (X,*,0) and (Y, Δ, 0) be PS– algebras. A mapping f: X → Y is said to be a homomorphism if f(x * y) = f(x) * f(y) for all x, y ∈ X.

Definition 5.2
Let (X,*, 0) and (Y, Δ, 0) be PS–algebras. A mapping f: X → Y is said to be an anti homomorphism if f (x * y) = f(y) Δ f(x) for all x, y ∈ X.

Definition 5.3
Let f: X → X be an endomorphism and μ be a fuzzy set in X. We define a new fuzzy set in X by μ_f in X as μ_f (x) = μ (f(x)) for all x in X.

Theorem 5.4
Let f be an endomorphism of a PS- algebra X. If μ is an anti Q- fuzzy R-closed PS-ideal of X, then so is μ_f.

Proof
Let μ be an anti Q-fuzzy R-closed PS-ideal of X. Now, μ_f (x * 0, q) = μ (f (x * 0, q))
≤ μ (f (x, q))
= μ_f (x, q), for all x, y ∈ X and q ∈ Q.
⇒ μ_f (x * 0, q) ≤ μ_f (x, q) Let x, y ∈ X and q∈ Q.
Then μ_f (x, q) = μ (f(x, q))
≤ max {μ ((f(y, q) * f(x, q)) , μ(f (y, q))]
= max {μ (f(y * x), q), μ (f (y, q))}
= max {μ_f (y * x, q), μ_f (y, q)}
∴ μ_f (x, q) ≤ max {μ_f (y * x), q), μ_f (y, q)}
Hence μ_f is an anti Q -fuzzy R-closed PS-ideal of X.

Theorem 5.5
Let f: X → Y be an epimorphism of PS- algebra. If μ_f is an anti Q-fuzzy R-closed PS- ideal of X, then μ is an anti Q-fuzzy R-closed PS-ideal of Y.

Proof
Let μ_f be an anti Q-fuzzy R-closed PS-ideal of X.
Let y ∈ Y and q∈ Q. Then there exists x ∈ X such that f(x, q) = (y, q). Now,μ (y * 0, q) = μ ((y, q) * (0,q))
= μ (f (x, q) * f(0,q))
= μ (f ((x, q) * (0,q)))
= μ_f ((x, q) * (0,q))
≤ μ_f (x, q)
= μ (f(x, q))
= μ (y, q)
∴ μ (y * 0, q) ≤ μ (y, q) Let y_1, y_2 ∈ Y and q∈ Q.
μ ((y_1,q)) = μ (f (x_1, q))
= μ_f (x_1, q)
≤ max {μ_f ((x_2, q) * (x_1, q)) , μ_f (x_2, q)]
= max {μ [f ((x_2, q) * (x_1, q))], μ (f(x_2, q))}
= max {μ [f (x_2, q)) * f (x_1, q)], μ (f(x_2, q))}
= max {μ [(y_2, q) * (y_1, q)], μ (y_2, q)}
∴ μ (y_1, q) ≤ max {μ [(y_2, q) * (y_1, q)], μ (y_2, q)}
⇒ μ is an anti Q-fuzzy R-closed PS-ideal of Y.

Theorem 5.6
Let f: X → Y be a homomorphism of PS- algebra. If μ is an anti Q-fuzzy R-closed PS- ideal of Y then μ_f is an anti Q-fuzzy R-closed PS-ideal of X.

Proof
Let μ be an anti Q-fuzzy R-closed PS-ideal of Y. Let x, y ∈ X and q ∈ Q.
μ_f (x * 0, q) = μ [f (x * 0, q)]
≤ μ [f(x, q)]
= μ_f (x, q)
⇒ μ_f (x * 0,q) ≤ μ_f (x, q).
μ_f (x, q) = μ (f (x, q))
≤ max {μ [f(y, q) * f(x, q)], μ (f (y, q))]
= max {μ [f(y * x, q)], μ (f (y, q))}
= max {μ_f (y * x, q), μ_f (y, q)}
∴ μ_f (x, q) ≤ max {μ_f (y * x, q), μ_f (y, q)}
Hence μ_f is an anti Q-fuzzy R-closed PS-ideal of X.

6. Cartesian Product of Anti Q-Fuzzy R-closed PS-ideals of PS–algebras
In this section, we introduced the concept of Cartesian product of anti Q-fuzzy R - closed PS-ideals of PS-algebra and established its properties.

Definition 6.1
Let μ and δ be the fuzzy sets in X. The Cartesian product μ x δ : X x X → [0,1] is defined by (μ x δ) (x, y) = min {μ (x), δ (y)}, for all x, y ∈ X.

Definition 6.2
Let μ and δ be the anti fuzzy sets in X. The Cartesian product
μ x δ: X x X → [0, 1] is defined by (μ x δ) (x, y) = max {μ (x), δ (y)}, for all x, y ∈ X.

Definition 6.3
Let μ and δ be the anti Q-fuzzy sets in X.

The Cartesian product $\mu \times \delta : X \times X \rightarrow [0,1]$ is defined by $(\mu \times \delta)((x, y),q) = \max\{\mu(x, q), \delta(y, q)\}$, for all $x, y \in X$ and $q \in Q$.

Theorem 6.4

If μ and δ are anti Q-fuzzy R-closed PS-ideals in a PS–algebra X, then μ x δ is an anti Q- fuzzy R-closed PS-ideal in X x X.

Proof

Let $(x_1, x_2) \in X \times X$ and $q \in Q$.
$(\mu \times \delta)((x_1 * 0, x_2 * 0), q) = \max\{\mu(x_1 * 0, q), \delta(x_2 * 0, q)\}$
$\leq \max\{\mu(x_1, q), \delta(x_2, q)\} = (\mu \times \delta)((x_1, x_2), q)$
$\therefore (\mu \times \delta)((x_1 * 0, x_2 * 0), q) \leq (\mu \times \delta)((x_1, x_2), q)$ Let $(x_1, x_2), (y_1, y_2) \in X \times X$ and $q \in Q$.
Now, $(\mu \times \delta)((x_1, x_2), q) = \max\{\mu(x_1, q), \delta(x_2, q)\}$
$\leq \max\{\max\{\mu(y_1 * x_1, q), \mu(y_1, q), \max\{\delta(y_2 * x_2), q), \delta(y_2, q)\}\}$
$= \max\{\max\{\mu(y_1 * x_1), q), \delta(y_2 * x_2), q)\}, \max\{\mu(y_1, q), \delta(y_2, q)\}\}$
$= \max\{(\mu \times \delta)(((y_1, y_2), q) * ((x_1, x_2), q)), (\mu \times \delta)((y_1, y_2), q)\}$
$\therefore (\mu \times \delta)((x_1, x_2), q) \leq \max\{(\mu \times \delta)(((y_1, y_2), q) * ((x_1, x_2), q)), (\mu \times \delta)((y_1, y_2), q)\}$. Hence, μ x δ is an anti Q-fuzzy R-closed PS- ideal in X x X.

Theorem 6.5

Let μ & δ be fuzzy sets in a PS-algebra X such that μ x δ is an anti Q-fuzzy R-closed PS- ideal of X x X. Then

1. Either $\mu(x * 0, q) \leq \mu(x, q)$ (or) $\delta(x * 0, q) \leq \delta(x, q)$ for all $x \in X$ and $q \in Q$.
2. If $\mu(x * 0, q) \leq \mu(x, q)$ for all $x \in X$ and $q \in Q$, then either $\delta(x * 0, q) \leq \mu(x, q)$ (or) $\delta(x * 0, q) \leq \delta(x, q)$
3. If $\delta(x * 0, q) \leq \delta(x, q)$ for all $x \in X$ and $q \in Q$, then either $\mu(x * 0, q) \leq \mu(x, q)$ (or) $\mu(x * 0, q) \leq \delta(x, q)$.

Proof: Straightforward.

Theorem 6.6

Let μ & δ be fuzzy sets in a PS-algebra X such that μ x δ is an anti Q-fuzzy R-closed PS- ideal of X x X. Then either μ or δ is an anti Q-fuzzy R-closed PS-ideal of X.

Proof

First we prove that δ is an anti Q- fuzzy R-closed PS-ideal of X.
Since by 6.5.5 (i) either $\mu(x*0, q) \leq \mu(x, q)$ or $\delta(x*0,q) \leq \delta(x, q)$ for all $x \in X$ and $q \in Q$. Assume that $\delta(x*0, q) \leq \delta(x, q)$ for all $x \in X$ and $q \in Q$.
It follows from 6.5.5 (iii) that either $\mu(x*0,q) \leq$ □(x, q) (or) $\mu(x*0,q) \leq \delta(x, q)$. If
$\mu(x*0,q) \leq \delta(x, q)$, for any $x \in X$ and $q \in Q$, then
$\delta(x, q) = \max\{\mu(x*0, q), \delta(x, q)\} = \max\{\mu(0,q), \delta(x, q)\} = (\mu \times \delta)((0, x),q)$
$\delta(x, q) = \max\{\mu(0, q), \delta(x, q)\}$
$= (\mu \times \delta)((0, x), q)$
$\leq \max\{(\mu \times \delta)[((0,y),q) * ((0,x),q)], (\mu \times \delta)((0, y),q)\}$
$= \max\{(\mu \times \delta)[((0*0,y*x), q)], (\mu \times \delta)((0, y),q)\}$
$= \max\{(\mu \times \delta)[((0, (y*x)),q)], (\mu \times \delta)((0, y),q)\}$
$= \max\{\delta((y*x),q), \delta(y, q)\}$
Hence δ is an anti Q- fuzzy R-closed PS-ideal of X.

Next we will prove that μ is an anti Q - fuzzy R- closed PS-ideal of X. Let
$\mu(x * 0,q) \leq \mu(x, q)$
Since by theorem 6.5.5 (ii), either $\delta(x * 0,q) \leq \mu(x, q)$ (or) $\delta(x * 0,q) \leq \delta(x, q)$. Assume that $\delta(x * 0, q) \leq \mu(x, q)$, then
$\mu(x, q) = \max\{\mu(x, q), \delta(x*0,q)\} = \max\{\mu(x, q), \delta(0,q)\} = (\mu \times \delta)((x,0),q)$
$\mu(x, q) = (\mu \times \delta)((x, 0), q)$
$\leq \max\{(\mu \times \delta)(((y,0),q) * ((x,0),q)), (\mu \times \delta)((y,0), q)\}$
$= \max\{(\mu \times \delta)(((y * x), (0*0),q)), (\mu \times \delta)((y,0),q)\}$
$= \max\{(\mu \times \delta)(((y * x), 0), q)), (\mu \times \delta)((y,0),q)\}$
$= \max\{\mu(y * x), \mu(y)\}$ Hence,
μ is an anti Q- fuzzy R-closed PS-ideal of X.

7. References

1. Akram M, Dar KH. On Fuzzy d-algebras, Punjab University Journal of Mathematics 2005;37:61-76.
2. Akram M, Dar KH, On K. homomorphisms of K - algebras, International Mathematical Forum 2007;2(46):2283-2293.
3. Abu Ayub Ansari M, Chandramouleeswaran M. Fuzzy translations of fuzzy β - ideals of β -algebras, International Journal of Pure and Applied Mathematics 2014;5(92):657-667.
4. Abu Ayub Ansari M, Chandramouleeswaran M. Normal fuzzy β-sub algebras of β-Algebras, Applied Mathematical sciences 2013;7(105):5213-5224.
5. Ahmad B, Fuzzy BCI-algebras. Journal of Fuzzy Mathematics 1993;2:445-452.
6. Ahn SS, Kim YH, Keum Sook So. Fuzzy BE-algebras, Journal of applied Mathematics and Informatics 2011;29:1049-1057.
7. Ahn SS, Kim HS. On QS – algebras, J. Chungcheong Math. Soc 1999;12:33-41.
8. Atanassov KT. Intuitionistic fuzzy sets, fuzzy sets and systems 1986;20(1):87-96.
9. Atanassov KT. Intuitionistic fuzzy sets theory and applications, Physica -Verlag, A Springer - Verlag company, Bulgaria, April 1999.
10. Bandaru RK. On BRK-algebras, International Journal of Mathematics and Mathematical Sciences 2012;(2012):1-12.
11. Biswas R. Fuzzy subgroups and Anti Fuzzy subgroups, Fuzzy sets andsystems 1990;35:121-124. Chaudhry M.A., Weakly positive implicative and weakly implicative BCI-algebra, Math. Japon 1990;35(1):141-151.
12. Chaudhry MA, Hafiz Fakhar–Ud-Din, On some classes of BCH –algebras, International Journal of Mathematics and Mathematical Sciences 2001;25(3)205-211.
13. Dar KHA. Characterization of Positive implicative BCK-algebras by self maps, Math. Japonica 1986;31(2):197-199.
14. Dar KH, Akram M. On Endomorphisms of BCH-algebras, Annals of University of Craiova, Math. Comp. Sci. Ser 2006;33,227-234.
15. Dar KH, Akram M. On K-algebra built on a group, Southeast Asian Bulletin of Mathematics 2005;29(1):41-49.
16. Dar KH, Akram M. Characterization of K(G)-algebras by self maps, Southeast Asian Bulletin of Mathematics 2004;28(4):601-610.

17. Dar KH, Akram M. On subclasses of K(G)-algebras, Annals of University of Craiova, Math. Comp. Sci. Ser 2006;33:235-240.
18. Dudek WA, Jun YB, Stojakovic Z. On fuzzy ideals in BCC-algebras, Fuzzy sets and System 2001;123:251-258.
19. Dudek WA, Jun YB. Fuzzification of ideals in BCC-algebras, Glasnik Matematicki 2001;36:127-138.
20. Dudek WA, Jun YB. Normalizations of fuzzy BCC-ideals in BCC-algebras, Mathematica Moravica 1999;3:17-24.
21. Hong SM, Jun YB, Kim SJ, Kim GI. Fuzzy BCI – sub algebras with Interval - valued membership functions, International Journal of Mathematics and Mathematical Sciences 2001;25(2):135-143.
22. Hu QP, Li X. On BCH - algebras, Math. Seminar Notes 1983;11(2):313-320.
23. Hussein Hadi Abbas and Hasan Mohammed Ali Saeed, On Closed BCH – Algebra with respect to an element of a BCH- Algebra, Journal of Kufa for Mathematics and Computer 2011;1(4):5-13.
24. Imai Y, Iseki K. on axiom systems of propositional calculi XIV, Proceedings of the Japan academy 1966;42:19-22.
25. Iseki K. Algebraic formulation of proportional calculi, Proceedings of the Japan academy 1965;41:803-807.
26. Iseki K, Tanaka S. An introduction to the theory of BCK – algebras, Math Japonica 1978;23:1-20.
27. Iseki K. On BCI-algebras, Math. Seminar Notes 1980;8:125-130.
28. Jun YB, Park CH. Fuzzy idelas of K(G)-algebras, Honam Mathematical Journal 2006;28:485-497.
29. Kim KH. on fuzzy dot sub algebras of d-algebras, International Mathematical forum 2009;4(13):645-651.
30. Kim CB, Kim HS. On BG – Algebras, Demonstratio Mathematica 2008;41(3):497-505.
31. Kordi A, Moussavi A. On fuzzy ideals of BCI-algebras, Punjab University Journal of Mathematics 2007;18:301-310.
32. Kyung Ho Kim. A Note on CI- algebras, Int. Math. Forum 2011;6(1):1-5.
33. Kyoung Ja Lee, Young Bae Jun, Myung Im Doh. Fuzzy Translations and Fuzzy Multiplications of BCK/ BCI - Algebras, Commun. Korean Math. Soc 2009;24(3):353-360.
34. Meng BL. CI-algebras, Scientiae Mathematicae Japonicae Online, e 2009, 695-701.
35. Megalai K, Tamilarasi A. Classifications of TM– algebra, IJCA special issue on computer aided soft computing techniques for imaging and biomedical applications, CASCT 2010, 11-16.
36. Megalai K, Tamilarasi A. Fuzzy Sub algebras and Fuzzy T - ideals in TM - algebra, Journal of Mathematics and Statistics 2011;7(2):107-111.
37. Meng J, Jun YB. BCK-algebras, Kyung Moon Sa Co. Seoul, Korea 1994.
38. Neggers J, Kim HS. On B-algebras, Math, Vensik, 2002;54:21-29.
39. Neggers J, Ahn SS, Kim HS. On Q-algebras, International Journal of Mathematics and Mathematical Sciences 2001;27:749-757.
40. Neggers J, Jun YB, Kim HS. On d-ideals in d-algebras, Math. Slovaca 1999;49(3):243-251.
41. Neggers J, Kim HS. On d-algebras, Math. Slovaca 1999;49:19-26.
42. Prabpayak C, Leerawat U. On Ideals and congurences in KU-algebras, Scientia Magna J 2009;5(1):54-57.
43. Priya T, Ramachandran T. Anti Fuzzy Ideals of CI-algebras and its lower level cuts, International Journal of Mathematical archive 2012;3(7):2524-25.