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The theoretical explanation for the sound barrier and its relation to light refraction

Robert J Buenker

Abstract
It is pointed out that application of the Doppler effect to sound waves created by airplanes leads to the conclusion that when they are accelerated to the speed of sound (Mach 1), the wavelength decreases to zero ($\lambda=0$). The de Broglie relation between momentum and wavelength ($p=h/\lambda$) which was used to predict electron diffraction therefore indicates that the momentum p of the carrier molecules of the waves must become unbounded ($p=mv=\infty$) at that point. It is argued that it is this singularity which is responsible for the phenomenon of sonic booms. The frequency of the waves is not affected by the motion of the airplane, however, consistent with an argument used by Einstein in his prediction of the gravitational red shift. A close relationship between this interpretation for the behavior of sound waves and the corresponding theory of light refraction is noted. It is shown that in both cases Einstein's mass-equivalence relation $E=mc^2$ is violated.

Keywords: doppler effect, snell's law, newton's second law of kinematics, de broglie relation, light refraction, hamilton's canonical equations, mass-energy equivalence

Introduction

Sonic booms occupy a special place in the history of the physical sciences. It is a phenomenon which has been experienced in everyday life. They can be heard while walking down the street without the aid of special equipment. The sound barrier responsible for them represented a special challenge to airplane pilots which was first overcome in a memorable flight by Yeager in 1947. Anyone could experience them firsthand during a supersonic flight of the Concorde over the Atlantic.

Yet, no consistent explanation for their existence has ever been given by theoretical physicists. It seems highly unlikely that relativity theory is required for this purpose since the speeds involved are much smaller than for light in free space. They originate at relatively low altitudes above the Earth, so the effects of gravitational fields can safely be ignored in searching for an answer as well. Maxwell's theory of electromagnetism seems irrelevant since no electrical or magnetic fields appear to play any significant role. Does the theory of quantum mechanics provide a possible clarification? Or does the more modern theory of quantum chromodynamics solve this puzzle? The discussion below is aimed at removing any uncertainty about why sonic booms occur.

Observation of Sound Waves

When an airplane passes a certain point, it produces sound waves with a constant speed v which possess a wavelength λ_0 and frequency ν_0. As the plane heads into the waves with speed w relative to their origin, the waves are compressed together, thereby resulting in a reduction in wavelength, as determined quantitatively by the Doppler effect, to have a value of $\lambda=(1-w/v)\lambda_0$. The frequency of the waves that reach the airplane is not affected, however, i.e. $\nu=\nu_0$, by its motion, since the same number of wave crests is emitted from the source per unit of time regardless of the value of w. The same argument about constant frequencies was given by Einstein [1, 2] in conjunction with his prediction of the gravitational red shift for light waves emitted near the sun's surface. When the airplane accelerates and the value of its speed $w<v$ relative to the original source of the sound waves increases, the wavelength λ decreases in accord with the Doppler formula but neither the speed v nor the frequency ν of the waves changes as a result.

As the value of w gets quite close to v, i.e.to Mach 1 in the scientific literature, it is clear from the Doppler formula that the value of the wavelength gradually approaches zero. At this point in the discussion, it is important to recall the Davisson-Germer electron diffraction experiment [3]. It was found that the result of passing 54 ev electrons through a nickel crystal is a wave pattern whose wavelength is quantitatively consistent with the de Broglie [4] quantum mechanical relation between momentum p and wavelength $p=h/\lambda$, where $h=6.625x10^{-34}$ Js. Planck's constant h also appears in the relation [5] between energy E and frequency ν, i.e. $E=h\nu$. Both relations are believed to be completely general, applying to both photons and particles with non-zero rest mass μ. In the present case, one is dealing with what one can loosely describe as "air molecules" as the carrier of the sound waves instead of electrons as in the Davisson-Germer example. In reality, air is composed of both O_2 and N_2 molecules plus small amounts of rare gases and CO_2. In the present discussion it is permissible to treat them as molecules with an averaged value of μ.

So, what happens as the airplane approaches Mach 1? First of all, since the wavelength λ is close to zero, the momentum p of the carrier molecules becomes unbounded ($p=\infty$) according to the de Broglie relation $p=h/\lambda$. The value of p changes with time during acceleration. As a result, a force F is generated by the motion, which in accord with Newton's Second Law, is equal to the time derivative of the momentum, i.e. $F=dp/dt$. The direction of this force is the same as that in which the airplane is headed.

What happens to this force? It clearly can have no effect on the molecules themselves since it has been generated *internal* to their motion. This is consistent with the fact that the speed of sound remains constant throughout, i.e. $dv/dt = 0$. Moreover, their energy E also does not change, which is consistent with the Planck relation [5] since the frequency of the sound waves is also not affected by the motion of the airplane. Instead, the force acts on its surroundings, which would account for the sonic boom phenomenon, and also on the gyrations experienced by the airplane in the Mach 1 range.

Yet, if the speed of the molecules continues to have the same value v as before, how can the momentum p change so substantially. This is theoretically possible from the definition of momentum as $p=mv$ *only* if the relativistic mass m of the carrier molecules is also unbounded ($m = \mu$ times ∞). It needs to be recognized, however, that this condition is inconsistent with Einstein's original prediction [6]:

$$m = \left(1 - v^2 c^{-2}\right)^{-0.5} \mu = \gamma\mu \quad , \qquad (1)$$

since v is finite and c is the speed of light in free space (299792458 ms^{-1}). It should be noted that eq. (1) is closely akin to Einstein's famous mass/equivalence relation:

$$E = mc^2 \quad . \qquad (2)$$

By squaring both sides of eq. (1) and multiplying by c^4, while defining the rest energy E_0 to have a consistent value of μc^2, the result is:

$$E^2 - p^2c^2 = E_0{}^2 \qquad (3)$$

which is another key relation in Einstein's theory [6]. Thus, if the interpretation in terms of the de Broglie and Planck relations is correct, it becomes necessary in this application to disregard key results of Einstein's theory of relativity.

Mass-Energy Equivalence and Light Refraction

There is precedent for combining relativity theory with applications of the de Broglie [4] and Planck [5] quantum mechanical relations. It is found in the phenomenon of light refraction, which has had a great impact on the development of physical theory over a period of several millennia. It is something that is easily observed with the naked eye and yet it took until the early 17th century before it was first possible to formulate a mathematical expression (Snell's Law of Sines) that successfully described it on a quantitative basis. Shortly thereafter, Newton [7] used light refraction to illustrate his Second Law and to support his *corpuscular theory* of light. However, his views clashed with those of Huygens and other proponents of the wave theory of light, especially in that the two theories led to opposite predictions of the change in the speed of light as it enters water from air [8].

Newton's arguments are based on the diagram shown in Fig. 1. He concluded that Snell's Law of Sines could be explained by assuming that there is a force *perpendicular* to the interface between the two media. On this basis, the component of momentum in the transverse direction should be the same in air/ free space as in water This leads to the following equation for the two momentum values p_1 and p_2 present on either side of the interface (θ_1 and θ_2 are the respective angles of incidence and refraction):

$$p_1 \sin\theta_1 = p_2 \sin\theta_2 \quad . \qquad (4)$$

The index of refraction n_i for a given medium is defined as the ratio of the two sine values indicated in Fig. 1. Specifically, if $p_1=p_0$ is the value of the momentum of the light particles *in free space* and $p_2=p_i$ is the corresponding value for the medium, the index of refraction n_i is equal to:

$$n_i = \frac{\sin\theta_1}{\sin\theta_2} \quad , \qquad (5)$$

so that

$$p_i = n_i p_0 \quad . \qquad (6)$$

By virtue of the definition given in eq. (5), the momentum of the light particles is thus *directly proportional to the index of refraction*, i.e.:

$$\frac{p_1}{n_1} = \frac{p_2}{n_2} \quad . \qquad (7)$$

The index of refraction is obtained experimentally by measuring the wavelength of light in the medium, not its momentum. In this case, there is an *inverse proportionality* involved, however, not the direct proportionality in eq. (6), namely:

$$\lambda_i = \frac{\lambda_0}{n_i} \quad (8)$$

If one combines eq, (7) with eq, (6), the result is:

$$p_i\lambda_i = p_0\lambda_0 , \quad (9)$$

i.e. pλ is the same for all media, which in turn is clearly consistent with the de Broglie relation (pλ=h) [4] discussed in Sect. II as potentially responsible for sonic booms.

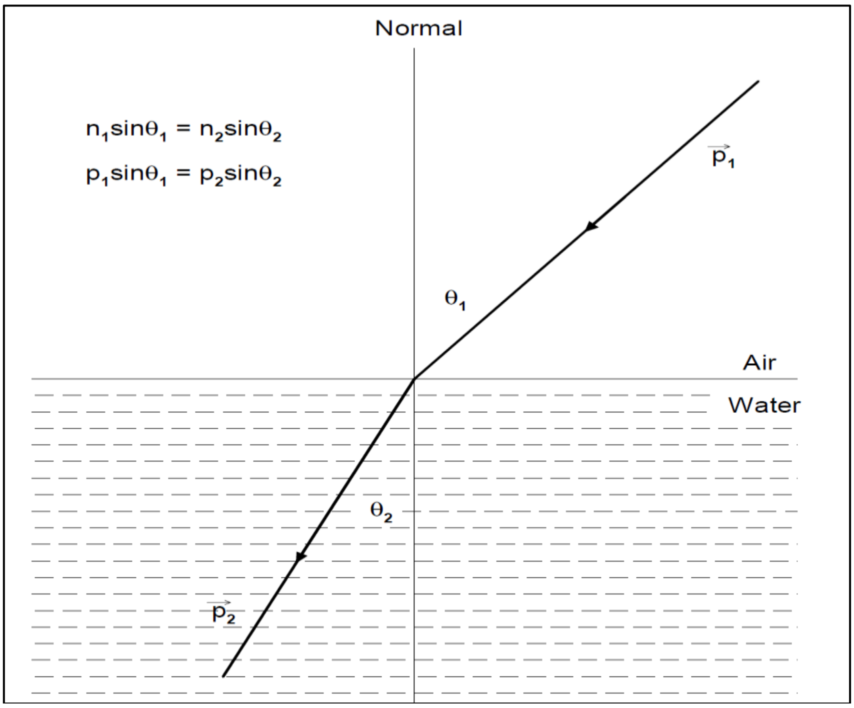

Fig 1: Diagram showing the refraction of light at an interface between air and water. The relation between angles of incidence θ_1 and refraction θ_2 in terms of the refractive indices n_i (Snell's Law of Sines) of the two media was viewed by Newton as a clear application of his Second Law of kinetmatics, according to which the component of the photon momentum p_i parallel to the interface must be conserved.

The Planck E=hν relation [5] is also consistent with light refraction observations. Both the energy and frequency of the light waves are unchanged as they pass into another medium. Newton surmised that the energy was constant because the monochromatic colors of light emerge unchanged at the opposite end of a prism. No interference of waves at the interface is grounds for believing that the frequency of the waves is also not changed. Hence, it can be concluded on that basis that both light and sound waves satisfy both the quantum mechanical relations [4, 5].
The situation with the velocity of light was certainly not so clear in the 18th century, however. As discussed above, Newton [7] took the view that his prediction of an increase in the momentum of the particles of light was a clear indication that the light speed in water was greater than in air. On the other hand, Huygens and his wave theory of light concluded that the light speed is smaller in water because the wavelength of the light waves is less there.
In 1834 Hamilton derived his Canonical Laws of Motion, the simplest of which is given below:

$$\frac{dE}{dp} = v \quad (10)$$

Ironically, the derivation is based on Newton's Second Law. Energy E is related to the applied force F = dp/dt on the object moving a distance Δx in the same direction:

$$dE = Fdx = \frac{dp}{dt}dx = dp\frac{dx}{dt} = vdp \quad (11)$$

For light in free space, v=c in eq. (10) for all wavelengths, so upon integration with respect to p, one obtains:

$$E = pc \quad (12)$$

Newton concluded in eq. (7) that p is proportional to n in refractive media, so on this basis it is possible to generalize eq. (12) [8-10] by replacing p with p/n:

$$E = \frac{pc}{n} \quad (13)$$

The true speed of light v_g in the refractive medium is obtained by applying eq. (10) to the definition of E in eq. (13):

$$v_g = \frac{dE}{dp} = \frac{c}{n_g} = \frac{c}{n} - pc\frac{\frac{dn}{dp}}{n^2} \quad (14)$$

By applying the de Broglie p=h/λ relation [4], one obtains the dependence of the light speed on wavelength (n_g is the group refraction index):

$$v_g = \frac{c}{n_g} = \frac{c}{n} + \lambda c \frac{\frac{dn}{d\lambda}}{n^2} \quad (15)$$

This relationship has been derived exclusively on the basis of Newton's corpuscular theory of light [7]. It indicates, however, that the speed of light in water is less than in air, contrary to what he assumed in the 18th century. As such, eq. (14) amounts to a correct prediction of the experiment carried out by Foucault in 1850 (more details of the history of physicists' reaction to the latter experiment are given elsewhere [8-10]). Moreover, the best measurements [11-13] that have been carried out subsequently indicate that eq. (14) is exact. In addition, the observed dependence of n_g on light frequency ν can be obtained as follows from eq. (13) by assuming the Planck energy-frequency relation E=hν:

$$n_g = c / v_g = \frac{d(pc)}{dE} = \frac{d(nE)}{dE} = n + \frac{Edn}{dE} = n + \frac{\nu dn}{d\nu}. \quad (16)$$

Stark [14,15] was the first to use the p=h/λ relation with respect to light in free space, before de Broglie [4] generalized it to all forms of matter. He made use of Planck's E=hν relation [5] in connection with eq. (12):

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}. \quad (17)$$

Stark also concluded on this basis that particles of light (photons [16]) in free space have inertial mass since by definition, p=mv=mc in the present case:

$$m = \frac{p}{v} = \frac{E}{vc} = \frac{h\nu}{c^2}. \quad (18)$$

For light in a refractive medium, the value of the photon's mass changes to:

$$m = \frac{p}{v} = \frac{\frac{nh\nu}{c}}{\frac{c}{n_g}} = \frac{nn_g h\nu}{c^2} \quad (19)$$

Accordingly, the value of mc^2 in this case is:

$$mc^2 = nn_g h\nu = nn_g E \quad (20)$$

As a result, it is clear that *Einstein's mass-energy equivalence relation of eq. (2) does not always hold*.
Consequently, there is no reliable deduction from relativity theory that rules out the situation discussed in Sect. II, namely where the particle momentum changes without the energy doing so as well. If the average mass m_0=μ of the air molecules changes by a factor of α to α m_0, for example, by analogy to eq. (20), with the original energy E having a value of E_0, it follows that:

$$mc^2 = \alpha m_0 c^2 = \alpha E_0 = \alpha E \quad (21)$$

Conclusion

When an airplane passes a given point, it generates sound waves with a definite speed (v). In accord with the Doppler effect, the plane heading into the waves at speed w causes a compression of their wavelength by a factor of (1- w/v). Consequently, as the plane accelerates, the value of the wavelength therefore steadily decreases. One knows from experiments with electron diffraction that there is a quantitatively well-defined relation between the momentum p of the electrons and the wavelength λ of the associated waves, namely the de Broglie equation, p = h/λ, where h is Planck's constant. Applying the de Broglie relation to the case of a rapidly moving airplane leads to the prediction that the momentum of the particles, in this case primarily O_2 and N_2 molecules, that carry the sound waves will approach a value of infinity as the speed of the plane nears that of sound (v=w, Mach 1). It then decreases from the peak value after Mach 1 is exceeded as the acceleration continues still further. This result is therefore consistent with both the Doppler effect and the de Broglie relation.
The value of the momentum increases over time toward Mach 1 as acceleration proceeds, so according to Newton's Second Law of Kinetics, F = dp/dt, there is a force acting in the direction of the airplane's motion. The closer the speed w of the airplane approaches Mach 1, the greater this force. This is an explanation for the occurrence of the sonic boom. This force acts on the airplane and on the surroundings in general, but not on the carrier molecules themselves. As a result, the speed of sound v itself is not affected by the momentum change, however. This is consistent with the fact that the frequency of the waves is not affected by the airplane's motion either. The peaks of the sound waves still keep coming at the same rate, independent of the airplane's motion. Einstein made use of the analogous conclusion in his successful prediction of the gravitational red shift. According to the other key quantum mechanical formula, Planck's Radiation Law E=hν, it therefore follows that the energy of the sound waves remains constant even though their momentum increases toward ever larger values.
The latter conclusion in not consistent with Einstein's mass-energy equivalence relation ($E=mc^2$), however. There is precedent for this inconsistency. in the phenomenon of light refraction. In that case, application of the de Broglie and Planck formulas leads to eq, (20), which states that the mass of the photons is given by eq. (19). As a consequence, it is found that the product of m with c^2 is equal to nn_gE, i.e. an extra factor of the product of the refractive index n and group refractive index n_g is required to obtain the observed relationship between mass and energy in this case. So it is clear that the $E=mc^2$ formula is by no means completely general and can be ignored in the present case involving sound waves.
It also is shown that the application of the two quantum mechanical formulas succeeds in accurately predicting the experimental value of the speed of light in water and other refractive media [see eqs. (14-16)]. It is noted that this result not only serves as a verification of the generality of the two quantum mechanical formulas, but also of the corpuscular theory of light espoused by Newton in the early 18th century as well. It is ironic that Hamilton's Canonical Laws (ca. 1834), which are instrumental in the latter derivations themselves, follow directly from Newton's Second Law of Kinetics. If these relationships had been acepted in Hamilton's lifetime, physicists could have been spared the

ubiquitous claims which continue to the present day, that light, contrary to what Newton assumed over three centuries ago, is not composed of particles/photons,

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