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Rebuttal of Fermi's denial of nuclear electrons: Part III XBPS calculations of the neutron's internal structure

Robert J Buenker

Abstract
Calculations employing the exponentially damped Breit-Pauli-Schrödinger (XPBS) equation which have been presented in previous work are analyzed with the goal of determining the possible causes for the hypothesized binding of the electron and anti-neutrino to the proton that is responsible for the meta-stability of the neutron. It is found that the main attractive effects come from the spin-same orbit and Darwin interactions between the proton and the electron and anti-neutrino, respectively. For sufficient bonding to occur, it is found that the charge-to mass ratio for the anti-neutrino must fall within the range of +0.5-0.7 a.u. It is found that the variation of the size of the proton charge distribution has a great influence on the degree of bonding with the lighter particles. The value of the proton-electron Coulomb energy indicates that the p^+e^- separation within the neutron is 10.2 fermi (3.62 α^2), which is well within the range expected by Fermi on the basis of the de Broglie momentum-wavelength relation. It is pointed out that the non-zero value of the charge-to mass ratio is still consistent with the known extreme penetrability of the neutrino through matter, as demonstrated by experiments carried out by Reines and Cowan. This is because the lack of electric charge and high speed prevent the neutrino from overcoming the centrifugal barriers encountered in the neighborhood of charged particles, without at the same time precluding collisions at very small separations. The fact that the neutron's magnetic moment is far less than that of the electron is consistent with observations of systems with tightly bound electrons in the Compton effect.

Keywords: neutron composition, neutrino penetrability, exponentially-damped Breit-Pauli-Schrödinger equation (XBPS)

Introduction
The calculations carried out with the XBPS method in the forgoing study[1] produce both a wavefunction for each of the $p^+e^-\bar{\nu}$ and $e^-\bar{\nu}$ systems as well as corresponding energy contributions of the various Hamiltonian operators. In the present work, these results will be analyzed to shed light on the basic question of how the proton is bonded to the two lighter particles to produce a meta-stable system with a relatively long lifetime. It is of special interest to see what the role of the non-zero value for the charge-to-mass ratio of the anti-neutrino plays in bringing about this tightly-bound structure. In addition, one would like to know what the relationship is between the wave function of the e^+e^- system which has been associated with the photon in previous work and that of the $e^-\bar{\nu}$ constituent assumed to be tightly bound within the neutron,

It is also of interest to understand how the properties of the theoretical $p^+e^-\bar{\nu}$ system agree with what is experimentally known about the neutron with which it is being associated. For example, although it is clear that the $p^+e^-\bar{\nu}$ composition is consistent with the electrical neutrality of the neutron, one also must expect that it cannot be ruled out on the basis of the relatively small value observed for its magnetic moment, given the much larger value of the electron's magnetic moment.

The Mechanism for Proton Bonding in the $p^+e^-\bar{\nu}$ System
A comparison of Figs. 1 and 3 of Ref. [1] helps to illustrate the origin of the proton binding process in the present model. The energy difference results for $e^-\bar{\nu}$ and $p^+e^-\bar{\nu}$ in the same (2s,2p) basis are plotted against the assumed q/m_o value for $\bar{\nu}$ in Fig. 1, and show that the

proton binding energy increases as this charge-to-mass ratio is reduced from its original value of +1.0 a.u. Because of the small q/m_o value of the proton it is to be expected that only terms in the XBPS Hamiltonian (Table 1 of Ref. [2]) which do not involve this quantity will be important. An analysis of the $p^+e^-\bar{\nu}$ total energy for the $\bar{\nu}$ q/m_o values of 1.0 and 0.5733 a.u. is given in Tables 1 and 2, respectively, inc1uding corresponding results for optimal treatments of the $e^-\bar{\nu}$ system in its lowest-energy 0^- state. The proton's kinetic energy contribution is relatively small because of its large mass, but at the interpartic1e distances involved, this quantity still works effectively against net binding. For the $\bar{\nu}$ q/m_o value of 0.5733 a.u. which leads to the experimental neutron total energy for the $p^+e\bar{\nu}$ system, it is found, for example, that the proton kinetic energy is 36401.887 hartree (Table 2), compared with a value of the proton-electron Coulomb attraction contribution of -5189.462 hartree. The latter result corresponds to a mean electron-proton distance of 3.62 α^2 bohr (10.2 fermi). There are two other XBPS terms for the proton which do not involve its own q/m_o value, however, which play a decisive role in the binding process, namely the spin-same-orbit and Darwin terms involving only the squares of either the q/m_o values for the electron or antineutrino (Table 1 of Ref. [2]). Consequently, these terms are of the same order of magnitude as their counterparts in the e^+e^- calculation (see Table 3 of Ref. [2]). The magnitudes of these contributions to the $p^+e^-\bar{\nu}$ energy depend very much on the characteristics of the various occupied spin-orbitals, however. A strong contribution toward proton binding comes from its spin-same-orbit interaction with the electron when the latter occupies a $p_{1/2}$ orbital while the proton occupies $s_{1/2}$ (cf. Table 1 of Ref. [1]). This attractive interaction is countered by the corresponding term involving $\bar{\nu}$, however, because the latter also occupies $p_{1/2}$ heavily and its q/m_o value is assumed to be of opposite sign to that of the electron. The Darwin term also is characterized by large contributions to the $p^+e\bar{\nu}$ total energy in which the proton is directly involved, however. In this case it is the antineutrino which provides an attractive force for the proton, however, which is countered only partially by the corresponding repulsive p^+e^- Darwin term contribution (Table 1 of Ref. [2]).

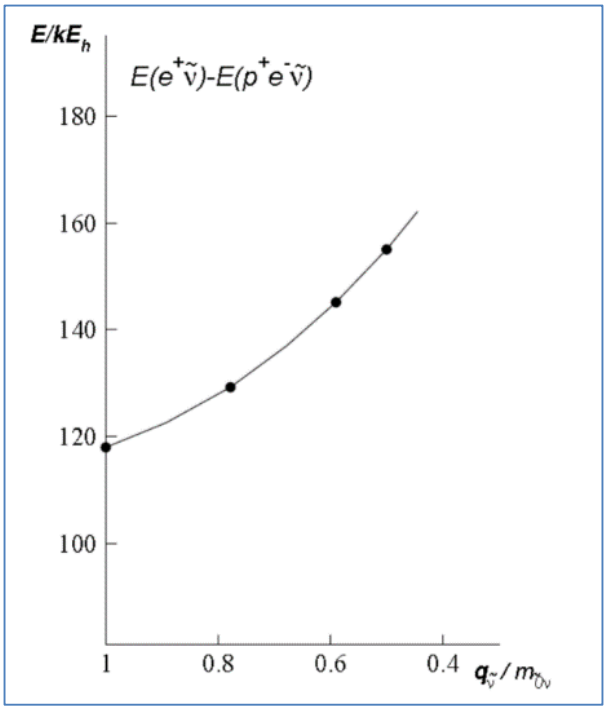

Fig 1: Difference of the computed total energies (in khartree) of the $e^-\bar{\nu}$ and $p^+e^-\bar{\nu}$ systems (with separately optimized scale factors η) as a function of the antineutrino charge-to-rest-mass ratio q/m_o (2s,2p basis).

Table 1: Energy contributions (in hartree) of various operators (see Table 1 of Ref. [2] for definitions) and particle combinations for (a) the $1/2^-$ ground state of the $p^+e^-\bar{\nu}$ neutron system and (b) the 0^- state of the isolated $e^-\bar{\nu}$ binary obtained by employing the 2s,2p basis with scale factor η = 0.115, exponential damping constant A = 1.054 a.u. and antineutrino q/m_o value of 1.0 a.u. for the XBPS Hamiltonian

a) Operator	p^+e^-	$p^+\bar{\nu}$	$e^-\bar{\nu}$	Total ($p^+e^-\bar{\nu}$)
Kinetic Energy	23276.671 (p^+)	1117472.413(e^-)	1133826.572 ($\bar{\nu}$)	2274575.656
Coulomb	-4305.801	0.000	0.000	-4305.801
Spin-same-orbit	-172833.434	48470.883	-395254.117	-519616.667
Spin-other-orbit	-31.731	27.278	-875528.194	-875532.646
Darwin Term	92782.662	-228814.351	19549.016	-116482.672
Orbit-orbit	-44.084	129.057	-450397.290	-450312.316
Spin-spin	-0.550	-0.132	-447017.344	-447018.026
Spin-spin δ	-0.223	-1.938	7381.956	7379.795
Total Energy				-131312.679

b) Operator				Total ($e^- \bar{\nu}$)
Kinetic Energy				2247935.736
Coulomb				0.000
Spin-same-orbit				-411800.053
Spin-other-orbit				-916668.797
Darwin Term				8317.250
Orbit-orbit				-480109.984
Spin-spin				-461579.506
Spin-spin δ				0.000
Total Energy				-13905.355

There is thus a fairly complex system of interactions available among the Breit-Pauli terms whose net energy contribution depends on two major factors: a) the character of the orbitals occupied respectively by the three particles and b) the assumed value of the antineutrino's charge-to-rest-mass ratio [since the corresponding spin-same-orbit and Darwin interactions with the proton depend on the square of this quantity (Table 1 of Ref. [2])]. The easiest means of grasping the influence of these various factors in the proton binding energy is to first examine the situation for q/m_o ($\bar{\nu}$) = +1.0 a.u. If the orbital occupations of the e^- and $\bar{\nu}$ particles were exactly equivalent, all the proton spin-same-orbit and Darwin interactions would exactly cancel one another. That would effectively leave only the proton kinetic energy and proton-electron Coulomb terms to determine the binding energy, which would mean that the $p^+e^-\bar{\nu}$ system is decidedly unstable relative to $e^-\bar{\nu}$. By assuming a wave function in which the electron has more $p_{1/2}$ character than $\bar{\nu}$, it is possible to shift the balance toward a more favorab1e binding situation, however, by virtue of the fact that the attractive proton-electron spin-same-orbit term then outweighs the repulsive $p^+\bar{\nu}$ contribution of the analogous type.

Table 2: Energy contributions (in hartree) of various operators (see Table I of Ref. [2] for definitions) and particle combinations for (a) the $1/2^-$ ground state of the $p^+e\bar{\nu}$ neutron system and (b) the 0^- state of the isolated $e^-\bar{\nu}$ binary obtained by employing the 2s,2p basis with scale factor η = 0.18, exponential damping constant A = 1.054 a.u. and antineutrino q/m_o value of 0.5733 a.u. for the XBPS Hamiltonian

a)	**Operator**	**p^+e^-**	**$p^+\bar{\nu}$**	**$e^-\bar{\nu}$**	**Total ($p^+e^-\bar{\nu}$)**
Kinetic Energy		36401.887 (p^+)	1346815.079 (e^-)	1453755.882 ($\bar{\nu}$)	2836972.849
Coulomb		-5189.461	0.000	0.000	-5189.461
Spin-same-orbit		-208544.881	50249.141	-465129.910	-623425.649
Spin-other-orbit		-58.267	54.775	-1007066.189	-1007069.681
Darwin Term		107014.110	-262604.473	17161.765	-138428.597
Orbit-orbit		200.486	-58.064	-523108.311	-522965.888
Spin-spin		-1.096	-0.248	-516918.459	-516919.802
Spin-spin δ		-0.387	-4.109	5928.285	5923.790
Total Energy					28897.559
b) Operator					Total ($e^-\bar{\nu}$)
Kinetic Energy					2789426.035
Coulomb					0.000
Spin-same-orbit					-481697.661
Spin-other-orbit					-1063806.525
Darwin Term					11500.864
Orbit-orbit					-558243.900
Spin-spin					-531802.164
Spin-spin δ					1127.425
Total Energy					166504.076

Examination of Table 1 shows clearly that heavy net attractive contributions actually arise for both the spin-same-orbit and Darwin terms as a result of such a polarization. As a result, the total binding energy of the proton to the $e^-\bar{\nu}$ complex is computed to be 117407 hartree (i.e. -131312.679-13905.355 hartree). By comparison, if only the proton kinetic energy and proton-electron Coulomb contributions were counted, a negative proton binding energy would result (Table 1).

There is another key factor in the $p^+e^-\bar{\nu}$ binding process, however, which is also important in the formation of tri-atomic systems in the realm of molecular physics. Some adaptation of the charge distribution of a diatomic component of the molecule is almost always crucial to the production of a stable tri-atomic system [3]. Generally speaking, the more stable the diatomic system, the higher the price to be paid for altering its charge distribution to accommodate bonding with a third atom. In the present context, it is important to recall that a quite small AO basis has been employed in these exploratory calculations, however, and so it can be anticipated that at least some of the $e^-\bar{\nu}$ system's affinity to polarize its charge distribution

is a consequence of its relatively poor representation at this level of theoretical treatment.

When the q/m_o value is decreased from unity for the antineutrino, several additional effects emerge. The $e^- \bar{\nu}$ complex gradually loses its stability in the process, and it thus becomes easier for the proton to bind to it as a direct consequence. The cancellation of the Breit-Pauli p^+e^- and $p^+ \bar{\nu}$ interactions is no longer perfect even if both e^- and $\bar{\nu}$ have equivalent orbital occupations because the key $(q/m_o)^2$ weighting factors now favor the electron. In addition, polarization effects are less likely to destabilize the $e^- \bar{\nu}$ complex, so that the types of changes in the isolated system's wavefunction which maximize binding with the proton are accomplished with less resistance than before. For the q/m_o value of 0.5733 a.u. required to obtain the experimental neutron rest mass for the $p^+e^- \bar{\nu}$ calculations employing the 2s,2p basis, it is found that the proton binding energy increases to 137600 hartree, over 20000 hartree greater than the value which results when a unit value for this quantity is employed.

The results of Table 2 show further that both the spin-same-orbit and Darwin interactions again produce a net attraction for the proton, namely 158296 and 155590 hartree, respectively (obtained by adding the corresponding p^+e^- and $p^+ \bar{\nu}$ results). These results are accomplished to a large extent by polarization of the $e^- \bar{\nu}$ complex, as can be seen from the following comparison. For this q/m_o value the minimal total energy obtained for the isolated $e^- \bar{\nu}$ system is 166504 hartree, whereas the corresponding energy value obtained employing the polarized $p^+e^- \bar{\nu}$ wavefunction is 311438 hartree (obtained by adding the e^- kinetic energy of 1346815.079 hartree to the $e^- \bar{\nu}$ results including the $\bar{\nu}$ kinetic energy), an increase of 144934 hartree. Again, while it is very likely that such polarization effects are greatly exaggerated by the use of such a small one-particle basis set in the present calculations, it at least seems conceivable that the combination of changes of this nature in the $e^- \bar{\nu}$ charge distribution along with the use of a $|q/m_o|$ value for the antineutrino which is smaller than that of the electron could lead to sufficient proton binding to produce a meta-stable $p^+e^- \bar{\nu}$ complex with the properties of the experimentally observed neutron. The effect of this polarization increases with the $\bar{\nu}$ charge-to-mass ratio, so these considerations make it at least qualitatively understandable why the proton binding energy increases as q/m_o decreases, as seen clearly from the result of Fig. 1.

There is another important aspect of the proton binding process yet to be discussed, however, namely the influence of the exponential damping factors in the XBPS Hamiltonian (Table 1 of Ref. [2]). These factors play a decisive role in obtaining an energy minimum for the $e^- \bar{\nu}$ binary (cf. Figs. 4-5 of Ref. [1]), but they also have a subtle influence on the proton's interactions. As we have already seen, the Breit-Pauli terms of greatest importance for the proton are the spin-same-orbit and Darwin interactions with e^- and $\bar{\nu}$ which are multiplied by the $(q/m_o)^2$ factors of the latter particles only. In these cases the corresponding damping factors are totally independent of the proton's momentum (Table 1 of Ref. [2]). By decreasing the size of the proton's orbital it is therefore possible to increase the magnitude of the un-damped Breit-Pauli expectation values, with their net attractive influence, without producing a corresponding decrease in the magnitude of the related exponential damping factors. As long as the charge distributions of the electron and antineutrino do not change at the same time, such an increase in the proton momentum thus generally leads to a net increase in the attractive contribution of the damped Breit- Pauli interactions to the proton-bonded system.

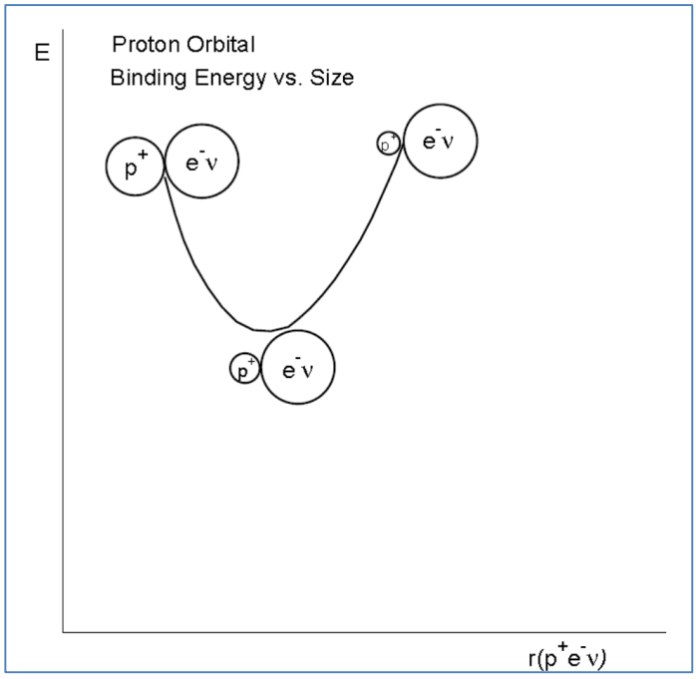

Fig 2: Schematic diagram showing the relationship between the size of the proton orbital and the stability of the $p+e^- \bar{\nu}$ neutron system. The large mass of the proton *vis-a-vis* the $e- \bar{\nu}$ species allows it to assume a relatively contracted charge distribution which helps to maximize the effect of its attractive short-range interactions with the lighter particles without greatly increasing its own kinetic energy, thus leading to the total energy minimum shown.

The possibilities are not unlimited for the proton, however, because its kinetic energy also increases as its charge distribution is made more compact (cf. Fig. 2). Because the Breit-Pauli terms vary as r^{-3} before damping effects are considered, while the proton kinetic energy increases only as $p^2 \approx r^{-2}$, it follows that in the interparticle distance range of interest ($r \approx \alpha^2$), the total binding energy at first increases with the proton's momentum. Eventually a point of diminishing returns is reached, however, because as the proton orbital shrinks in size, the values of the Breit-Pauli integrals for fixed e^- and $\bar{\nu}$ probability distributions begin to change more and more slowly, ultimately approaching the point-charge limit commonly employed in atomic calculations using the Breit-Pauli method. The proton kinetic energy continues to increase at roughly the same rate, however, and so at some point an optimum proton size is reached. Nonetheless, this effect represents a distinct advantage for the proton *vis-a-vis* lighter particles, and will be seen to play an increasingly more important role in the theory of nuclear binding which emerges from consideration of the properties of the XBPS Hamiltonian.

To illustrate the proton's tendency to assume a relatively more compact charge distribution than the electron and antineutrino, an additional series of calculations has been carried out in which two s-type Gaussian functions have been added to the original 2s,2p one-particle basis employed above. Optimization of the new exponents (α_1 and α_2) while holding those of the other functions fixed at their optimal values for the $e^-\bar{\nu}$ wavefunction shows that the proton prefers relatively high values for these quantities (2.56×10^8 and 8.0×10^7 a_0^{-2}). These values are 8.0 and 2.5 times greater than that of the largest exponent in the optimal 2s,2p $e^-\bar{\nu}$ basis, indicating a significant contraction of the proton wavefunction, as expected. The new basis functions succeed in lowering the total $p^+e^-\bar{\nu}$ energy by nearly 60000 hartree relative to the 2s,2p result given in Table 2. In order to be consistent with the general calibration procedures adopted earlier, it is necessary to readjust the respective value of the exponential damping constant A and the antineutrino q/m_o value so as to produce minimal energy results for the e^+e^- and $p^+e^-\bar{\nu}$ systems of the desired values. On this basis A is increased only slightly to a value of 1.0568 a.u. while the $\bar{\nu}$ q/m_o result is adjusted downward to 0.5375 a.u. A summary of the energy contributions for the $p^+e^-\bar{\nu}$ $1/2^-$ state found in this basis set are given in Table 3 for comparison with the corresponding 2s,2p data discussed first (Table 2). The shrinking of the proton charge distribution is most easily recognized from the magnitude of the kinetic energy obtained in the two treatments. This quantity increases by 79% as a result of the basis set expansion, as compared to relatively small changes in the corresponding values for

Table 3: Energy contributions (in hartree) of various operators (see Table 1 of Ref. [2] for definitions) and particle combinations for (a) the $1/2^-$ ground state of the $p^+ e^-\bar{\nu}$ neutron system and (b) the 0^- state of the isolated $e^-\bar{\nu}$ binary obtained by employing the 4s,2p basis with scale factor $\eta = 0.16$, exponential damping constant A = 1.0568 a.u. and antineutrino q/m_o value of 0.5375 a.u. for the XBPS Hamiltonian

a) Operator	**p^+e^-**	**$p^+\bar{\nu}$**	**$e^-\bar{\nu}$**	**Total ($p^+e^-\bar{\nu}$)**
Kinetic Energy	65280.208 (p^+)	1339330.432(e^-)	1478062.82 t(\j)	2882673.461
Coulomb	-5517.636	0.000	0.000	-5517.636
Spin-same-orbit	-250929.141	70545.115	-463958.811	-644342.837
Spin-other-orbit	-67.206	61. 902	-984257.224	-984262.528
Darwin Term	117676.768	-352977.701	15413.212	-219887.721
Orbit-orbit	-66.116	228.315	-492670.535	-492508.336
Spin-spin	-2.195	-0.350	-511881.100	-511883.645
Spin-spin δ	-0.474	-10.788	5302.617	5291.355
Total Energy				29562.111
b) Operator				Total ($e^-\bar{\nu}$)
Kinetic Energy				2759022.924
Coulomb				0.000
Spin-same-orbit				-475143.463
Spin-other-orbit				-1042436.569
Darwin Term				11692.897
Orbit-orbit				-536596.507
Spin-spin				-520943.516
Spin-spin δ				1286.444
Total Energy				196882.208

The electron and antineutrino. The net attractions of the proton spin-same-orbit and Darwin terms are also increased to 180384 and 235301 hartree respectively, representing a total of 101794 hartree additional binding to help offset the increase in proton kinetic energy. By construction, the total energies in both calculations are nearly identical, from which it is clear that the shrinking of the proton charge distribution is also paid for at the price of additional destabilization of the $e^-\bar{\nu}$ complex.

Another key point to consider in the present $p^+e^-\bar{\nu}$ calculations is the way in which the translational energy is treated. Because of the heavy mass of the proton, it follows

that the relativistic translational energy for a given $p^+e^-\bar{\nu}$ wave-function is much smaller than that for an equivalent e^+e^- species. We have seen in Sect. 3.4 of Ref. [2] that the use of $\langle E_o \rangle = \langle H-T \rangle$ as criterion for the basis set optimizations leads to a notably higher damping constant A (1.7725 a.u. in the 5s, 5p basis) than when only $\langle H \rangle$ is used (1.0775 a.u.). If the higher of these A values is used in the $p^+e\bar{\nu}$ calculations (or the corresponding value for the 2s, 2p basis), it is impossible to obtain a low enough total energy for this system to satisfy the requirement that its rest mass be equal to that of the neutron. This situation is an artifact of the small (2s, 2p) basis employed, however, because the exact solutions of the XBPS must be eigenfunctions of both H and T and the lowest total energy eigenvalue must correspond to a vanishing translational energy. Under the latter condition there would be no need to distinguish between the two different optimization procedures employed above, since they would necessarily lead to identical results ($E_{min} = E_o$), At this stage of development, however, it is necessary to deal with the fact that the approximate wavefunctions generated at the present level of treatment invariably have large expectation values of the translational momentum operator for the system at hand, at least when such compact charge distributions are involved as are known to characterize the internal structure of nuclei.

Under the circumstances, the decision to focus on the expectation value of the total energy in comparing the stabilities of systems containing different numbers of protons must carry with it the recognition that this choice favors heavier particles over lighter ones. Another way of seeing this is to recall that the value of the exponential damping constant A must be significantly smaller than variational arguments indicate it should be in order to obtain total energies for the particle-antiparticle binary systems which correspond to binding energies of $2m_oc^2$. As the quality of the basis set improves, the effective value of A must increase to maintain this condition until it ultimately assumes the ideal result corresponding to the exact solution of the XBPS for the translation-less binary systems.

To illustrate this tendency calculations have been carried out for the e^+e^- system employing a 3s,2p,2d basis. They lead to a value for A of 1.2647 a.u., as compared to that of 1.0775 a.u. obtained with the 5s,5p basis. Since the proton binding energy in the $p^+ e^-\bar{\nu}$ calculations considered thus far has been shown to arise primarily from terms which contain the corresponding damping fact factors in which the constant A appears, it seems highly likely that the degree of binding will be strongly influenced by such developments.

As usual, the binding energy of the e^+e^- system is held at -37557.773 hartree in the new basis, for example, but the degree to which it or its $e^-\bar{\nu}$ analog can bind a proton can be expected to decrease as the value of the damping factor increases. This is tantamount to concluding that the computed proton binding energy for a given system will generally decrease as the level of theoretical treatment is improved within the XBPS model. Moreover, this expectation is also consistent with the discussion given earlier regarding the relationship between the stability of the $e^-\bar{\nu}$ complex and its susceptibility to polarization by neighboring protons. In other words, as the basis set is improved there is a growing tendency for the $e^-\bar{\nu}$ system to more strongly resist having its charge distribution altered relative to its isolated state.

There is a competing factor which tends to insure that the minimal $p^+e^-\bar{\nu}$ total energy can always be adjusted to the value corresponding to the neutron at rest, however, namely the dependence of this quantity on the q/m_o value assumed for the antineutrino as the quality of the one-partic1e basis is improved. In the present case it is found, for example, that it is necessary to increase this value to 0.63 a.u. to obtain the desired energy result to a suitable approximation. The corresponding energy contributions are given in Table 4, similarly as for the other basis sets discussed earlier. Since the Gaussian exponents for the 3s,2p,2d basis have been optimized for the e^+e^- system, the results of these calculations are most meaningfully compared to those employing the 2s,2p basis (Table 2), i.e. without the benefit of specially optimized proton s functions. As expected, the use of a significantly larger damping constant leads to a reduction in the net binding associated with the proton spin-same-orbit and Darwin terms (74427 and 70107 hartree respectively), 46% of the previous total. The proton's kinetic energy is correspondingly smaller in the 3s,2p,2d computations as well, 56% of the former value. The proton-electron Coulomb energy is also smaller, reflecting a general tendency to keep these two particles further apart on the average as a result of the addition of d functions to the basis set.

Table 4: Energy contributions (in hartree) of various operators (see Table 1 of Ref. [2] for definitions) and particle combinations for the $1/2^-$ ground state of the $p^+e\bar{\nu}$ neutron system obtained by employing the 3s,2p,2d basis with scale factor $\eta = 0.111$, exponential damping constant $A = 1.2648\ e^{-1}$ and aritineutrino q/m_o value of 0.63 a.u. for the XBPS Hamiltonian

Operator	p^+e^-	$p^+\bar{\nu}$	$e^-\bar{\nu}$	Total ($p^+e^-\bar{\nu}$)
Kinetic Energy	20354.545(p^+)	1068402.098(e^-)	1140129.945($\bar{\nu}$)	2228886.588
Coulomb	-3856.027	0.000	0.000	-3856.027
Spin-same-orbit	-94307.843	19879.681	-363207.789	-437635.951
Spin-other-orbit	-23.085	19.111	-757175.358	-757179.332
Darwin Term	48531. 942	-118639.460	8357.925	-61749.593
Orbit-orbit	-16.656	145.966	-559254.673	-559125.363
Spin-spin	-0.370	-0.176	-383654.253	-383654.799
Spin-spin δ	-0.179	-1.489	2345.886	2344.218
Total Energy				28029.737

Particularly when one considers the capacity of the proton to assume a more contracted charge distribution when extra functions are included in the basis which can be specifically optimized for it, it appears feasible to construct a suitably quantitative theory of nuclear binding on the basis of the model under discussion. At least these results show that it is far from proven that the "nuclear electrons" which Fermi spoke of in introducing his theory of beta decay[4] actually cannot exist within the confines of a bound nucleus. Before discussing further computations employing the XBPS Hamiltonian, however, it is well to consider other experimental information regarding the neutron which has been claimed to lend support to the hypothesis of the disappearing electron.

Comparison of the Properties of the $p^+e^-\bar{\nu}$ System with Those Known for the Neutron

The calculations discussed above have been suggested by the model of a neutron as a composite system formed by its known decay elements. The lowest-energy $p^+e\bar{\nu}$ system is found to be a doublet, consistent with what is known for the neutron. As a product of three fermions, it is a fermionic system itself, in agreement with Pauli's original interpretation of beta decay processes [5]. The charge distribution found to be optimal for the $p^+e^-\bar{\nu}$ system corresponds to nuclear dimensions, in fulfillment of another obvious requirement, but more extensive calculations are highly desirable in order to be more quantitative on this point. A three-way partnership is suggested in forming the neutron which is reminiscent of the excimer concept in molecular physics. The fact that the system corresponds to a local energy minimum which lies above that of its separated particles is clearly consistent with the known meta-stability of the neutron. This aspect of nuclear binding (weak interaction) will be taken up in more detail in following work dealing with the structure of the deuteron.

A key to the binding of the three particles together is the assumption of a positive q/m_o value for the antineutrino, but one with a smaller absolute value than that of the electron. One has the picture of an $e^-\bar{\nu}$ system which is an imperfect copy of the e^+e^- mass-less binary first discussed. Its relative instability makes it more attractive to the proton than e^+e^- itself, and in effect the proton binding that results can be looked upon as an attempt to compensate for what is otherwise missing in the $e^-\bar{\nu}$ bond *vis-a-vis* either of its e^+e^- or $\nu\bar{\nu}$ counterparts. The electrical neutrality of the antineutrino guarantees that the $p^+e^-\bar{\nu}$ system has no net charge, but this brings us to a far more delicate matter. The neutron possesses a magnetic dipole moment which is negative and of the order of the nuclear Bohr magneton [5, 6]. As such it is far smaller in absolute magnitude than what one would expect for a system containing an electron.

To examine this point it is helpful to consider an experiment in which the hypothetical $p^+e^-\bar{\nu}$ system is subjected to a magnetic field in order to determine this quantity. If we assume that the magnetic moment of the combined system is equal to the algebraic sum of the individual moments of its constituents, the measured value must be expected to be nearly equal to that of the electron alone. This assumption works quite well in dealing with molecular properties and has also been found to be acceptable for nuclei, as for example in the comparison of the deuteron's magnetic moment with those of its constituent proton and neutron [7]. If we look more closely at the way magnetic moments are measured, however, we note that for this additivity principle to hold in the $p^+e^-\bar{\nu}$ case, it is essential that the electron be just as apt to rotate in response to the torque applied by a magnetic field in this tri-atomic system as it is in its free state, or alternatively when it is weakly bound to a positive atomic or molecular ion. Especially since the present calculations indicate that the binding process requires a particular spin orientation between all three particles in the optimum $p^+e^-\bar{\nu}$ resonance state, it seems far from obvious that the above condition is fulfilled in the present instance. In order for the electron spin to change its orientation, it seems much more likely that the system as a whole must rotate, and that would mean that a much larger mass is involved than for a lightly bound electron located in an outer shell of an atom. A similar situation is well known in the study of the Compton effect [8], for example. The modified wave observed in x-ray radiation from atoms is observed only when an essentially free electron is involved in the interaction, in which case the change in wavelength $\Delta\lambda$ is found to be inversely proportional to the electronic mass. When an inner-shell electron does the scattering, however, it is as if the whole atom is involved in the interaction. Consequently, the Compton scattering law indicates the production of an essentially unmodified wave, i.e. $\Delta\lambda \sim 0$. The calculations discussed above suggest that the electron in the $p^+e^-\bar{\nu}$ resonance system is very tightly bound to both the proton and the antineutrino constituents, and thus that the model of an inner-shell electron in the preceding analogy is far more appropriate.

On this basis one is led to expect that the $p^+e^-\bar{\nu}$ system has a magnetic moment of an extremely heavy electron, i.e. negative in sign, but of the order of a nuclear bohr magneton, which is at least approximately what is observed for the neutron [6, 9]. The additivity principle should not work at all well in this case because the interaction of the particle spins is intimately involved in the internal bonding process, unlike the case when predominantly Coulomb interactions are involved. In subsequent work we will come back to this point when we discuss the muon magnetic moment, which is almost exactly what one would expect for an electron having this particle's rest mass. The overall situation is complicated for the proton and neutron, however, because of the pion cloud known to surround each of them [10-13]. The observed neutron magnetic moment is roughly double [6, 9, 12] that which one would expect from the above "clamped-electron" model, but such a discrepancy is at least potentially understandable in terms of the environment otherwise assumed for the neutron. Moreover, the fact that high-energy electron scattering experiments [14] have indicated elastic form factors which correspond to a small but not necessarily vanishing neutron charge radius are also seemingly consistent with the present model's assumed tri-atomic composition for this neutral system.

Conclusion

The wave-function corresponding to the experimental neutron binding energy is relatively easy to analyze because the proton is seen to occupy a single s-type orbital almost exclusively in the small basis employed. It is multiplied with the singlet (αß-ßα) combination for $e^-\bar{\nu}$ which is very reminiscent of the 0^- state preferred by the e^+e^- system. The M_J=-1/2 component of the $1/2^-$ state can be obtained by inverting all α (and ß spins), which in effect means that the resulting proton with ß spin is then bound to the same 0^- $e^-\bar{\nu}$ structure as before.

The known q/m_o value of the proton is so small as to rule out its participation in XBPS interactions in the range of interparticle separations typical for nuclear binding for which this quantity appears as a coupling constant. This circumstance practically eliminates any chance of such Breit-Pauli terms as the spin-spin, orbit-orbit and spin-other-orbit interactions being directly involved in the binding of protons, but it leaves open a distinct possibility that the spin-same-orbit and Darwin terms with electrons and antineutrinos can affect them in a significant manner. One knows from observations of atomic spectra, for example, that the electron undergoes a spin-same-orbit interaction with the nucleus which does not depend on the mass of the heavier particle, but rather on the square of the mass of the electron itself. Preliminary calculations with the XBPS model demonstrate that such a proton-electron interaction can be enormously attractive in the desired range of inter-particle separation. The opposite effect is found for the proton-antineutrino spin-same-orbit interaction, but the calculations demonstrate that the most stable state of the $p^+e^-\bar{\nu}$ system adjusts its charge distribution to obtain a significant net binding between the proton and the $e^-\bar{\nu}$ complex.

In addition to this polarization effect, another dominant factor responsible for proton binding is its attractive Darwin term interaction with the antineutrino, which easily outweighs the effect of the corresponding proton-electron repulsive contribution. By decreasing the q/m_o value of $\bar{\nu}$ below that of the positron, it is found that the total energy of the most stable state of the $p^+ e^-\bar{\nu}$ system can be adjusted to become equal to that known experimentally for the neutron, i.e. 28781.31 a.u. higher than that of its separated component particles. This occurs for a q/m_o value of +0.63 a.u. for the antineutrino in a 3s2p2d basis, i.e. about two-thirds that of the electron. A value of unity for this quantity gives a much lower energy for the $p^+e^-\bar{\nu}$ system, showing it to be bound by 62528.34 a.u. relative to its separated constituents (employing the same basis), a change of 91309 a.u. or 2.48 MeV, so it is clear that these results are quite sensitive to the numerical value chosen for this charge-to-rest-mass ratio.

The last comparison offers a clear perception of the mechanism of proton binding in the present model. The $e\bar{\nu}$ complex can be looked upon as a clone of the prototype e^+e^- massless binary, especially when q/m_0 for $\bar{\nu}$ is assumed to be the same as for the positron. Because of the nature of the coupling constants in the spin-same-orbit and Darwin terms in the XBPS Hamiltonian, the proton can be strongly attracted to $e^-\bar{\nu}$, especially if the charge-to-rest -mass ratio of $\bar{\nu}$ is relatively large. Reducing the value of this quantity below unity necessarily causes the stability of the $e^-\bar{\nu}$ complex to be lessened, but at the same time it increases the magnitude of the net proton binding energy to this system. The symmetry of the resulting $p^+e^-\bar{\nu}$ metastable state is consistent with this interpretation. The proton finds itself predominantly in an $s_{1/2}$ orbital, while the $e^-\bar{\nu}$ species prefers the 0^- character exhibited by both e^+e^- and $\nu\bar{\nu}$ in their respective lowest states, giving an overall $1/2^-$ symmetry to the resulting $p^+e^-\bar{\nu}$ wave function.

There is an indication that the computed proton binding energy is overestimated in the computational treatment employed, since improvements in the AO basis tend to make it more difficult for the proton to polarize the $e^-\bar{\nu}$ charge distribution. On this basis it can be expected that the q/m_o value for the antineutrino needed to give the experimental neutron binding energy in the exact XBPS treatment is somewhat larger than found in the best calculations carried out in the present study. Moreover, it seems likely that as the one-particle basis becomes more flexible, that the description of the tight-binding phanton states as well as that of the $e^-\bar{\nu}$ complex will allow more effective use of the short-range Breit-Pauli attractive interactions, thereby drawing the two component fermions somewhat closer together at an improved level of treatment. This conclusion is consistent with the results of the different e^+e^- optimizations discussed in Ref [2]. for which the influence of translational energy is explicitly taken into account.

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