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Rebuttal of Fermi's denial of nuclear electrons: Part II: Adapting the XBPS method to the description of the Neutron's internal structure

Robert J Buenker

Abstract
The assertion that no potential can exist which is capable of holding an electron within the small volume of a neutron, with the requisite 500 MeV kinetic energy expected for it on the basis of the de Broglie p=h/λ relation, is considered in detail. The present work assesses this claim on the basis of recent theoretical developments which make use of the exponentially damped Breit-Pauli-Schrödinger (XPBS) equation. It is pointed out that the known extreme penetrating power of the neutrino does not constitute proof that the particle must possess a null charge-to-mass ratio. The XBPS Hamiltonian operator is thereupon adapted so as to include potentially significant short-range interactions involving the neutrino and its anti-particle. Calculations are presented which show that a neutrino charge-to mass ratio which is within the 0.5-0.7 a.u. range lead to a minimum energy for the neutron which is in good agreement with the observed value of +0.7825 MeV relative to that of its constituent particles (proton, electron and anti-neutrino) separated to infinity.

Keywords: Neutron composition, neutrino properties, creation-annihilation hypothesis, Exponentially-Damped Breit-Pauli-Schrödinger (XBPS) equation

Introduction
In preceding work [1], it has been argued that a theoretical model based on the exponentially damped Breit-Pauli-Schrödinger (XBPS) equation might be capable of explaining how electrons can be confined to the small radius of a nucleus. This supposition is supported in large part by the fact that calculations [2, 3] employing the same model have demonstrated that an electron and positron can be bound to one another so strongly that they form a mass-less system which can reasonably be identified with the photon. To accomplish this objective, it is clear that neutrinos must have charge-to mass ratios that allow them to participate significantly in the Breit-Pauli short-range interactions such as spin-orbit coupling and the Darwin term. How this can be possible given the capacity of neutrinos to pass nearly undetected through matter for very long distances is an obvious question that needs to be dealt with in a convincing manner.

Extending the XBPS Model to Neutrino Interactions
In Sect. 6 of Ref. [1], it was remarked that if the rest mass of the neutrino $m_{0\nu}$ is exactly zero, its charge-to-rest-mass ratio might still be non-vanishing by virtue of its lack of electric charge. By providing the XBPS Hamiltonian given in Table 1:

Designation		Operator
Relativistic Kinetic Energy (one-particle only	KE	$\left(p_i^2\alpha^{-2}+m_{0i}^2\alpha^{-4}\right)^{\frac{1}{2}}-m_{0i}\alpha^{-2}$
Coulomb	C	$q_iq_jr_{ij}^{-1}$
Spin-same-orbit (exponentially damped)	SsO	$-\frac{\alpha^2}{2}G(i,j)$ $\times\left\{\left(\frac{q_i}{m_{0i}}\right)^2\exp\left(-2A\alpha^2\left\lvert\frac{q_i}{m_{0i}}\mathbf{p}_i\right\rvert\right)\times\left(\mathbf{r}_{ij}\times\mathbf{p}_i\cdot\mathbf{s}_i\right)r_{ij}^{-3}\right.$ $\left.+\left(\frac{q_j}{m_{0j}}\right)^2\exp\left(-2A\alpha^2\left\lvert\frac{q_j}{m_{0j}}\mathbf{p}_j\right\rvert\right)\times\left(\mathbf{r}_{ji}\times\mathbf{p}_j\cdot\mathbf{s}_j\right)r_{ij}^{-3}\right.$, $G(i,j)=\begin{cases}1, & \text{if } \frac{q_iq_j}{m_{0i}m_{0j}}>1\\ -1, & \text{if } \frac{q_iq_j}{m_{0i}m_{0j}}<1\end{cases}$
Spin-other-orbit (exponentially damped)	SoO	$-\alpha^2\left(\frac{q_i}{m_{0i}}\right)\left(\frac{q_j}{m_{0j}}\right)$ $\times\exp\left(-A\alpha^2\left\lvert\frac{q_i}{m_{0i}}\mathbf{p}_i\right\rvert\right)\exp\left(-A\alpha^2\left\lvert\frac{q_j}{m_{0j}}\mathbf{p}_j\right\rvert\right)$ $\times\left(\mathbf{r}_{ji}\times\mathbf{p}_j\cdot\mathbf{s}_i+\mathbf{r}_{ij}\times\mathbf{p}_i\cdot\mathbf{s}_i\right)r_{ij}^{-3}$
Darwin Term (exponentially damped)	D	$-\pi\frac{\alpha^2}{2}G(i,j)\delta(r_{ij})$ $\times\left\{\left(\frac{q_i}{m_{0i}}\right)^2\exp\left(-2A\alpha^2\left\lvert\frac{q_i}{m_{0i}}\mathbf{p}_i\right\rvert\right)\right.$ $\left.+\left(\frac{q_j}{m_{0j}}\right)^2\exp\left(-2A\alpha^2\left\lvert\frac{q_j}{m_{0j}}\mathbf{p}_j\right\rvert\right)\right\}$,
Orbit-orbit (exponentially damped)	OO	$-\frac{\alpha^2}{2}\left(\frac{q_i}{m_{0i}}\right)\left(\frac{q_j}{m_{0j}}\right)$ $\times\exp\left(-A\alpha^2\left\lvert\frac{q_i}{m_{0i}}\mathbf{p}_i\right\rvert\right)\exp\left(-A\alpha^2\left\lvert\frac{q_j}{m_{0j}}\mathbf{p}_j\right\rvert\right)$ $\times\left[\left(\mathbf{p}_i\cdot\mathbf{p}_j\right)r_{ij}^{-1}+\left(\mathbf{r}_{ij}\cdot\left(\mathbf{r}_{ij}\cdot\mathbf{p}_i\right)\mathbf{p}_jr_{ij}^{-3}\right)\right]$
Spin-spin (exponentially damped)	SS	$-\alpha^2\left(\frac{q_i}{m_{0i}}\right)\left(\frac{q_j}{m_{0j}}\right)$ $\times\exp\left(-A\alpha^2\left\lvert\frac{q_i}{m_{0i}}\mathbf{p}_i\right\rvert\right)\exp\left(-A\alpha^2\left\lvert\frac{q_j}{m_{0j}}\mathbf{p}_j\right\rvert\right)$ $\times\left[\left(\mathbf{s}_i\cdot\mathbf{s}_j\right)r_{ij}^{-3}+3\left(\mathbf{r}_{ij}\cdot\mathbf{s}_i\right)\left(\mathbf{r}_{ij}\cdot\mathbf{s}_j\right)r_{ij}^{-5}\right]$

Spin-spin δ (exponentially damped)	SSδ	$-\frac{8\pi\alpha^2}{3}\left(\frac{q_i}{m_{0i}}\right)\left(\frac{q_j}{m_{0j}}\right)$ $\times\exp\left(-A\alpha^2\left\lvert\frac{q_i}{m_{0i}}\mathbf{p}_i\right\rvert\right)\exp\left(-A\alpha^2\left\lvert\frac{q_j}{m_{0j}}\mathbf{p}_j\right\rvert\right)$ $\times\mathbf{s}_i\cdot\mathbf{s}_j\delta(\mathbf{r}_{ij})$

Table 1. Definition of quantum mechanical operators present in the exponentially damped Breit-Pauli Hamiltonian employed throughout the present study (α is the fine-structure constant; atomic units employed throughout) [4]. The indices i and j are used generically to represent two interacting particles; the quantities q_i (q_j) and m_{oi} (m_{oj}) are the electric charges and rest masses of the i^{th} (j^{th}) particle, A is the exponential damping constant, and $\mathbf{p_i}$ ($\mathbf{p_j}$), $\mathbf{s_i}$ ($\mathbf{s_j}$) and $\mathbf{r_{ij}}$ are the standard vectorial symbols for the linear and spin angular momenta of a single particle and the distance between the i^{th} and j^{th} particles, respectively.

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Note that there is an error in the original version of this table [4] namely the exponential factors appear more often than above.

with such q/m_0 factors in the various exponentially damped Breit-Pauli terms, it is then conceivable at least in theory for the neutrino to undergo attractive interactions with other particles. In the present section, we will test this possibility by means of explicit calculations.

The first question that arises is clearly what value to use for $q_\nu/m_{o\nu}$. Consideration of the scaling arguments discussed in Ref. [3] shows, however, that this choice is not critical for the $\nu\bar{\nu}$ system itself. If we simply assume that the charge-to-mass ratios for the neutrino and antineutrino are equal but of opposite sign, we obtain a Hamiltonian (Table 1) with the following representative terms:

$$H(p,r,q_\nu,m_{o\nu}) = p\alpha^{-1} - \left(\frac{q_\nu}{m_{o\nu}}\right)^2 \alpha^2 r^{-3} F(p,q_\nu,m_{o\nu}) \quad . \qquad \text{II.1}$$

This operator is identical in form to that employed in Ref. [3] except that all the terms with factors of q_i or m_{oi} which do not appear as ratios of one another are missing because of the assumed vanishing magnitudes of the neutrino's charge and rest mass.

With this special form of the XBPS Hamiltonian it is possible to scale the coordinates in a slightly different manner than before, namely with

$$p' = \frac{q_\nu}{m_{o\nu}}\left(\frac{Q}{M_o}\right)^{-1} p$$

and

$$r' = \left(\frac{q_\nu}{m_{o\nu}}\right)^{-1}\frac{Q}{M_o} r \quad , \qquad \text{II.2}$$

so as to obtain the following result:

$$H(p,r,q_\nu,m_{o\nu}) = \left(\frac{q_\nu}{m_{o\nu}}\right)^{-1}\frac{Q}{M_o}$$

$$x\left(p'\alpha^{-1} - \frac{Q^2}{M_o^{\,2}}\alpha^2 r'^{-3} F(p',Q,M_o)\right) \qquad \text{II.3}$$

$$= \left(\frac{q_\nu}{m_{o\nu}}\right)^{-1}\frac{Q}{M_o} H(p',r',Q,M_o)$$

where H (p′, r′, Q, M_o) is the corresponding Hamiltonian in the primed coordinate system for particles with a different value of $|q/m_o|$. The operations are exactly the same as in the mass scaling procedure of Ref. [3], but the relative charge-to-mass ratios are involved instead of relative masses.

It must be recalled, however, that the simple relation between the two different Hamiltonians in this case only holds for $q_\nu = m_{o\nu} = 0$. Under these conditions the lowest possible energy eigenvalue for the $\nu\bar{\nu}$ system, i.e. $2m_{o\nu}c^2$. is exactly zero as well. As a result, a wave-function ψ(r) satisfying the corresponding Schrödinger equation with this eigenvalue can be converted by means of the above coordinate transformation [eq. II.2] into an eigenfunction

$\psi(r')$ of the primed Hamiltonian, which by virtue of the above scaling property also possesses the desired vanishing energy. In short, if we can find a Hamiltonian of this type which gives E = 0 as the lowest eigenvalue for the $\nu\bar{\nu}$ system with a given $|q_\nu/m_{o\nu}|$ value, we can repeat the process for any other charge-to-mass ratio. It can be noted that for this situation to hold, it is essential that the argument of the damping exponential operator F also contain a q/m_o factor.

The next point that merits discussion is whether it makes sense to talk of a bound system with the same energy as its separated products, in this case E=0. The answer is clearly yes as long as the combined and dissociated systems are separated by a reasonably large energy barrier.

With reference to Fig. 1 of Ref. [3] for the e^+e^- system, it is easy to construct an analogous total energy curve for the $\nu\bar{\nu}$ system which satisfies this requirement (Fig. 1). Because of the absence of the long-range Coulomb interaction in this case, we must expect that the energy first goes up as the neutrino and antineutrino approach each other from a long distance ($r^{-1}=0$ in the diagram). The kinetic energy itself rises more slowly with decreasing separation than for the e^+e^- system because it varies linearly with momentum for systems with no rest mass. As a result, one doesn't expect the atomic-like potential minimum for the e^+e^- system in this case, but rather that the energy should increase steadily until very short inter-particle separations of roughly $r \cong \alpha^{3/2}$ (for $q_\nu/m_{o\nu} = 1.0$ a.u.). A maximum seems likely at that point, similarly as for e^+e^-, because the Breit-Pauli attractive short-range interactions start to change more rapidly than the kinetic energy. After this point it is necessary that the exponential damping halt the attractive tendency when the total energy has again reached the value of zero. The kinetic energy begins to dominate once more at still smaller separations, so that a potential well can be formed in which the $\nu\bar{\nu}$ system can exist indefinitely in the absence of external forces.

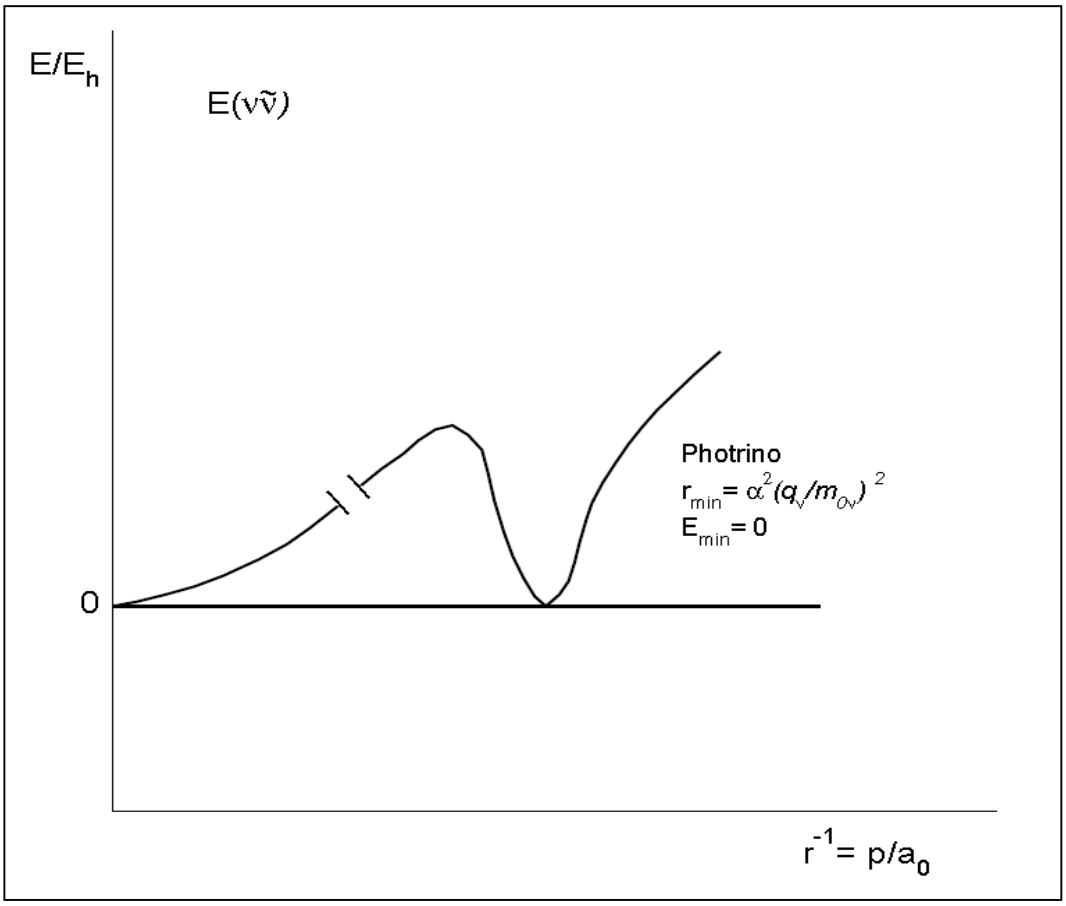

Fig 1: Schematic diagram showing the proposed variation of the $\nu\bar{\nu}$ system's internal energy as a function of the reciprocal of the distance between the two constituent particles. Only one minimum is expected, in contrast to the e^+e^- case shown in Fig. 1 of Ref. [3], at which point the total energy vanishes exactly, i.e. corresponding to a binding energy of $2m_{o\nu}/c^2$ for the tightly bound $\nu\bar{\nu}$ (photrino) system.

The present scaling argument tells us that it really does not matter what value is chosen for $|q_\nu/m_{o\nu}|$ to demonstrate this effect, but to obtain the most straightforward possible comparison with the e^+e^- system, it is reasonable to take it to be unity. If the former 5s,5p basis is employed (cf. Ref. [3]) in the corresponding XBPS calculation it is found that the zero-energy minimum occurs for a damping constant A which is only slightly smaller than the corresponding e^+e^- and p^+p^- value (Fig. 2). This result is easily understood if one uses the optimum scale factor and A value derived from the e^+e^- calculations, for which the lowest-energy eigenvalue is $-2m_{oe}c^2$. This means that the damped Breit-Pauli terms (Table 1 of Ref. [3]) will have the same magnitudes for the $\nu\bar{\nu}$ system (with $|q_\nu/m_{o\nu}|=1.0$ a.u.) as for the e^+e^- system. The Coulomb component is missing, however, as are the m_oc^2 terms in the kinetic energy. The two electronic $m_{oe}c^2$ terms appearing with minus signs in the kinetic energy expression for e^+e^- make up the entire difference in the assumed binding energies of the two systems, so we can effectively ignore them in the following discussion.

As a result, there are only two effects remaining which can cause a non-zero binding energy for $\nu\bar{\nu}$ when employing

the same damping constant as before for e^+e^-. One of these favors the latter system (Coulomb attraction), while the other favors $\nu\bar{\nu}$ [i.e. pc vs. $(p^2c^2 + m^2c^4)^{1/2}$]. Of these, the Coulomb effect is larger in magnitude for the inter-particle distance range in question, but the energy difference (2680 hartree) is relatively small compared to the e^+e^- binding energy (37557.73 hartree). Thus to obtain an exactly zero binding energy value for the $\nu\bar{\nu}$ system in the same basis set, it is only necessary to lower the value of the damping constant A by 0.0007 a.u..

This result ignores translational effects, however, and so it is interesting to carry out the optimizations based on the E_o expectation value, <H>-<T>, rather than on that of the total energy alone. When this is done the required value of A must increase in order to obtain the same binding energy as before with e^+e^-. If the same value of A (1.7725 a.u.) is employed which gives the correct $2m_{oe}c^2$ binding energy value of -37557.7 hartree for e^+e^- using the $<E_o>$ criterion, a total internal energy of 897.2 hartree results for $\nu\bar{\nu}$, which is about three times smaller than in the analogous treatment employing the optimal A value (1.0775 a.u.) for e^+e^- based on the <H> computations.

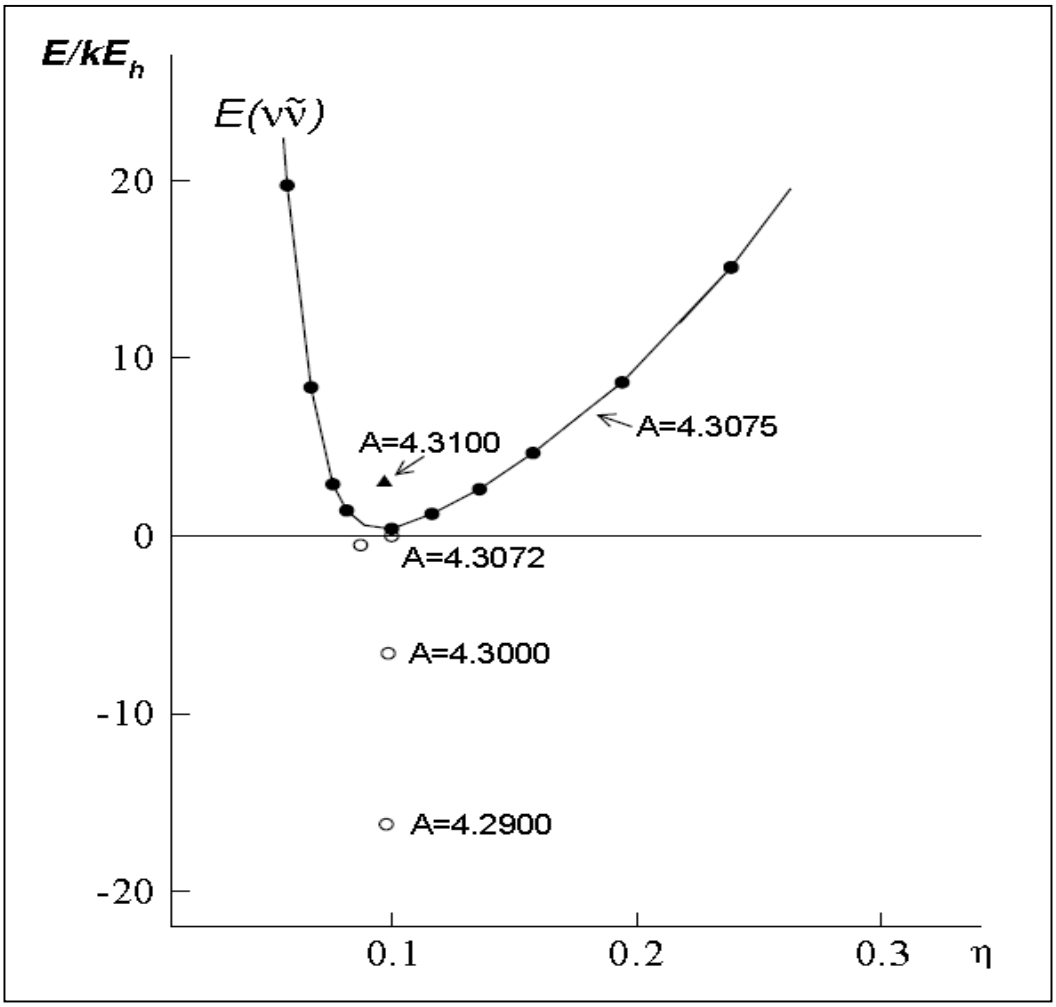

Fig 2: Variation of the computed total energy (in khartree) of the $\nu\bar{\nu}$ system as a function of the 5s,5p basis set scaling factor η in the XBPS treatment for various values of the damping constant A. The horizontal line at the center of the diagram corresponds to the null rest energy ($2m_{o\nu}c^2$) of the system. A value of A is sought which leads to this energy result for the optimum choice of η. Results for several other A values are also shown for comparison. Note that the values of A in the diagram are 4.0 times larger than those given in original text due to a difference in definition.

In view of the fact that the present full CI calculations fall short of providing exact solutions of the Schrödinger equations under consideration, it seems justified to conclude that for all practical purposes the same damping constant in the XBPS Hamiltonian leads to minimal binding energies for the three particle-antiparticle binary systems studied which are each equal to $-2m_oc^2$, in accordance with the expectations of STR. This result is exact for e^+e^- and p^+p^-, but only accurate to a good degree of approximation for the $\nu\bar{\nu}$ system based on the calculations carried out thus far. Moreover, the second scaling theorem discussed above [eq. II.3] shows that this situation holds for any choice of the charge-to-mass ratio assumed for the neutrino. The latter finding thus offers a means of describing more than one kind of neutrino in the present model, consistent with what has been inferred from the results of various experiments[5] and other theoretical considerations[6] (see Sect. 3 of Ref. [1]).

Extreme Penetrating Power of Neutrinos

The total energy curve of Fig. 1 for a $\nu\bar{\nu}$ interaction derives its attractive characteristics from the short-range Breit-Pauli terms in the XBPS Hamiltonian by virtue of an assumed non-zero charge-to-mass ratio for the constituent particles. This assumption needs to be reconciled with the observation that neutrinos have essentially no magnetic moment. In at

least one sense the total energy variation shown in Fig. 1 is consistent with the experimental findings related to this issue, however, namely with the lack of ionization exhibited by neutrinos and especially their ability to penetrate essentially unhindered through dense matter [7].

For a particle to cause ionization it is necessary that it be attracted to an electron (or other charged system) to a close proximity. Because of the lack of a Coulomb interaction, the total energy for the hypothetical $\nu\,\bar{\nu}$ system should rise as the two constituents come closer together. Before the attractive Breit-Pauli terms can reverse this trend, it is necessary that the neutrinos approach each other very closely, to at least $r = \alpha$. The possibility of undergoing a long-range attractive force thus clearly distinguishes electrons (cf. Fig. 1 of Ref. [3]) from neutrinos (Fig. 1). If we look upon ionization as requiring some kind of orbital motion for the colliding (point) particles, this distinction can be crucial since it means that the centrifugal force must exceed any opposing attractive force in the $e^-\bar{\nu}$ or $\nu\,\bar{\nu}$ interactions at all but extremely small inter-particle separations. In this case, a hyperbolic trajectory would be expected almost universally. There is always a large centrifugal barrier which needs to be overcome, and the particles must come very close to one another before this is feasible, even if a fairly large q/m_o value is assumed for the neutrino.

An automobile with such a total energy profile could race down the highway with reckless abandon, certain to be repelled whenever it came very close to any other object. The only danger would be that it might approach something so closely at extremely high speed that the short-range attractive force would finally be able to overcome the repulsive force. Such a picture is very reminiscent of the behavior of neutrinos in the Reines-Cowan experiment,[7] in which an extremely small cross section for neutron formation of $\sim 10^{-43}$ cm^2 is observed.

The conclusion that a non-zero q/m_o value necessarily implies a non-zero magnetic moment would also seem to overlook the possibility that the mass of a neutrino is only zero at rest. The ratio of the charge to the *relativistic* mass is a more relevant quantity in determining the results of magnetic interactions, and this is exactly zero whenever the neutrino is in motion because no corresponding change in its electronic charge occurs as a result. The dominant formula used in describing cyclotron dynamics, $p = mv = qBr$, leads to the conclusion that a charge-less particle cannot achieve orbital motion with a finite radius, for example. The key assumption in formulating the XBPS Hamiltonian is that the coupling constants are ratios of charge to rest mass, and without this provision the short-range minimum in either Fig. 1 or 2 would not be possible. In general, it should not be forgotten that virtually everything one knows about electromagnetism stems from experiments with individually charged particles (see also Sect. 6 of Ref. [1]), even if the macroscopic system being observed possesses zero *net* electric charge.

Support for this view also comes from the classical experimental result that the magnetic force is no more short-ranged than its electric counterpart. The force per unit length on an electric wire, for example, varies as the product of its current and that of a neighboring wire and is inversely proportional to the first power of the distance between them.[8] There is little hint of a short-range effect in this behavior. The theoretical explanation for this result lies in the (nearly) constant velocities of the electrons involved in such experiments, which implies that the quantity $mvr/r \equiv l/r$ is constant as well. In quantum mechanical interactions the more typical situation is that l (or s) is constant, so that an energy term of order l^2r^{-3} varies as the inverse cube of the inter-particle distance rather than as r^{-1}. The fact that particles don't always move at constant velocities opens up the possibility that related short-range interactions also exist, however, as suggested by the appearance of the Breit-Pauli operators [9]. For the present, however, we can conclude that a non-zero charge-to-rest-mass ratio for the neutrino does offer a way of rationalizing its extreme penetrability through dense matter, while at the same time leaving open the possibility of its undergoing a significantly strong interaction at very short inter-particle separations.

Results of Calculations for Other Binary Systems

There are 21 distinct binary systems that can be formed from the proton, electron and neutrino and their respective antiparticles. Of these, three are of the particle-antiparticle type already discussed, while the other 18 divide into pairs related to one another by charge conjugation. One such example is p^+e^-, which has the same Hamiltonian as p^-e^+ by virtue of the fact that the charges of the respective constituents appear as products of one another in all interactions (or as an absolute value in the exponential damping functions). Six of the remaining pairs are diagonal cases involving two identical particles. No binding is expected as a result, in accord with the results of explicit calculations. The various Breit-Pauli terms are attractive for certain angular momentum states, so this is not such an obvious result. The strength of the attraction is never enough to outweigh the kinetic energy contributions, however. Another three pairs are easily dispensed with as well, namely $p^+\nu$, $p^+\,\bar{\nu}$ and p^+e^+ and their charge-conjugated partners. If we assume a negative q/m_o value for ν, the $p^+\nu$ system is seen to be quite similar to p^+e^-, at least as long as the absolute value of the charge-to-mass ratio is close to unity. In all cases it is assumed that the damping constant A in a given basis is the same as has been found in the e^+e^- calculations discussed in Ref. [3].

This leaves two such pairs of binary systems unconsidered, $e^-\nu$ and $e^-\,\bar{\nu}$ and their charge-conjugated positron systems. Of these, only the ones with q/m_o values of opposite sign for their constituents are interesting in view of the experience with the other binary systems. Because of the fact that electrons and antineutrinos commonly appear together as decay products of neutrons, it is tempting to associate a positive q/m_o value with $\bar{\nu}$. The similarity to e^+e^- is expected to be particularly great if this quantity is assumed to be unity, in which case the results for the e^+e^- system shown in Table 2 again become pertinent.

Table 2: Energy contributions (in hartree) of various operators (see Table 1 for definitions) for the 0_g^- ground state of the e^+e^- system obtained employing the 5s,5p basis with scale factor $\eta = 0.095$ and exponential damping constant A = 1.0775 a.u. for the XBPS Hamiltonian.

Operator	Expectation Value
Kinetic energy	1992262.978
Coulomb	-3233.894
Spin-same-orbit	-378340.098
Spin-other-orbit	-830993.650
Darwin Term	8889.119
Orbit-orbit	-407592.397
Spin-spin	-418648.776
Spin-spin δ	0.000
Total energy	-37656.721

Especially if the translational energy is excluded from consideration, it has been found above that e^+e^- and $\nu\bar{\nu}$ are characterized by very nearly the same wave-function in their respective lowest-energy states. The maximum binding energy of the $e^-\bar{\nu}$ system can thus be anticipated to fall midway between the corresponding values for e^+e^- and $\nu\bar{\nu}$. This expectation is very nearly fulfilled in explicit calculations with the XBPS Hamiltonian in which it is assumed that q/m_o=1.0 a.u. for the $\bar{\nu}$ species. A binding energy of -15872.194 hartree, i.e. nearly one-half of the e^+e^- binding energy, is computed when the same 5s,5p basis is employed which was found to be optimal for e^+e^- and $\nu\bar{\nu}$ (see Fig. 2).

Since it seems certain that no such bound $e^-\bar{\nu}$ system actually exists (it would correspond to the mutual annihilation of the two particles in the conventional description for particle-antiparticle pairs), it can be concluded that such a high value for the $\bar{\nu}$ charge-to-mass ratio is unacceptable. We can eliminate any computed binding with the electron by decreasing the above q/m_o value, however, and repeating the scaling optimization procedure for the corresponding $e^-\bar{\nu}$ system (while again maintaining the damping constant A at its previous value). The resulting minimal energies are plotted in Fig. 3 as a function of the assumed q/m_o value. There is a simple relationship between this quantity and the optimum scaling parameter η, namely they are found to be inversely proportional to one another. This result is consistent with the second scaling theorem discussed in Sect. 2, which shows that the optimal value of η is inversely proportional to the square of the $|q/m_o|$ value assumed for the neutrino in a treatment of the hypothetical $\nu\bar{\nu}$ system.

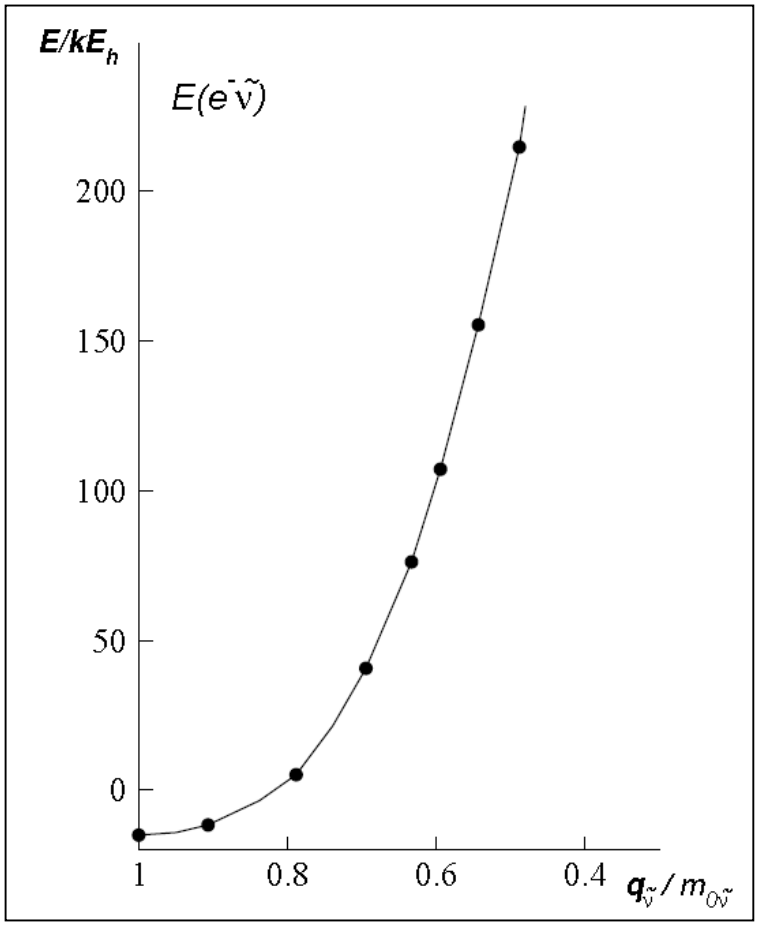

Fig 3: Variation of the minimum value of the computed total energy (in khartree) of the $e^-\bar{\nu}$ system (obtained by optimizing the scale factor η) in the XBPS treatment employing a 5s,5p basis set, given as a function of the antineutrino charge-to-rest-mass ratio q/m_0. The value of the

exponential damping constant A assumed throughout (1.0775 a.u.) is taken from the results of the analogous e^+e^- calculations in the same basis.

On the basis of the above considerations it appears likely that the $|q/m_o|$ value assumed for the neutrino and antineutrino cannot be greater than 0.7 a.u. and still avoid the prediction of a bound $e^-\bar{\nu}$ system in nature. A value exceeding zero by a good margin is still tenable on this basis, however, since it would allow for a $\nu\bar{\nu}$ system with a zero binding energy which is nonetheless separated by a large barrier from its dissociation products. The above scaling arguments show that the inter-particle distances involved must decrease as the value of $|q_\nu/m_{o\nu}|$ is lowered. It also should be noted that the symmetry of the lowest-energy $e^-\bar{\nu}$ state is also found to be 0^-, so that the analogy with the e^+e^- tight-binding state appears to retain its validity over a wide range of $|q_\nu/m_{o\nu}|$ values. There is a tendency to emphasize the $p_{1/2}$ character of the electronic function over $s_{1/2}$ as the antineutrino charge-to-rest-mass ratio decreases, as seems reasonable from the nature of the Breit-Pauli terms and the requirement of minimizing the total energy for the resultant $e^-\bar{\nu}$ system (the charge-conjugation symmetry is clearly missing for this binary system).

Calculations of the $p^+e^-\bar{\nu}$ System; Nuclear Binding in the XBPS Model

The questioning of the creation-annihilation hypothesis considered in Refs. [2, 3] has led to an alternative interpretation of particle-antiparticle interactions in terms of a Schrödinger equation whose Hamiltonian contains momentum-dependent short-range operators. According to this model the electron and positron can be bound so strongly to one another that there is a total loss of mass relative to their respective free-particle states. Since nuclear binding processes are well known to be accompanied by distinctions in the total masses of products and reactants, in accordance with the predictions of STR, it is natural to speculate that the same types of interactions might be involved in the e^-e^+ interaction. The exponential form of the attractive potentials employed in the XBPS Hamiltonian follows at least partially from this line of reasoning.

There is another similarity connecting these two types of phenomena as well, however, which again is tied up with the supposition that matter can be created and destroyed by the gain or loss of energy. As discussed in Ref. [2], the accepted view of the role of the electron and antineutrino in nuclear interactions is that they are created whenever a neutron decays but that they are not present in the bound nuclei themselves [10] The alternative interpretation which will be pursued in the present work is that the neutron is akin to a tri-atomic molecule, i.e. it is composed of a proton, an electron and an antineutrino. It has *negative* binding energy and is thus analogous to an excimer system commonly encountered in molecular physics studies. A key element in the ensuing theoretical model is that neutrinos are capable of being strongly attracted by other particles, but only at very short range. The $e^-\bar{\nu}$ model system treated in the last section is the starting point of this investigation.

The XBPS Hamiltonian (Table 1) has a single free parameter, the exponential damping constant A, and it has been fixed by the requirement that the binding energy of each of the three particle-antiparticle binary systems be $2m_oc^2$ in a full CI treatment employing a given one-particle basis set. Otherwise, all that is needed to completely define a system's Schrödinger equation are the electric charges and rest masses of the component particles. In the case of the antineutrino $\bar{\nu}$ these are assigned to be zero in each instance, but the possibility that their ratio has a non-zero value is left open as a means of explaining this particle's role in the nuclear binding process. The actual value for the antineutrino's charge-to-rest-mass ratio has not been specified as yet, primarily because it has been found that the requirement of a vanishing binding energy for the $\nu\bar{\nu}$ binary system can *be satisfied for any choice of this quantity's magnitude.* A potential solution to this problem is provided by the "tri-atomic hypothesis" for the neutron's structure, however. Accordingly, it will simply be required that the magnitude of the antineutrino's q/m_o value be such so as to lead to the experimental total energy of the neutron when solving the XBPS for the $p^+e^-\bar{\nu}$ system to which it is assumed to correspond in the present model. Specifically, the binding energy of the latter system should be equal to -28758 hartree (-0.7825 MeV). This value corresponds to the difference in the rest masses of the proton-electron combination and that of the meta-stable neutron.

The first series of calculations carried out to investigative this hypothesis employs the simple 2s,2p Gaussian basis mentioned in Sect. 3.3. of Ref. [3] (exponents of 2.0×10^8 and 1.0×10^8 a_o^{-2} in both cases). A scale factor η is then defined as before to be optimized so as to obtain the minimum energy possible for a given choice of $q_\nu/m_{o\nu}$. Strictly speaking, only a local minimum is of interest in this case, since the desired energy is greater than that of the separated products, but the range of η involved is anticipated to be in the same neighborhood as for the e^+e^- calculations considered earlier. The minimization is carried out in terms of the total energy, although the magnitude of the corresponding translational energy will also be determined. Accordingly, the value chosen for A is 1.0540 a.u., since it gives the correct binding energy in the analogous treatment for the e^+e^- system. For all q/m_o values of $\bar{\nu}$ it is found that the lowest energy eigenvalue in the interesting range of η occurs for a state of $1/2^-$ symmetry. A minimum with close to the desired total energy (+28900 hartree) occurs for a scale factor η of 0.18 and a q/m_o value of + 0.5733 a.u. The corresponding variation of energy with η is shown in Fig. 4, while the dependence of the minimal total energy as a function of the assumed q/m_o value for $\bar{\nu}$, i.e. as obtained by optimizing η in each case, is given in Fig. 5.

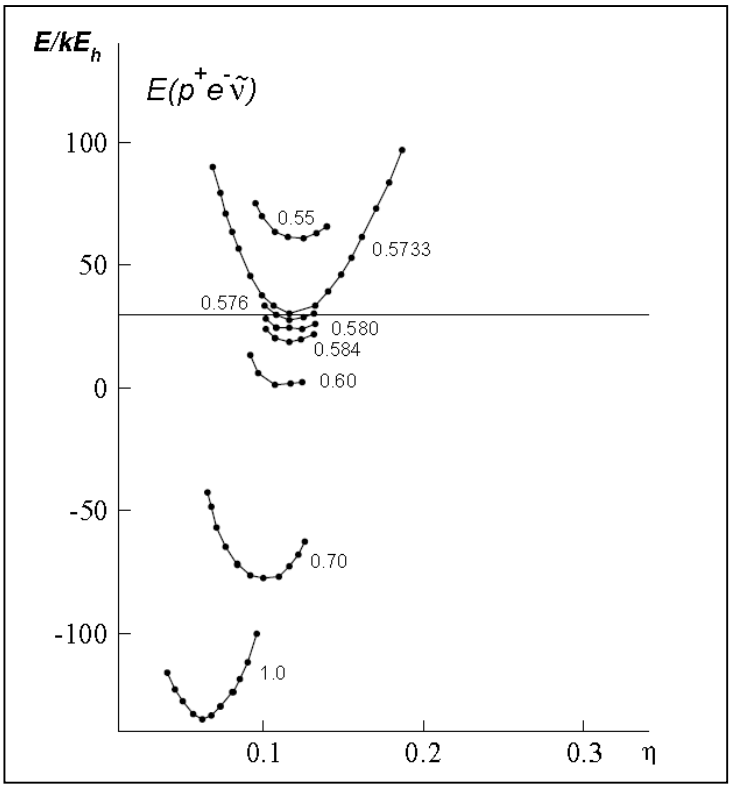

Fig 4: Variation of the computed total energy (in khartree) of the $p^+e^-\bar{\nu}$ (neutron) system as a function of the 2s,2p basis set scaling factor η in the XBPS treatment for various values of the antineutrino charge-to-rest-mass ratio q /m_o. The horizontal line at the center of the diagram corresponds to the experimental total energy of the neutron at rest. The value of the exponential damping constant A assumed in each case (1.054 a.u.) is taken from the results of the analogous e^+e^- calculations in the same basis.

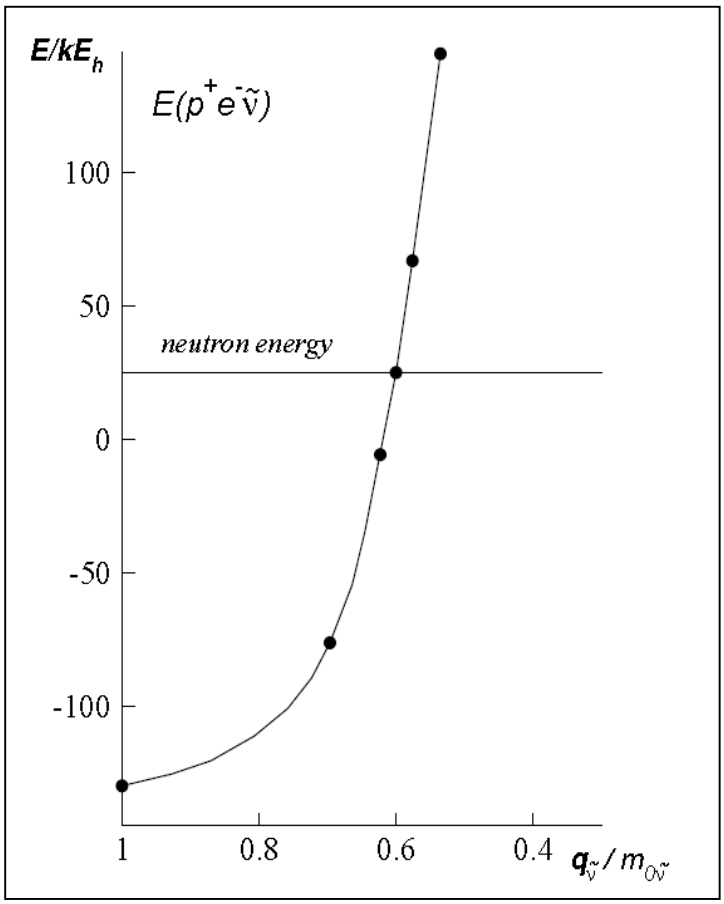

Fig 5: Variation of the minimal $p^+e^-\bar{\nu}$ system's computed total energy (taken from Fig. 3, 2s,2p basis) as a function of the antineutrino charge-to-rest-mass ratio q/m_o. The horizontal line at the center of the diagram corresponds to the experimental total energy of the neutron at rest.

Although the above calculations are fairly crude because of the small number of functions in the one-particle basis employed, they nonetheless illustrate a number of features of the present theoretical model which can be expected to be retained as the level of computational treatment is improved. To begin with, the fact that a minimum in total energy is always found demonstrates that the short-range potential employed is capable of binding the above three particles together within a relatively small volume. The computed binding energy is relatively sensitive to the choice of the $\bar{\nu}$ q/m_o, and indicates that for values close to +1.0 a.u, the $p^+e^-\bar{\nu}$ system would be more stable than the neutron actually observed experimentally. The corresponding optimal η value increases rather quickly as q/m_o decreases. There is thus a tendency for the constituent particles to draw more closely together at the same time that the total energy is increasing during the same variation. This is very similar to the experience noted for the $e^-\bar{\nu}$ system discussed in Sect. IV (see Fig. 3), which in turn can be anticipated based on the coordinate-scaling arguments of Sect. II. It is also clear from comparing Figs. 3 and 5 that the rate of increase in total energy caused by lowering the antineutrino's q/m_o value is somewhat greater for the $e^-\bar{\nu}$ binary system than for tri-atomic $p^+e^-\bar{\nu}$.

The wave-function corresponding to the experimental neutron binding energy is shown for the present basis in Table 3 (SCF orbitals and full CI coefficients). It is relatively easy to analyze these results because the proton is seen to occupy a single s-type orbital almost exclusively in the small basis employed. For each product of three spatial orbitals, there are two doublet states possible, constructed from the three spin products (M_J=+1/2): ααß, αßα and ßαα (spins given in order of $p^+e^-\bar{\nu}$, α for m_J=1/2 and ß for m_j= -1/2 for both $s_{1/2}$ and $p_{1/2}$ spin-orbitals. In the calculations the two doublets are represented by the linear combinations: $\chi_1=(2/3)^{1/2}$ααß–$6^{-1/2}$(αßα+ßαα) and $\chi_2=2^{-1/2}$ (αßα-ßαα). In this basis the preferred spin combination is $(3/4)^{1/2}$ χ_1+(1/2)χ_2, which thus reduces to $\chi_{opt}=2^{-1/2}$ (ααß-αßα). In essence the proton therefore almost always has α spin, while the $e^-\bar{\nu}$ pair forms a singlet (αß-ßα) combination which is very reminiscent of the 0^- state preferred by the e^+e^- system (cf. Sect. 3.3 of Ref. [3]) and the other particle-antiparticle binaries studied, as well as $e^-\bar{\nu}$ itself. The $p_{1/2}$ $s_{1/2}$ configurations are again preferred for the $e^-\bar{\nu}$ complex, just as in the absence of the proton, but the corresponding $s_{1/2}$ $p_{1/2}$ product also makes a substantial contribution in each case. This type of polarization of the wave-function is greater for the p^+ $e^-\bar{\nu}$ system, however. Clearly the M_J=-1/2 component of the $1/2^-$ state can be obtained by inverting all α (and ß spins), which in effect means that the resulting proton with ß spin is then bound to the same 0^- $e^-\bar{\nu}$ structure as before.

Table 3: Self-consistent field (a) and selected CI (b) coefficients for the XBPS calculation (order 176) of the $1/2^-$ ground state of the $p^+e^-\bar{\nu}$ neutron system employing a 2s,2p basis (α_1 = 0.36 x 10^8 a_0^{-2} and α_2 = 0.18 x 10^8 a_0^{-2}) with scale factor η=0.18, exponential damping constant A=1.054 a.u. and $\bar{\nu}$ q/m_o value of 0.5733 a.u.

a)				**SCF Coefficients**			
Orbital		**p+**		**e⁻**		$\bar{\nu}$	
		α_1	α_2	α_1	α_2	α_1	α_2
$1s_{1/2}$		1.62047	-1.88386	-1.23308	2.15739	-0.01823	-2.48485
$2s_{1/2}$		-0.72534	2.37670	1.99702	-1.47876	1.01666	2.26743
$1p_{1/2}$		1.52029	-1.26858	-0.47744	-1.92162	-0.55928	1.89942
$2p_{1/2}$		-0.67147	1.86271	1.38257	1.41742	1.44199	-1.35692
$1p_{3/2}$		1.52029	-1.26858	-0.76754	1.82523	-0.23080	-1.96655
$2p_{3/2}$		-0.67147	1.86271	1.58427	-1.18771	1.19238	1.58076
b)		Configuration (orbital occupations)			CI ($1/2^-$) Coefficients *		
	p^+	e^-	$\bar{\nu}$				
	$1s_{1/2}$	$1p_{1/2}$	$1s_{1/2}$		0.572975	0.331858	
	$1s_{1/2}$	$2p_{1/2}$	$1s_{1/2}$		-0.211011	-0.122144	
	$2s_{1/2}$	$1p_{1/2}$	$1s_{1/2}$		0.082410	0.047579	
	$1s_{1/2}$	$1p_{1/2}$	$2s_{1/2}$		-0.242427	-0.140535	
	$1s_{1/2}$	$1s_{1/2}$	$1p_{1/2}$		0.274353	0.158440	
	$1s_{1/2}$	$1s_{1/2}$	$2p_{1/2}$		0.141213	0.081532	
	$1s_{1/2}$	$2s_{1/2}$	$1p_{1/2}$		0.255488	0.147615	
	$1s_{1/2}$	$2s_{1/2}$	$2p_{1/2}$		0.279461	0.161382	
	$1s_{1/2}$	$2p_{1/2}$	$2s_{1/2}$		0.196547	0.113694	
	$1p_{1/2}$	$1p_{1/2}$	$1p_{1/2}$		-0.023543	-0.040888	
	$2s_{1/2}$	$1s_{1/2}$	$1p_{1/2}$		0.058872	0.033968	
	$2s_{1/2}$	$2s_{1/2}$	$1p_{1/2}$		0.061997	0.035741	
	$2s_{1/2}$	$2s_{1/2}$	$2p_{1/2}$		0.055680	0.032099	
	$1p_{3/2}$	$1s_{1/2}$	$1s_{1/2}$		-0.054191		
	$1p_{3/2}$	$2s_{1/2}$	$1s_{1/2}$		-0.057859		
	$1p_{3/2}$	$1p_{1/2}$	$1p_{1/2}$		-0.067169		
	$1p_{3/2}$	$1p_{1/2}$	$2p_{1/2}$		-0.060622		

*Two configuration-state functions are needed to span the spaces corresponding to (1/2) (1/2) (1/2) occupations (as defined in text), whereas only one such function is needed for the corresponding (3/2) (1/2) (1/2) species; all configurations with at least one CI coefficient whose absolute value exceeds 0.04 are listed.

Conclusion

Examination of the coupling constants in the XBPS Hamiltionian shows that the only way that it can lead to significant interactions for neutrinos is if their charge-to-rest-mass ratios are different from zero. This condition can only be met if the rest mass of the charge-less neutrino is itself exactly zero, something which is at least consistent with all experimental investigations as yet undertaken to measure this quantity. Accordingly, the sign of the corresponding Breit-Pauli interactions is determined on the basis of the product of the respective q/m_0 values of the interacting particles rather than for the product of their electric charges alone.

The corresponding $\nu\,\overline{\nu}$ total energy curve (Fig. 1) can be thought of as containing a double minimum at E=0, one for a tightly bound system at very short range and the other for the particles separated to infinity. Because of the lack of a (long-range) Coulomb interaction for neutrinos such a theoretical energy curve climbs immediately as the particles approach each other from quite large distances, contrary to what occurs for the approach of a charged particle to its antiparticle. In the latter case the internal kinetic energy of the system varies more slowly with distance in this range than does the Coulomb attraction of the particles, an effect which is crucial to the success of the original Bohr model of the hydrogen atom. It can be reasonably argued that the failure of the neutrino to participate in similarly long-range attractive interactions is responsible for the extremely penetrating nature of its radiation, making it effectively incapable of overcoming centrifugal barriers separating it from other particles. Nonetheless, the short-range XBPS interactions leave open the possibility the neutrino can overcome the large centrifugal barrier at some point and collide with charged particles as a result.

Subsequent calculations for the $\nu\,\overline{\nu}$ system have shown that the same value for the damping constant A which gives the desired binding energy of $2m_oc^2$ for the e^+e^- and p^+p^- pairs also works to a good approximation for $\nu\,\overline{\nu}$, i.e. with zero binding energy relative to the separated neutrino and antineutrino constituents. The agreement is particularly good if translational effects are considered explicitly in the optimization procedure. The XBPS Hamiltonian has a distinctive scaling property (see eqs. II.1-3) which shows that the desired zero binding energy of the $\nu\,\overline{\nu}$ system holds independently of which $|q/m_o|$ value is assumed for the neutrinos. This result is consistent with the existence of more than one type (flavor) of neutrino in nature, but it also leaves the choice of this quantity's value open for another purpose, such as to satisfy the requirement of accurately computing the measured binding energy of the neutron relative to its decay products at rest.

It is possible to understand the observed lack of conventional magnetic interactions for neutrinos on a similar basis (Sect. 6 of Ref. [1]). From the point of view of a charged particle, all electromagnetic interactions are effectively purely Coulomb because magnetic contributions to the energy are always computed to be of vanishing magnitude in its own inertial system. Thus a non-zero qIm_o value is not necessarily inconsistent with the measurement of a vanishingly small magnetic moment for the neutrino. At the same time, it can explain why such particles undergo strong interactions at extremely small interparticle separations, as exemplified in the original Reines-Cowan antineutrino capture experiments. The fact that relatively high centrifugal barriers must be overcome before coming into a suitable range for which the damped Breit-Pauli terms can dominate is also consistent with the extremely small cross sections observed for such reactions. Moreover, the possibility of its participating in short-range interactions offers an explanation for the presence of an antineutrino in the meta-stable neutron, as demanded by the hypothesis of particle balance in all physical transformations. To obtain suitably strong binding with the electron, the other light component of the neutron in this view, it is necessary to assume that $\overline{\nu}$ has a positive q/m_o value, i.e. opposite to that of the electron (and also to that of its antiparticle).

The wave-function corresponding to the experimental neutron binding energy is relatively easy to analyze because the proton is seen to occupy a single s-type orbital almost exclusively in the small basis employed. It is multiplied with the singlet ($\alpha ß$-$ß\alpha$) combination for $e^-\,\overline{\nu}$ which is very reminiscent of the 0^- state preferred by the e^+e^- system. The M_J=-1/2 component of the $1/2^-$ state can be obtained by inverting all α (and ß spins), which in effect means that the resulting proton with ß spin is then bound to the same 0^- $e^-\,\overline{\nu}$ structure as before.

The qualitative picture which emerges from these considerations is thus that the proton experiences a net attraction to the $e^-\,\overline{\nu}$ complex primarily because the antineutrino's charge-to-rest-mass ratio is somewhat smaller in absolute value than that of the electron. This condition makes the $e^-\,\overline{\nu}$ system significantly less stable than e^+e^- in its lowest state, but it also makes the former binary more attractive to protons than its electron-positron counterpart. This attraction is insufficient to produce a bound $p^+e^-\,\overline{\nu}$ system, however, i.e. to bind a single proton, and this circumstance is assumed to be responsible for the known instability of the neutron.

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References

1. Buenker RJ. Rebuttal of Fermi's Denial of Nuclear Electrons: Part I Historical Background Intern. J. Appl. Res 2021;7(5):181-192; ISSN Online: 2394-5869.
2. Buenker RJ. Production of photons in positronium decay: Critique of the creation-annihilation hypothesis: Part I ITS Applied Physics 2021;4(1):11-34; ISSN: 2664-0821.
3. Buenker RJ. Production of photons in positronium decay: Critique of the creation- annihilation hypothesis: Part II ITS Applied Physics, 2021;4(1):35-69; ISSN: 2664-0821.
4. Buenker RJ. Exponentially damped Breit-Pauli Hamiltonian for the description of positronium decay and other high-energy processes, Mol. Phys. 1992;**77**:1095-1122 see also http://arxiv.org, physics/0504070.
5. Danby G, Gaillard JM, Goulianos KL, Ledermann M, Mistry N, Schwartz M *et al*. Observation of High-

Energy Neutrino Reactions and the Existence of Two Kinds of Neutrinos. Phys. Rev. Lett 1962;9:36.
6. Foster B. (editor), Electron-Positron Annihilation Physics. Adam Hilger, Bristol 1990;179:206.
7. Reines F, Cowan CL. Detection of the Free Neutrino: a Confirmation. *Science* 1956;124:3212, 103-104; Free Antineutrino Absorption Cross Section. I. Measurement of the Free Antineutrino Absorption Cross Section by Protons. *Phys. Rev.* 1959, 113, 273; Reines F.; Cowan, C.L.; Harrison, F.B.; McGuire, A.D.; Kruse. H.W. Detection of the Free Antineutrino. *Phys. Rev.* 1960; 117:159.
8. Shortley G, Williams D. Elements of Physics, Third edition, Prentice-Hall, Englewood Cliffs, N.J., 1961, 770.
9. Bethe HA, Salpeter EE. *Quantum* Mechanics of One- and Two-Electron Atoms, Springer, Berlin 1957, 181.
10. Fermi E. *Ric. sei.* 2, No. 12, translation quoted by G. Wentzel, Quantum Theory of Fields. in: Theoretical Physics in the Twentieth Century, edited by M. Fierz and V.F. Weisskopf; Interscience, New York 1960, 69.