International Journal of Applied Research 2022; 8(1): 422-423

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 8.4
IJAR 2022; 8(1): 422-423
www.allresearchjournal.com
Received: 05-11-2021
Accepted: 07-12-2021

Lakhan Singh
Department of Mathematics,
D.J. College, Baraut, Baghpat,
Uttar Pradesh, India

Corresponding Author:
Lakhan Singh
Department of Mathematics,
D.J. College, Baraut, Baghpat,
Uttar Pradesh, India

On the structure equation $F^6 + F^4 + F^2 = 0$

Lakhan Singh

Abstract
In this paper we have studied various properties of the F – Structure $F^6 + F^4 + F^2 = 0$. Nijenhuis tensor, metric F-Structure, Kernel have also been discussed.

Keywords: Differentiable manifold, projection operators Nijenhuis tensor, Metric, kernel, tangent and normal vectors

Introduction
Let M^n be a C^∞ differentiable manifold and F be a (1, 1) tensor of class C^∞ defined on M^n by
(1.1) $F^6 + F^4 + F^2 = 0$

We define the operators l and m on M^n by
(1.2) $l = F^6,\ m = I - F^6$

from (1.1) and (1.2) , we have
(1.3) $l + m = I,\ l^2 = l,\ m^2 = m,\ lm = ml = 0,$
$F^2 l = lF^2 = F^2,\ F^2 m = mF^2 = 0$

Let
(1.4) $M = \{m - F^6, m - F^5, \ldots m - F, m + F, m + F^2, \ldots m + F^6\}$
(1.5) $L = \{l - F^6, l - F^5, \ldots l - F, l + F, l + F^2, \ldots l + F^6\}$

We study some properties of some elements of M and L

Theorem (1.1) we define (1, 1) tensors by 2
(1.6) $p = m + F^3,\ q = m - F^3$
(1.7) $\alpha = l + F^3,\ \beta = l - F^3$
(1.8) $\gamma = l + F^2,\ \delta = l - F^2,$

Then we have
(1.9) $pq = m - l,\ p^2 = q^2 = I,\ p^2 - p - q + I = 2l$
(1.10) $\alpha^n = 2^{n-1}\alpha,\ \beta^n = 2^{n-1}\beta$
(1.11) $3\gamma^2 + \delta^2 = 0$

Proof: We prove only (1.11)
Using (1.1), (1.2), (1.3) and (1.8)
(1.12) $\gamma^2 = (l + F^2)(l + F^2)$
$= l^2 + lF^2 + F^2 l + F^4$
$= l + F^2 + F^2 + F^4$
$= F^6 + F^4 + F^2 + F^2$, thus
(1.13) $\gamma^2 = F^2$

And
(1.14) $\delta^2 = (l - F^2)(l - F^2)$
$= l^2 - lF^2 - F^2 l + F^4$
$= l - F^2 - F^2 + F^4$

$= F^6 + F^4 + F^2 - 3F^2$

(1.15) $\delta^2 = -3F^2$,

From (1.13) and (1.15) we get (1.11).

2. Nijenhuis tensor

The Nijenhuis Tensor corresponding to F, *l* and m be denoted by N_F, N_l and N_m respectively and defined by

(2.1) $N_F (X, Y) = [FX, FY] + F^2 [X, Y] – F [FX, Y] – F [X, FY]$

(2.2) $N_l (X, Y) = [lX, lY] + l^2 [X, Y] – l [lX, Y] – l [X, lY]$

(2.3) $N_m (X, Y)=[mX, mY] +m^2[X,Y] – m[mX, Y] – m[X, mY]$

Theorem (2.1) For the F - structure F satisfying (1.1), we have

(2.4) $N_F^2 (mX, mY)= F^4 [mX, mY]$

(2.5) $F^2 N_F^2 (mX, mY) = l [mX, mY]$

(2.6) $N_l (mX, mY) = l[mX, mY]$

(2.7) $N_m (lX, lY) = m [lX, lY]$

(2.8) $N_l (lX, mY) = N_m [mX, lY] = 0$

Proof: Using (1.2), and (1.3) in (2.1), (2.2) and (2.3) we get the result.

3. Metric F – Structure

Let the Riemannian metric g is such that

(3.1) $\grave{F} (X, Y) = g (FX, Y)$ is symmetric, then

(3.2) $g (FX, Y) = -g (X, FY)$ and

(3.3) $\grave{m} (X, Y) = g (mX, Y) = g (X, mY)$

Theorem (3.1) For the F-Structure satisfying (1.1) we save

(3.4) $g (F^3X, F^3Y) = g (X, Y) – \grave{m} (X, Y)$

Proof: Using (1.2), (1.3), (3.2) and (3.3) we get the results.

Definition 3.1: Let us define

(3.5) Ker F = {X: FX = 0}

Theorem (3.2) for the F-structure satisfying (1.1) we have

(3.6) $\text{Ker } F^2 = \text{Ker } F^4 = \text{Ker } F^6$

Proof Using (1.1) we have $F^8 = F^2$, $F^{10} = F^4$, $F^{12} = F^6$

Let $X \in \text{Ker } F^2 \Rightarrow F^2 X = 0$

$\Rightarrow F^{10} X = 0$

$\Rightarrow F^4 X = 0$

$\Rightarrow X \in \text{Ker } F^4$

Thus

(3.7) $\text{Ker } F^2 \subseteq \text{Ker } F^4$

Let $X \in \text{Ker } F^4 \Rightarrow F^4 X = 0$

$\Rightarrow F^8 X = 0$

$\Rightarrow F^2 X = 0$

$\Rightarrow X \in \text{Ker } F^2$

Thus

(3.8) $\text{Ker } F^4 \subseteq \text{Ker } F^2$

From (3.7) and (3.8), we get

(3.9) $\text{Ker } F^2 = \text{Ker } F^4$,

Proceeding similarly we get $\text{Ker } F^4 = \text{Ker } F^6$, and hence (3.6).

References

1. Bejancu. A, CR-submanifolds of a Kaehler Manifold l, proc. Amer. Math. Soc. 1978;69:135-143.
2. Demetropoulou-Psomopoulou D, Andreou F. Gouli, On Necessary and sufficient conditions for an N-Dimensional Manifold to admit a Tensor Field f(≠0) of type (1,1) satisfying) $f^{2v+3} + f = 0$ Tensor (N.S.). 1985;42,252-257.
3. Blair DE, Chen BY. On CR- submanifolds of Hermitian Manifolds. Israel Journal of Mathematics. 1979;34(4):353-363.
4. Goldberg SI, Yano K. On normal globally framed f-manifolds. Tohoku Math. 1970;I(22):362-370.
5. Goldberg SI. On the Existance of Manifold with an F-structure. Tensor (N.S.). 1972;26:323-329.
6. Upadhyay MD, Gupta VC. Integrability conditions of a structure f_λ satisfying $f^3 + \lambda^2 f=0$, publications mathematics. 1977;24(3-4):249-255.
7. Yano K. On structure defined by a tensor Reid f of type (1,1) satisfying $f^3 + f = 0$, Tensor (N.S.). 1963;14:99-109.
8. Yano K, Kon M. Differential geometry of CR-submanifolds, Geometriate Dedicala. 1981;10:369-391.
9. Das LS, Nivas R, Singh A. On CR-structures and F-structures satisfying $F^{4n} + F^{4n-1} \ldots\ldots + F^2 + F = 0$, Tesor N.S. 2008;70:255-260.
10. Das LS. On CR-structure and F(2K+1.1)- structure satisfying $F^{2k+1} + F = 0$, JTSI, India. 2004;22:1-7.
11. Lakhan Singh. On the cyclic group related to the F - structure equation $\sum_{k=1}^{p} F^k = 0$. Naveen shodh sansar oct to Dec 2020. E - Journal vol I , Issue XXXII , ISSN 2320-8767.