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Study on mathematical model of COVID-19 transmission in Uttar-Pradesh

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Abstract
Differential equations have a vital role in studying many problems of various fields. Presently, they have been applied in studying COVID-19 regarding its transmission, control and vaccination. Here an existing model of pandemic has been applied to this disease using the data available on Uttar-Pradesh government site. Then an analysis has been made to find the effect of various measures including vaccination in controlling the disease. The results are also statistically tested. The aim of this study is to provide a mathematical base for the present pandemic from the point of view of its controlling measures. The result has been found positive i.e., measures are effective.

Keywords: COVID-19, epidemic, SIR model, transmission rate, reproduction number, statistical analysis

Introduction
An unknown virus was suspected to have emerged into the human population in Wuhan in late December 2019 through a report of several pneumonia cases by a local hospital. Chinese health authorities took actions immediately and had detected a new virus relevant to outbreak of disease, which was known as Novel Coronavirus (nCoV) infected pneumonia by 8 January 2020 and later be designated as COVID-19 by WHO. The first human case of Coronavirus disease (COVID-19) has been detected in 2019 and it was supposed to be caused by a newly identified virus, a mutated form of Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2) on 11 February 2020 by WHO. The novel disease was declared pandemic by World Health Care Organization on 11 March 2020.

On March 11, 2020, the rapid increase of cases in the world led the WHO Director-General to announce that the outbreak could be characterized as a pandemic. By then more than 1,18,000 cases had been reported in 114 countries, and 4291 deaths had been recorded.

By mid-March 2020, the WHO European Region had become the epicentre of the epidemic, reporting over 40% of globally confirmed cases. As of April 28, 2020, 63% of global mortality from the virus was from the Region.

As of October 7, 2020, there were approximately 36,295,345 confirmed cases of COVID-19, and approximately 1,058,080 deaths reported worldwide.

In India, three cases of COVID-19 were identified in Kerala on January 2020. On March 2, 2020, two other positive cases were identified who also returned from abroad. The total number of cases was 50 on March 10, 2020, and the first death due to COVID-19 was reported on March 12, 2020. Afterwards, COVID-19 spread in the nation at a very rapid rate. At that time, India has the highest number of COVID-19 cases in Asia and the second most confirmed cases in the world after the United States. On August 29, 2020, India recorded one-day increase in COVID-19 cases, with 78,761 cases surpassing the previous record of 77,368 cases recorded in the US on July 17, 2020. India had one-day record for having the highest number of cases, *i.e.*, 97,894, as of September 17, 2020, during the first wave. As of September 30, 2020, according to the Ministry of Health and Family Welfare (MoHFW), Govt. of India, a total of 6,225,763 COVID-19 cases were reported in 35 states/union territories. These include 5,187,825 people who were cured/discharged and the 97,497 mortality cases. Hospital/home isolation of reported cases, contact tracing, and home quarantine were still in progress.

Each affected country was taking preventive measures to combat the pandemic by avoiding transmission or slowing down the spread depending on the current state in each country.
It appears that a detailed study is extremely necessary for COVID-19 regarding the management of COVID- 19 on the basis of mathematical text which can provide a scientific base. Such models may be useful for other epidemics spreading at other places in future. Also, its study is useful in controlling the similar type of disease and taking suitable measures like vaccination, social distancing, sanitization and other medical requirements. It can be applied at other places also. Due to its importance, a study on the transmission of COVID-19 has been made in this work using SIR Model with the following objective.
Data have been obtained from the government site for the period from 12 March 2020 to 9 March 2022 for the state of Uttar-Pradesh.

Methods Data

The data of COVID-19 in the study were mainly obtained from Ministry of health and family Welfare, Government of India, and various websites of Indian government agencies.

Mathematical formulation of the model

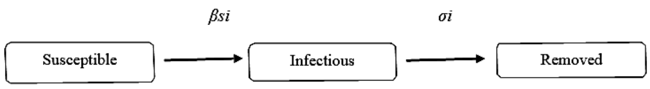
Mathematical modelling of infectious diseases was initiated by Bernoulli in 1760. The work of Kermack and McKendrick, published in 1927, had a major influence on the modelling framework. Their SIR model and some of extensions of SIR model, such as SIR, SIS, SEIR, and so on is still used to model epidemic of infectious diseases.
According to the known characteristics of the COVID-19 pandemic, we assume that each person is in one of the following compartments:
Basic Reproduction Number (Ro)**:** The basic reproduction number Ro is an essential indicator in epidemiology to represent whether an infectious disease develops a pandemic. Under the SIR model, Ro represents the average number of secondary cases transmitted by a single infected individual that is placed into fully susceptible population. A value of Ro >1 implies an exponential growth in the number of cases of the disease in the population, while a value of Ro < 1 implies an exponential decay in number of cases of the disease in the population. When a value of Ro = 1 implies the disease will stay alive and stable, but there won't be an outbreak or an epidemic.

$Ro = \beta/\gamma$

Susceptible (S)**:** Those who have not contracted the virus but might be infected as transmission from as infected individuals.
Infected (I)**:** Those who have already contracted the disease.
Removed (R)**:** The virus likely leads to one of two directions either the person recovers or dies. Here, β is transmission rate and γ is removal rate.

Susceptible-Infected-Removed (SIR) model

The SIR model is a mathematical model also known as compartmental disease model to describe spread in a population. Here, the considered population belongs to any one of three compartments: Susceptible (*S*), Infected (*I*) or Removed (*R*). This model was first constructed by Kermack and McKendrick, it comprises of following set of time related nonlinear ordinary differential equations to simulate the growth of a disease.



Assumed that the population to be continuous and differentiable function of time for solving the equations used in this model. The rate equations describing the model are:

$$ds\ dt = -\beta si \quad (1)$$
$$di\ dt = \beta si - \gamma i \quad (2)$$
$$dr\ dt = \gamma i \quad (3)$$

Where,
S(t) is the number of susceptible individuals, *I(t)* is the number of infected individuals, *R(t)* is the number of removed individuals,
β is the transmission rate of the disease,
γ is the removal rate.

We denote by *S(t)*, *I(t)* and *R(t)* the population of these classes at time *t* and *N* is the total population of Uttar-Pradesh.

$S(t) + I(t) + R(t) = N$

for all *t.*

The initial conditions are $S(T{=}0), I(T{=}0), R(T{=}0) \geq 0$. Let it be respectively *So*, *Io* and *Ro*. $s = S/N$, $i = I/N$, $r = R/N$, *β* is the transmission rate, *γ* is the removal rate.

$s + r + t = 1$

Assuming that the whole population is initially susceptible. Therefore, $s = S/N$ is similarly equal to 1. Hence equation (2) reduces to;

$$di\ dt = \beta i - \gamma i \quad (4)$$
$$di\ dt = (\beta - \gamma)i \quad (5)$$

In order to estimate the proportion of infected population at any time *t*, we integrate equation (5) from 0 to *t.*
We therefore have;

$$i(t) = io\ ex\ p[(\beta - \gamma)t] \quad (6)$$
$$I(t) = Io\ ex\ p[(\beta - \gamma)t] \quad (7)$$

Equation (7) gives the total proportion of infectives present at time *t*.

$$i_o = \frac{I_o}{N}$$

Taking *I*0= number of initial positive cases in Uttar-Pradesh after which the infection spread locally.
Using these derived relations (7), the number of infected people per day has been calculated and suitable
graphs have been plotted.
The value of reproduction rate has also been calculated using the standard formula.

A comparative study and mathematical analysis have been done using the data provided by the government site and calculated by the derived relation.

Also, a statistical analysis has been made for the data available on government site and calculated by using the above relation for infected number of cases.

A critical analysis has been made in the next chapter after drawing the graphs for a suitable interval of time.

Statistical Analysis

Test of Hypothesis about the population Mean:

When the population distribution is normal and standard deviation σ is unknown then the "z" statistic is defined as:

$$Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \quad (8)$$

Follows the z-distribution with (n-1) degree of freedom

Where,

$\bar{x}$ = sample mean

μ = hypothesised population mean

n = sample size

and s is the standard deviation of the sample calculated by the formula:

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} \quad (9)$$

The null hypothesis to be tested is whether there is a significant difference between $\bar{x}$ and μ.

If the calculated value of z exceeds the table value of z at a specified level of significance, the null hypothesis is rejected and the difference between $\bar{x}$ and μ is regarded significant. If the calculated value of z is less than the table value, the difference between $\bar{x}$ and μ is not considered to be significant.

Result and Discussion

Table 1: Table of observations taken from government data and model value

Duration	Infected number of persons (government data)	Infected number of persons (model value)	% Error	Z-test	Result
12/03/2020 to 09/06/2020	1313.411	1313.82	2.3012	3.715E-05	non-significant
10/06/2020 to 07/09/2020	27628.722	27622.67	0.1014	-3.198E-05	non-significant
08/09/2020 to 06/12/2020	38041.22	38029.61	0.0229	-7.282E-05	non-significant
07/12/020 to 13/03/2021	8944.622	8943.91	0.14279	-1.171E-05	non-significant
14/03/2021 to 13/06/2021	110386.511	110083.6	0.67175	-0.0002964	non-significant
14/06/2021 to 16/09/2021	1550.889	1556.739	0.47999	0.00032681	non-significant
18/09/2021 To 24/12/2021	132.967	133.3399	0.58747	0.00098879	non-significant
25/12/2021 to 09/03/2022	30327.04	30122.9	0.99843	-0.0006834	non-significant

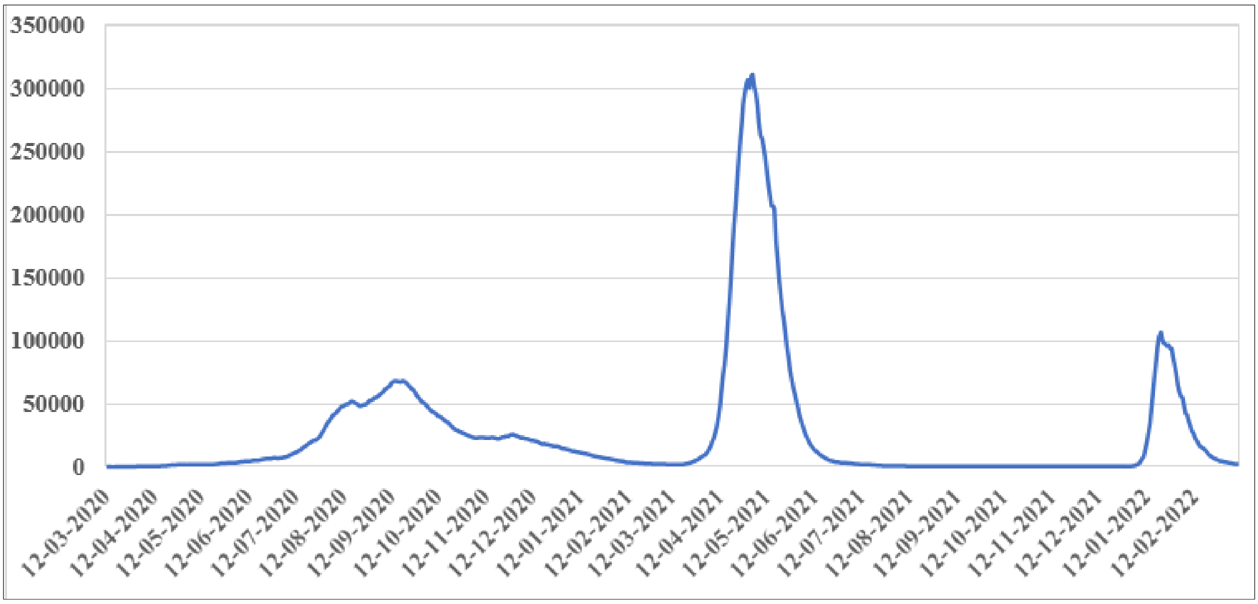


Fig A: Graph of Number of Infected people from government data

Here, X-axis shows the date and Y-axis shows the number of infected persons.

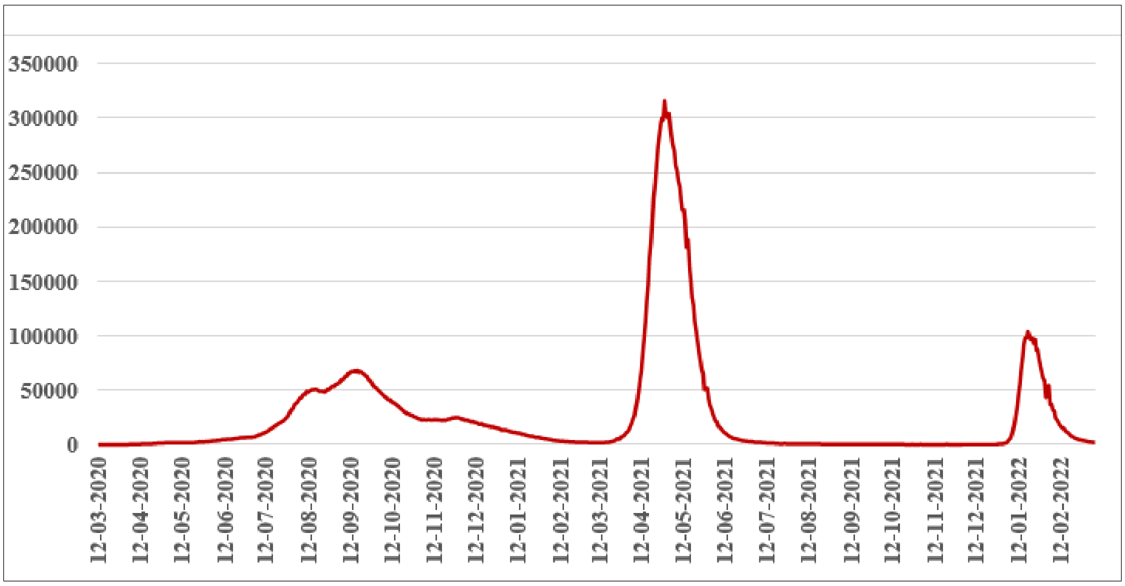


Fig B: Graph of Number of Infected people from model value

Here, X-axis shows the date and Y-axis shows the number of infected persons.
Using these graphs, it is observed that the protocols of COVID-19 have shown positive results in controlling the pandemic.

[April-June 2020]: In absence of vaccinations and lack of medical facility particularly during first wave, the measures like lockdowns, closure of institutions, social distancing, masking etc, have been found effective. This is visible from the graph plotted using the government data and model values. Since the model values are showing more infected cases in comparison to government data.

[March-May 2021]: Again, in second wave infected cases have been observed more from model value than that of government data but it is worth to mention that vaccination was started, more facilities were made available and more public awareness was stressed. Despite partial relaxation in lockdown, opening of organisations, free movement of people and transportation system were allowed, the infected cases as per model value should be found more in comparison to government value, but it did not happen.

- For Uttar-Pradesh, with $\beta = 0.103093$ and $\gamma = 0.091345$, the basic reproduction number was found as,

$$R0 = \beta/\gamma = 1.128615$$

Z-test

z-test table value for 90 degrees of freedom at 5 percent level of significance is 1.96

$$|z_{tab}| = \pm 1.96$$

z-test values for each period of three months are coming non-significant as we compare z-table values 1.96 with z-calculated values which indicates our model is giving the approx. observation values as we get from the government site.
Microsoft Excel is used to determine the P-value based on z-statistics. P-value is the probability of null hypothesis not being rejected. The null hypothesis of z-statistics is that there is no significant difference between data values and SIR Model values. How ever since % errors are less, hence z-statistics value is extremely low for SIR model values.

Conclusions

From this study of COVID-19, it has been found that

- It is observed that mostly the number of infected people is higher in case of the data of model in comparison to government data. This indicates that the measures taken by the government are appropriate during this pandemic.
- Effect of vaccination has been found positive.
- Laxity in COVID-19 protocols resulted in tremendous loss of life.

Recommendations

- Study can be extended for the whole nation depending upon the corresponding physical situations.
- More parameters may be included in the study.
- Mathematical model may be used in controlling other infectious diseases of similar nature.
- Results of this study may be useful in proper management and control of the present pandemic by estimating the infected people, required medical aid and covid protocols.

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