International Journal of Applied Research 2022; 8(2): 596-597

International Journal of Applied Research

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2022; 8(2): 596-597
www.allresearchjournal.com
Received: 10-12-2021
Accepted: 27-01-2022

Lakhan Singh
Department of Mathematics,
D.J. College, Baraut, Baghpat,
Uttar Pradesh, India

Correspondence
Lakhan Singh
Department of Mathematics,
D.J. College, Baraut, Baghpat,
Uttar Pradesh, India

On the structure equation $F^{4K} + F^{K} = 0$

Lakhan Singh

Abstract
In this paper we have studied various properties of the F – Structure equation $F^{4k} + F^{k} = 0$, k being positive integer. Nijenhuis tensor, metric F-Structure, kernel, tangent and normal vectors have also been discussed.

Keywords: Differentiable manifold, almost complex structure, projection operators, Nijenhuis tensor, metric, kernel, tangent and normal vectors

1. Introduction
Let M^n be a C^∞ differentiable manifold and F be a C^∞ (1, 1) tensor defined on M^n by

(1.1) $F^{4k} + F^{k} = 0$

We define the operators l and m on M^n by

(1.2) $l = - F^{3k}$, $m = I + F^{3k}$

Where, I denotes the identity operator.
From (1.1) and (1.2) we have
(1.3) $l+m=I$, $l^2 = l$, $m^2 = m$, $lm = ml = 0$
$F^k l = lF^k = F^k$, $F^k m = mF^k=0$

Theorem (1.1) we define (1, 1) tensors by 2
(1.4) $p = m + F^{2k}$, $q = m – F^k$
(1.5) $\alpha = m - F^k$, $\beta = m + F^k$
(1.6) $\gamma = l - F^{2k}$, $\delta = l + F^{2k}$

then we have
(1.7) $P^2 = q$, $q^{-2} = p$, $pq = p^{-3} = q^3 = I$
(1.8) $\alpha^2 = \beta^2$, $\beta^4 = \alpha, \alpha^3 = \beta^6 = I$
(1.9) $\delta^2 - v^2 = 4F^{2k}$

Proof using (1.1), (1.2,), (1.3), (1.4), (1.5), and (1.6) we get the results.

Theorem (1.2) Let rank ((F)) = n and k be even then
(1.10) $l = I$, $m = 0$, $\{F^{3k/2})$ is an almost complex structure.

Proof: From the result.
(1.11) Rank of F + Nulity of F = Dim (M^n)
$\Rightarrow$ Nulity of F = 0
$\Rightarrow$ Ker F contains only O

(1.12) Thus FX = 0 has the only solution X = 0

Let $FX_1 = FX_2$

Thus
(1.13) $F(X_1 - X_2) = 0$,

From (1.12) and (1.13) we get

(1.14) $X_1 = X_2$

Thus F is 1 – 1, also an operator on a finite dimensional differentiable manifold, if 1 – 1 is onto also therefore F is invertible. Thus

(1.15) $(F^K)^{-1}$ exists

From (1.1) and (1.15) we have

(1.16) $F^{3k} + I = 0$

$\Rightarrow F^{3k/2}$ is an almost complex structure.

2. Nijenhuis tensor

Let N_F, N_l, N_m denote the Nijenhuis tensors corresponding to the operators, F, l and m respectively then.

(2.1) $N_F(X, Y) = [FX, FY] + F^2 [X,Y] - F[FX, Y] - F[X, FY]$
(2.2) $N_l(X, Y) = [lX, lY] + l^2 [X,Y] - l[lX, Y] - l[X, lY]$
(2.3) $N_m(X, Y) = [mX, mY] + m^2 [X,Y] - m[mX, Y] - m[X, mY]$

Theorem (2.1) For the structure F satisfying (1.1), we have
(2.4) $N_F{}^k(mx, my) = F^{2k} [mX, mY]$
(2.5) $F^K N_F{}^k(mx, my) = -l [mX, mY]$
(2.6) $N_l(mX, mY) = l [mX, mY]$
(2.7) $F^k N_F{}^k(mX, mY) + N_l(mX, mY) = 0$
(2.8) $N_m(lX, lY) = m [lX, mY]$
(2.9) $N_m(lX, mY) = 0$

Proof: Using (1.2), 1.3), (2.1), (2.2) and (2.3) we get these results we prove only (2.4)

(2.10) $N_F{}^k(mX, mY) = [F^k mX, F^k mY]$
$+ F^{2K} [mX, mY]$
$- F^K [F^k mX, mY]$
$- F^K [mX, F^k mY]$
$= F^{2K} [mX, mY]$

3. Metric F – Structure

Let the Riemannian metric g satisfies
(3.1)' $F(X, Y) = g(FX, Y)$ is symmetric, then
(3.2) $g(FX, Y) = g(X, FY)$ and
{ F, g} is called metric F – structure.

Theorem (3.1) with the structure F Satisfying (1.1), we have
(3.3) $g(F^{3k} X, F^{3k}Y) = g(X, Y) - 'm(X, Y)$ where
(3.4) $'m(X, Y) = g(mX, Y) = g(X, mY)$

Proof: Using (1.1), (1.2), (1.3), (3.2) and (3.4)
(3.5) $g(F^{3k}X, F^{3k}Y) = g(X, F^{6k}Y)$
$= g(X, l^2Y)$
$= g(X, lY)$
$= g(X,(\mathbf{I} - m)Y)$
$= g(X, Y) - g(X, mY)$
$= g(X, Y) - 'm(X,Y)$

4. Kernel, Tangent and Normal Vectors

We define
(4.1) $\mathrm{Ker}\, F = \{X: FX = 0\}$
(4.2) $\mathrm{Tan}\, F = \{X: FX \parallel X\}$
(4.3) $\mathrm{Nor}\, F = \{X: g(X, FY) = 0, \forall Y\}$

Theorem (4.1) with the structure F satisfying (1.1)
(4.4) $\mathrm{Ker}\, F^K = \mathrm{Ker}\, F^{4K}$
(4.6) $\mathrm{Tan}\, F^K = \mathrm{Tan}\, F^{4K}$
(4.6) $\mathrm{Nor}\, F^K = \mathrm{Nor}\, F^{4K}$

Proof: Using (1.1), (4.1) (4.2) and (4.3) we get the results.

5. References

1. Bejancu A: On semi invariant submanifolds of an almost contact metric manifold. An Stiint University, A.I.I. Cuza Lasi Sec. Ia Mat (Supplement) 1981;27:17-21.
2. Prasad B. Semi - invariant submanifolds of a Lorentzian Para-sasakian manifold, Bull Malaysian Math. Soc (second series). 1988;21(1):21-26
3. Careres F. Linear invariant of Riemannian product manifold, Math Proc. Cambridge Phil. Soc. 1982;91:99-106.
4. Endo Hiroshi: On invariant submanifolds of connect metric manifolds, Indian J Pure Appl. Math. 1991 June;22(6):449-453.
5. Pandey HB, Kumar A. Anti - invariant some manifold of almost para contact manifold. Prog of Maths. 1987;21(1).
6. Yano K. On a structure defined by a tensor field f of the type (1,1) satisfying f3+f = 0. Tensor N.S. 1963;14:99-109.
7. Nivas R, Yadav S. On CR- structure and Fλ (2v +3,2) - HSU - structure satisfying F2v+ 3 + λ r F2 = 0, Acta Ciencias Indica. 2012;37(4):645.
8. Abhishek Singh, Ramesh Kumar Pandey, Sachin Khare. On horizontal and complete lifts of (1,1) tensor fields F satisfying the structure question F (2k+S,S) = 0. International Journal of Mathematics and soft computing. 2016;6(1):143-152. ISSN 2249-3328.