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Dr. Davinder Suri
Faculty Member, IBS Mumbai, Maharashtra, India

Dr. Dimple Pandey
Faculty Member, IBS Mumbai, Maharashtra, India

Dr. Madhu Iyengar
Faculty Member, IBS Mumbai, Maharashtra, India

Corresponding Author:
Dr. Madhu Iyengar
Faculty Member, IBS Mumbai, Maharashtra, India

Portfolio risk and return of Sensex stocks using Markowitz and single index model

Dr. Davinder Suri, Dr. Dimple Pandey and Dr. Madhu Iyengar

Abstract
Investment is an agreement for current outflow of money over a period in anticipation of future inflow that will compensate for changes in purchasing power of money as well as the uncertainty with respect to inflow of money in the future. Risks in investments stem from uncertainty that future return may be different from future expected return. Higher the return expected by an investor, higher is the risk he has to bear. Diversifying investments that is investing in more than one financial asset can reduce risk and increase return. Harry Markowitz and later William Sharpe introduced the Modern Portfolio Theory (MPT) putting forth that by identifying appropriate financial assets, maximizing overall returns within an acceptable level of risk. This case study, using the SENSEX 30 stocks, aims to highlight the benefit gained by investors by diversification of financial assets into a portfolio while elucidating the two models in detail.

Keywords: Covariance, index, markowitz, portfolio, return, risk, sharpe, systematic

Introduction

"It's not whether you're right or wrong that's important, but how much money you make when you're right and how much you lose when you're wrong." — George Soros

Savings are postponement of current consumption, for a larger future consumption. In contrast when the amount available for current consumption is less than the current needs, negative savings or borrowings become necessary. The lender expects a rate of return for foregoing his current consumption and the borrower will need to return in excess of what he has borrowed.

The trade-off between current consumption and future consumption helps an investor to decide whether consumption or investment gives higher utility. The relation between the amount of current consumption that can be exchanged for future consumption is linked to the concept of time value of money or rate of return. For example, if foregoing Rs. 100 today gives an income of Rs.105 after one year, the rate of interest is 5 percent. This rate of interest is also referred to as the real rate of interest, wherein the expectation in decline of purchasing power of money will result in higher expectation of future income, causing a rise in expected return and rate of interest. In addition, when the realization of the future amount is uncertain, the borrower will expect to be compensated with a higher amount, and this higher interest rate is referred to as the risk premium.

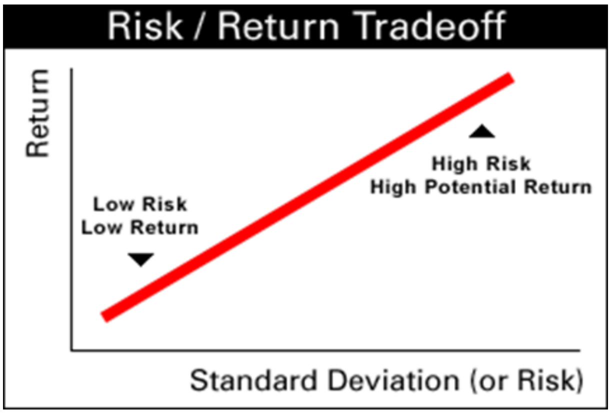
Based on the above, it can be said that an investment is an agreement for current outflow of money over a period in anticipation of future inflow that will compensate for changes in purchasing power of money as well as the uncertainty with respect to inflow of money in the future. This applies to investment in all asset classes including equity, bonds, commodities, or real estate by various groups of investors including retail investors, institutional investors or government. In all investments the trade-off is between a certain amounts being invested today in return for an expected amount in the future. Hence managing investments is a process of monitoring and evaluating the investments based on the changes in the macroeconomic environment.

The desire to postpone current consumption for higher consumption in the future is on account of varied investment motives by both individuals and institutions. Motives for investments by individuals include: buying a house, financing one's child's education, saving for old age, capital for a venture later etc. For institutions, generally, the goals are about

generating the promised return for investors or generating the maximum possible return for all subscribers.
Hence, the bottom line is that current consumption is sacrificed for a larger consumption in the future based on the expectation of returns on investments.

What is risk?
Risks in investments stem from uncertainty that future return may be different from future expected return. This variability in return can be segregated into three components: business risk, financial risk, and liquidity risk.
Business risk arises from the nature of business of the borrower on account of changes in price of raw materials and finished goods, changes in demand and supply of raw materials and finished goods, changes in wage rates, changes in fuel costs, changes in tax laws, changes in operating costs amongst other factors that affect business performance and profitability causing variability in returns.
Financial risk arises with the financing mix used by the borrower company. In the case the borrowing company has financed itself through 100 percent equity, it has no obligatory payments to be made and hence poses less risk.
Liquidity risk refers to the uncertainty of the ability of an investor to exit from an investment when desired. This exit route depends on the secondary market where the securities are traded. When an investor approaches the secondary market for liquidity, time taken for liquidation and price realization are two primary concerns. In care or illiquidity, price reduction may be required to reduce time taken for liquidation leading to price concession. Investments like treasury bills have highly liquid markets whereas real estate may take considerable time and effort to buy and sell.
Hence, to sum up, variability in returns termed as risk in investments that cause expected income to vary from actual income may be on account of business, financial or liquidity risk.

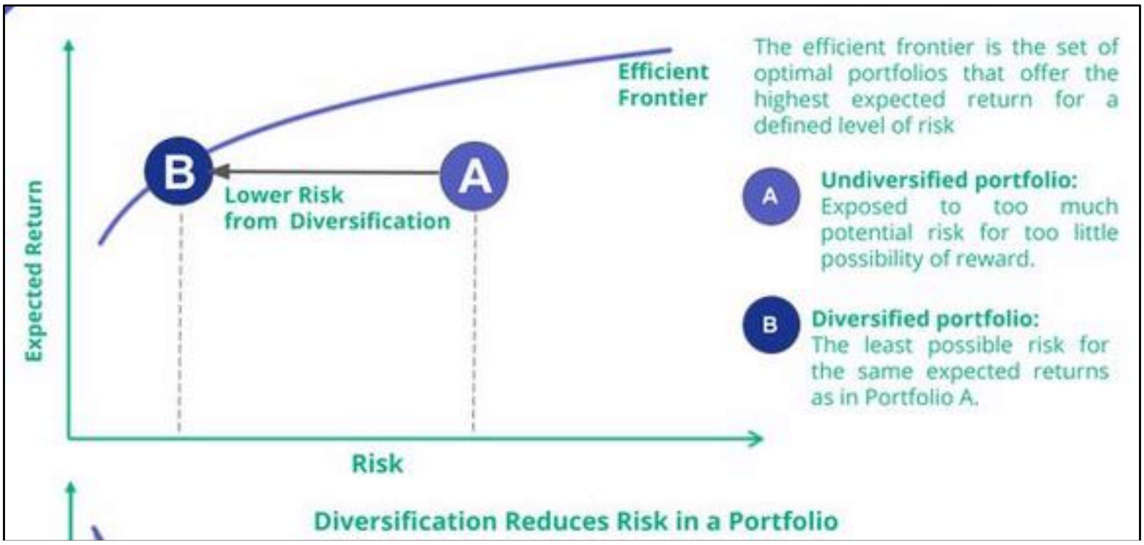
Risk and return are important factors to be considered for investment. They are two inseparable dimensions of investments that are interlinked with each other. Risk and return go hand in hand. Higher the return expected by an investor, higher is the risk he has to bear. If one wants more returns, he/she would have to take more risk. This relationship between risk and return is known as risk return trade-off. By trade-off it is meant how much extra returns one would want for additional risk.



Source: Risk-Return Tradeoff - E-Finance Management

Fig 1: Graph between risk vs return tradeoff

In order to overcome this risk-return trade-off, that is to manage to get the highest return for the given risk, investing in more than one financial asset is advised. Diversifying investments that is investing in more than one financial asset can reduce risk and increase return. Harry Markowitz and later William Sharpe introduced the Modern Portfolio Theory (MPT) putting forth that by identifying appropriate financial assets, maximizing overall returns within an acceptable level of risk, would permit investors to overcome the risk-return tradeoff.



Source: What is Efficient Frontier in Portfolio Theory?

Fig 2: Graph between expected retrun vs diversification reduces risk in a portfolio

Markowitz (1952) introduced the concept of optimal portfolio based on forecasted future returns and an appropriate covariance matrix of equity returns. Sharpe (1963) simplified portfolio creation using the concept of market risk and also reduced the number of calculations required for portfolio creation.

This case study aims to highlight the benefit gained by investors by diversification of financial assets into a portfolio as compared to investment in a single asset, understand the two models- Markowitz Model and Single Index Model and compare the risk and returns of the portfolio generated by both models.

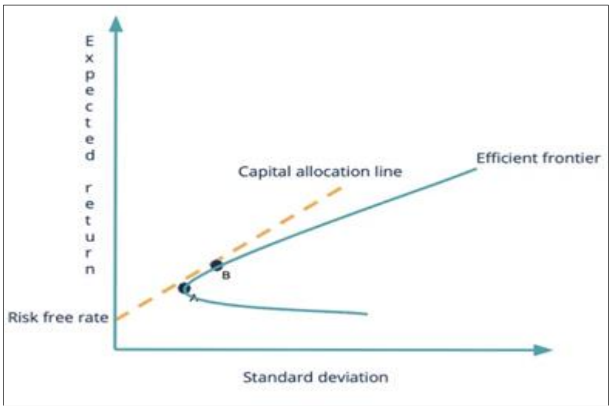
Markowitz Model

Harry Markowitz developed a basic portfolio model and considered 'Variance' as a measure of risk and expected return as payoff from investment. When an investor makes an investment in a single stock they are exposed to higher risk. For an investor to take advantage of risk return trade off and also to diversify risks of investment they should invest in a group of stocks so that even if few stocks are not performing others stocks returns may compensate for the same. This concept of making investment in multiple stocks to minimize risk was introduced by Markowitz. He advocated that an investor should make investments in stocks which are less correlated to each other for better risk return trade-off Markowitz's approach's main assumption was risk-averse investors. This means that investors will accept higher risk only when higher returns are expected from investment. The traditional maxim of 'do not put all your eggs in one basket' was given a formal and scientific shape by Harry Markowitz in his model of portfolio selection. Individual securities are inefficient and unless combined with other securities we cannot have risk reduction. Diversification of portfolio by combining securities would lead to (a) either greater return for the same risk or (b) lower risk for the same return.

Efficient Portfolios and Efficient Frontier

The investment scenario consists of a large number of financial assets/securities that are available for investment. As an input to find the efficient frontier we need expected return and standard deviation of each of the assets. For finding superior portfolios or efficient frontier, we assume rational investment behavior as follows: 1. Investors prefer securities that give greater return for the same risk. 2. Investors prefer the securities with lower risk for the same return.

An efficient frontier is a set of funding portfolios which might be expected to offer the best returns at a given level of chance. A portfolio is said to be efficient if there's no other portfolio that gives better returns for a decrease or equal quantity of risk. The Optimal portfolio desired must satisfy the following conditions: (i) It must lie on the efficient frontier (ii) It must maximize utility for the investor.



Source: CFI - Modern Portfolio Theory

Fig 3: Graph between risk free rate vs standard deviation

Minimum Variance Portfolio

An investor can use diversification to minimize risk and maximize profits and also create minimum variance portfolio based on correlation coefficient between stocks. Stocks need to be selected in such a way that the correlation coefficient between stocks is negative.

Sharpe Single Index Model

Modern Portfolio Theory developed by Harry Markowitz in 1950 is a mean variance criterion for selecting optimum portfolios for an investor. The application of this model requires a large number of input data for calculations. For instance, if there are n securities in a portfolio, the Markowitz Model requires n expected returns, n variance terms, and n (n–1)/2 covariance terms. Then we need to reduce the inputs necessary for Markowitz Portfolio Optimization for making the theory operational.

To improve upon and also to simplify the Markowitz model and reduce the covariance estimates, William Sharpe developed a simple and elegant model called the Single Index Model or Market Model. This Model aims to simplify the Markowitz model by reducing the data in a substantive manner. He assumed that the securities not only have individual relationships, but they are related to each other through the market index.

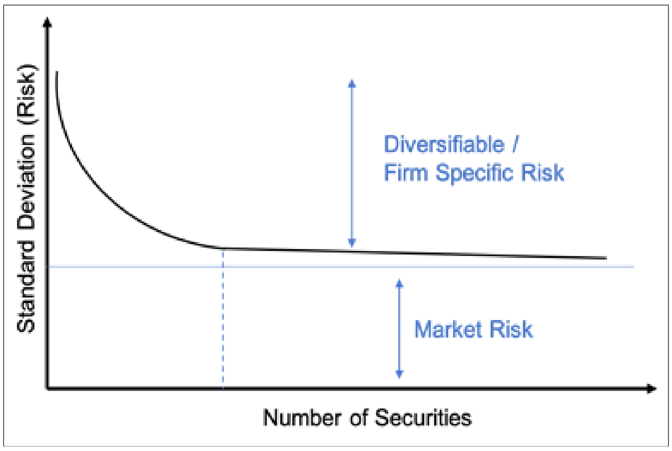
The portfolio theory perceives the investment risk from a different perspective where total risk could be divided as under: Total Risk = Market Risk + Unique Risk.

Market Risk or Systematic Risk

As we know that systematic risk comprises the proportion of total risk that occurs on account of external factors. Systematic risk arises due to fluctuations in the market and therefore it is called market risk. This kind of risk is unpredictable, and it is inbuilt in the system that cannot be avoided. The systematic risk that exists in the total risk cannot be eliminated even through the diversification of the portfolio. Therefore, this risk is observed in well-diversified portfolios. The risk factors comprise systematic risk include, equity risk, interest rate risk, currency risk and commodity risk.

Unsystematic Risk

Unsystematic risk arises on account of internal operational and administrative deficiencies of an organization. These kinds of risks are firm specific and therefore they are also known as unique risks. This risk primarily affects the specific business entity and not all the firms in general and therefore they could be controlled and diversified. The examples of such risks could be administrative and operational inefficiencies in the organization, competitors, regulatory norms impacting the business, mismanagement etc.



Source: Diversification and Portfolio Risk

Fig 4: Graph between standard deviation (Risk) vs Number of securities

Portfolio Risks

Different risks impacting portfolio is:

- Market Risk- Any change in macro-economic factors such as interest rates, inflation, currency exchange rates can lead to changes in the value of the portfolio.
- Liquidity Risk- Risk of not being able to liquidate your portfolio for immediate cash requirements. Eg- Gold is highly liquid and real estate is highly illiquid.

Table 1: Data monthly average returns and standard deviation for nifty stocks from January 2017 to February 2023

Sr. No.	Name of Security	Monthly Return(%)	Standard Deviation(%)	Sr. No.	Name of Security	Monthly Return(%)	Standard Deviation(%)
1	Asian Paints	1.673	7.16	16	M&M	0.781	11.42
2	Axis Bank	1.364	9.85	17	Maruti	0.895	8.44
3	Bajaj Finance	3.367	13.11	18	Nestle	1.74	5.25
4	Bajaj Finserve	1.705	17.56	19	NTPC	0.244	7.28
5	Bharti Airtel	0.978	7.98	20	Power Grid	0.27	6.49
6	HCL Technologies	1.381	7.98	21	Reliance Industries	1.803	10.88
7	HDFC Ltd.	1.139	7.31	22	SBI	1.564	11.13
8	HDFC Bank	0.716	8.56	23	Sun Pharma	0.965	8.81
9	HUL Ltd.	1.654	6.18	24	Tata Motors	0.967	16.26
10	ICICI Bank	1.992	9.04	25	Tata Steel	0.325	15.43
11	IndusInd Bank	1.088	14.47	26	TCS	1.092	9.26
12	Infosys	1.238	9.56	27	Tech Mahindra	1.631	8.78
13	ITC	0.762	6.58	28	Titan	3.047	9.28
14	Kotak Bank	1.361	7.57	29	Ultratech	1.21	7.62
15	L&T	0.92	8.51	30	Wipro	0.477	10.74

Questions for Discussion

1. Calculate Risk and Return of the Portfolio using Markowitz Model.
2. Calculate Risk and Return of the Portfolio using Sharpe Single Index Model.
3. Comment on the information intensity of Markowitz Model and appropriateness of Sharpe Single Index Model.
4. Compare results and comment.

Teaching Note

Synopsis of case study

The case study can be used in the course Portfolio Management and Mutual Funds as conceptual as well as numerical base to teach MPT including Markowitz and Sharpe Single Index Model. The case has a detailed explanation of the models and uses SENSEX 30 stocks data to calculate the Portfolio Risk and Return using both the models. Calculating individual stock and portfolio returns and also individual stock and portfolio risk. Appreciating the importance of correlation coefficient of returns between each pair of stocks in the portfolio and its central importance in reducing risk of portfolio. Appreciating the need to Sharpe Index Model given the information intensity of Markowitz model and the fact that the market itself is central to all stocks in this model as opposed to Markowitz where each pair of stock has a relation. Portfolio return and risk can be calculated using either of the methods with the central theme being that risk can be reduced by diversifying the portfolio.

Learning Outcomes

1. Understanding the need for portfolio creation
2. Understanding the two models under MPT
3. Familiarize with the theory and concepts at the foundation of the two models
4. Use of Excel in calculation of Risk and Return.

Questions under discussion

1. Calculate Risk and Return of the Portfolio using Markowitz Model.

First the concept

Modern Portfolio Theory: An Introduction

Traditional Approach to portfolio management was based on two-fold functions, one determining the objectives of the portfolio and then selecting the securities to be included in the portfolio. Much before formulating the objectives, constraints need to be specified, common constraints are income needs, liquidity, time horizon, safety, tax etc. Then followed by selection of portfolio, based on asset classes and optimal asset mix.

Traditional approach to portfolio construction has some basic assumptions. Individuals prefer more returns to less, but to reach a higher level of return, the investor has to take higher risk. The investor's ability to take risk determines the portfolio mix. Investor selects the industries appropriate to his investment objectives. The industry selection depends upon its growth, yield etc. EIC analysis is undertaken to identify the stocks to be included in the portfolio.

People's perception of investment and building a portfolio was very different in the 1930s, much before the advent of portfolio theory. Investment basically was considered laying bets on stocks that were thought to be good ones. Information centricity was not as robust and the prices on the market on tickers could not capture the story in a complete manner.

Due to "The Great Depression", investment in Capital Markets was perceived for people who are either wealthy or derisive. The tightening of accounting regulations did help but in a small way to do away with this perception.
A valuation method popularly called the Dividend Discount Model emerged during 1938 (John Burr Williams) and became the most popular measure to identify a good stock and acquire the same at the most optimal price. Benjamin Graham was one of the forerunners in analysis of information through Fundamentals of the company.
But risk as a parameter was not the focus till a 25-year-old grad student in Operations Research, Harry Markowitz transformed the world of investments and finance. His seminal Journal of Finance article "Portfolio Selection" revolutionized the field of finance (William F Sharpe in Foreword of the book Mean Variance Analysis in Portfolio Choice and Capital Markets). The article mathematically proved two old axioms: "nothing ventured, nothing gained" and "don't put all your eggs in one basket."
Modern Portfolio Theory is based on the premise that markets are efficient, and places more importance on the process of portfolio selection. The selection of stocks is based on analysis of risk and return. From the list of stocks in a leading stock exchange, few stocks are chosen with the objective of maximizing return or minimizing risk. Asset Allocation is based on risk perceptions of investors. Post asset allocation, investors choose either to actively manage a portfolio or opt for passive strategy.
The interpretations of the article led people to the conclusion that risk and not the best price, should be the thrust of any portfolio selection and construction.
Portfolio theory, propounded by Harry Markowitz, is the beginning of portfolio risk calculation and a formal method to prove diversification reduces risk. Diversification is the core concept of Modern Portfolio Theory and goes with the premise and wisdom of "never putting all your eggs in one basket".
Markowitz MPT of portfolio selection is a normative theory which has the assumption about market and investors. Investors are rational, Information centricity, Efficient Markets.
Two critical dimensions, the return or expected return and risk or variance and base of portfolio selection is on overall risk return characteristics, as opposed to simply compiling portfolios with securities with individually attractive risk-reward characteristics.

Calculation of Risk and Return
The return of a single stock is the Arithmetic average of returns over a period of time. Many times, holding period return is considered for calculating individual stock returns. For ex ante return, expected value is considered to be the return.
Return of an individual security is nothing but the mean /average/expected value of data points.

Return of the portfolio (collection of individual securities) is the weighted average of the individual securities' returns
The return on a portfolio of *n* securities is the weighted average of weights and return of individual securities in a portfolio.
$R_p = å\, w_i R_i$

The expected return on a portfolio is the weighted average of the expected returns on the individual securities in the portfolio.

$E(Rp) = å\, w_i E(R_i)$

Risk is uncertainty that an investment will earn its expected rate of return or not. (Returns deviating from investment). Thus, risk refers to the possibility that the actual outcome of an investment will deviate from its expected outcome.
Standard Deviation is one of the most popular and accepted measures of risk. Thus, risk of an individual security is its standard deviation or variance with its return/expected return.
Although the expected return on a portfolio is the weighted average of the individual securities in the portfolio, risk of portfolio is not the weighted average of the risks of the individual securities in the portfolio. Exception to this is when returns from securities are uncorrelated.
The standard deviation of a portfolio is not a linear combination of the individual asset standard deviations, risk of an asset is not risk in isolation, but the contribution of each asset to the risk of aggregate portfolio. Thus, risk of a portfolio is not the mere summation of risks of the individual securities. Thus, the risk of a portfolio is not the addition of the risk of individual securities, as the risk can be reduced through diversification depending on the covariance /correlation of the securities.
Symbolically,

$s_p^2 \neq å\, w_i^2 s_i^2$

Risk of the portfolio is measured using variance and standard deviation of the return on and n-security portfolio

$$s_p^2 = å\, å\, w_i\, w_j\, r_{ij}\, s_i\, s_j \qquad (1)$$
$$s_p = å\, å\, w_i\, w_j\, r_{ij}\, s_i\, s_j^{½}$$

To compute portfolio risk, information on weighted individual security risks and weighted co movements between securities included in the portfolio are required.

Significance of Correlation coefficient
The most prominent feature of the above formula (1) is the correlation coefficient between the stocks. The portfolio standard deviation depends on (i) variances of the individual assets and (ii) co variances between all the assets in the portfolio.
Covariance and Coefficient of Correlation are theoretically similar the sense that both of them are descriptive measure but the former depicts the co movement between two variables, while the latter measures the degree of association between them. Mathematically, the relationship between the two measures is:

$r_{ij\,/}rij = s_{ij\,/}\, s_I\, s_j$
or
Cov ij = [Correl Coef (rij) * S.D i * S.D j] or symbolically
$s_{ij\,=}\, rij * s_{I\,*}\, s_j$

The correlation shows the strength of the relationship between the returns generated by the two stocks over a period.

The correlation coefficient ranges from -1 to +1. The two variables under study can either be directly associated with each other, thus will have a positive correlation coefficient, or inversely associated with each other, showing a negative coefficient of correlation. The greater the value towards 1, the higher the association, the closer the value to -1, the greater the inverse relation. The number following the symbol expresses how strongly, moderately or not strongly the variables are associated. Negative correlation reduces portfolio risk

The strength of the co-movements between the returns of securities is measured by covariance which is an absolute measure and strength of association is measured using coefficient of correlation which is a relative measure.

Covariance measure helps us to understand the degree to which the returns of the two securities vary or change together. If the covariance measure is positive, then it can be implied that the returns of the two securities move in the same direction. If covariance measure is negative, it can be inferred that the returns of the two securities move in opposite direction. The formula to compute covariance between any two securities I and j is:

$\text{Cov}(R_i, R_j) = \{\sum(R_i - \bar{R}_i))(R_j - \bar{R}_{j})\} / (n - 1)$ for ex post returns

$\text{Cov}(R_i, R_j) = \{\sum(R_i - E(R_i))(R_j - E(R_j)\, p\}$ for ex ante returns

If the number of securities in a portfolio is very large, the impact of covariance in the total risk of portfolio will be higher and the risk of individual security will be lower.

The individual securities which form the part of the portfolio have different returns (measured using arithmetic average or expected value, risk measured using Standard Deviation and degree of correlation coefficient with each another. The return of portfolio is calculated using weighted average of individual securities, but the risk is not a mere weighted average of individual risk of securities but something less due to the presence of co variance.

Thus the diversifying helps reduce risk.

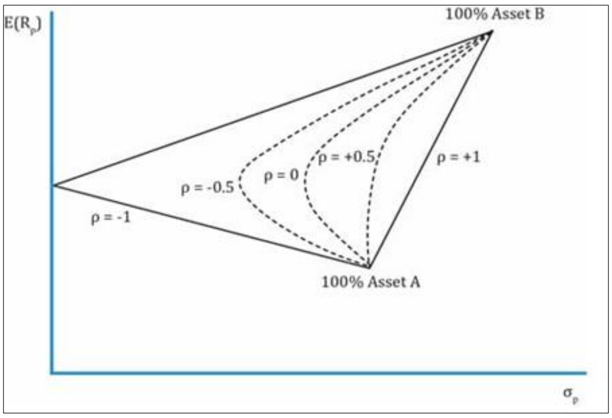
Diversification also works when assets have positive correlation with each other. The risk of portfolio will be lower as more and more assets are added. For a two-security portfolio, comprising of varied returns and risks, the risk reduction will be different even with similar weights due to the presence of a measure that is correlation.

If the two securities move perfectly in tandem with each other, their correlation coefficient, r will be +1, then diversification may not work. It is risk return graph showing traditional trade off, higher the risk, higher the return. Thus it will help us to construct a two asset portfolio with different weighs along the straight line towards the North East corner of the graph.

If the two securities are not related to each other at all, that is correlation coefficient, r = 0, there is a possibility that two asset portfolio might have a lower risk than holding a single asset.

If the two securities have correlation coefficient, r = + 0.5, the risk can be further reduced.

While a negative correlation coefficient between two securities will result in further reduction of risk. If the two assets are perfectly inversely related to each other, the standard deviation, which is risk of the portfolio can even be reduced to near zero.



Source: Investment Analysis & Portfolio Management -> Prasanna Chandra (McGraw Hill)

Fig 5: Investment analysis and portfolio management

Negative correlation reduces portfolio risk. Combining two assets with -1.0 correlation reduces the portfolio standard deviation to zero only when individual standard deviations are equal.

From the formula for calculating portfolio risk, it can be seen that as the number of securities n increases, the importance of risk of an individual security decreases, but the significance of covariance relationship increases.

Numerical Example

The following example of considering Asset A and Asset B with return 12 % and 20 %, risk 20 % and 40 % with correlation coefficient of -0.2, will help in understanding the benefit of diversification.

Variance of the portfolio of two securities =

$\sigma^2 p = w_1^2\, \sigma^2{}_1 + w_2^2\, \sigma^2{}_2 + 2\text{cov}_{12}\, w_1 w_2$

w_1= weight of security 1

w_2 = weight of security 2

$\text{cov}_{12} =$ Covariance between two securities or $r * \sigma_1 * \sigma_2$

$\sigma^2{}_1 =$ variance of security 1

$\sigma^2{}_2 =$ variance of security 2

Expected Return	12%	20%
SD	20%	40%

Correlation Coefficient is -0.2

Source: Prasanna Chandra-> “Investment Analysis and Portfolio Management”.

Table 2: Investment analysis and portfolio management

Portfolio	Proportion of A w_A	Proportion of B w_B	Expected return $E(R_p)$	Standard deviation σ_p
1 (A)	1.00	0.00	12.00%	20.00%
2	0.90	0.10	12.80%	17.64%
3	0.759	0.241	13.93%	16.27%
4	0.50	0.50	16.00%	20.49%
5	0.25	0.75	18.00%	29.41%
6 (B)	0.00	1.00	20.00%	40.00%

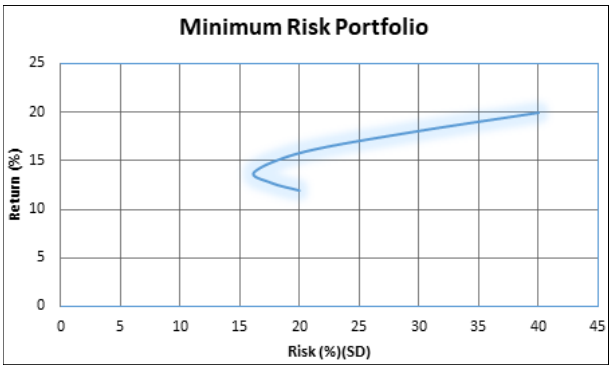


Fig 6: Graph between Return (%) vs Risk (%) (SD) at minimum risk portfolio

As can be viewed from the graph, the minimum possible risk is when 76% of the fund is invested in A and 24 % in B. The risk being 16.27% with a return of 13.93 %. Benefit of diversification arises when correlation between the two securities is less than 1. Had the correlation between the two stocks been 1, the investor can be better off investing in one of them depending on the risk perception.

No investor would be in the range of lower return than 13.93% and bear a higher risk, thus points below the minimum risk portfolio representing higher risk becomes redundant. A set of portfolios beginning from Minimum Portfolio, depicting a feasible combination of A and B, resulting in higher returns with some additional risk, is called the Efficient Set.

Every portfolio on the EF has either higher return for equal risk or lower risk for equal return than the portfolio that lies beneath the EF

This is an illustration for a set of portfolios containing two securities.

An investor can include as many securities in the portfolio as he considers right.

The concept of building portfolios by combining securities and drawing an envelope curve that contains all the best possible combinations can be extended to portfolios with more than two securities.

Let's take a small example to better understand the computation of portfolio return and risk. In order to simplify, we will consider a portfolio comprising two stocks X and Y and we have the following information:

Std Dev X = 8

Std Dev Y = 5

Std Dev market = 6

Wx = 0.6

Wy = 0.4

CorXY = 0.62

CorXM = 0.8

CorYM= 0.74

Let's calculate Portfolio Risk from given information using the Markowitz Model:

Variance of the portfolio of two securities = $\sigma^2 p = w12\ \sigma^2{}_1 + w_2{}^2\ \sigma^2{}_2 + 2cov_{12}\ w_1w_2$

= 0.6^2*64+0.4^2*25+2*0.62*0.6*0.4 = 38.944

2. Calculate Risk and Return of the Portfolio using Single Index Model

First the concept:

Sharpe Single Index Model

In 1963, William F. Sharpe had developed a simplified Single Index Model (SIM) for portfolio analysis based on Harry Markowitz's idea of diversification to minimize risk while reducing the number of calculations making the model less tedious. The Sharpe Single Index Model relates the return in a security to a single Market index in contrast to the Markowitz Model that relates each security to every other security in the portfolio. SIM computes estimates of a security's return as well as of the value of index. This premise that each security is linked to the market index assuming that the market index is used as a surrogate for other individual securities in the portfolio and hence each other immensely simplifies computations.

The Single Index model was the much-needed simpler methodology to compute portfolio return and risk. Sharpe's Single index will require a total of 3n+2 data estimates of expected returns, 50 variance terms, and 1225 covariance terms.

The single-index model each security returns to that of that market index as it considers the market index as a representation of all macroeconomic factors that comprise the systemic risk that affects all security returns. This model, further, states that the return of any stock can be divided into two parts – i) *unsystematic risk*: expected excess return of an individual security due to firm-specific factors and the unexpected microeconomic events that affect only the firm, denoted by α alpha and ii) systematic risk: the return due to macroeconomic events that affect the market, denoted by β Beta coefficient.

The relation of each individual security with the Market index can be denoted using the regression analysis. Regression is a statistical technique that estimates the relation between two or more variables where the relationship between a dependent variable against independent variables is tested. A regression line output is in the form of $Y = A + BX$ where Y is the dependent variable, X the independent variable, B is the slope of the line and A is the intercept on the Y axis. In the Sharpe Single Index model, this output is known as the 'characteristic line'.

This equation in the Sharpe SIM is $Ri = a + \beta im\ Rm + ei$ where where Ri is the holding period return on security i, Rm is the holding period return on the market index, a Alpha is the Y axis intercept signifying the return on the security when only unsystematic risk is considered, β Beta reflects the systematic risk and ei is the residual component, not captured by the above variables. Hence the Characteristic Line depicts the return of an individual security as a function of the market return.

The term $\beta im\ Rm$ embodies the return of an individual security on account of return of market index embodying all macroeconomic events including interest rates or the cost of labor, causes the systemic risk that affects the returns of all stocks and ei signifies changes in return on account of firm-specific factors including unexpected microeconomic events that affect the returns of specific firms, such as the demise of important personnel or the low credit rating on debt, that would affect the firm. It is the unsystematic risk that can be brought down to zero by way of diversification of portfolio.

The relation between each pair of securities in the portfolio is estimated through covariance. Generally the securities have a positive covariance because they all respond similarly to macroeconomic factors. Covariances are the result of differing responses to macroeconomic factors.

Under SIM, the covariance of each stock can be found by multiplying their betas by the market variance: Cov (Ri, Rk) = βiβkσ2.
When the Sharpe model puts forth this relation, the covariance calculations of a security with each and every security in the portfolio is greatly reduced. Armed with this relation, of each and every security with market index and then linked to each other, covariance of two securities can be estimated with the help of only the betas of the individual securities and the market variance. Hence, the Single Index model reduces significantly the count of calculations that would otherwise have to be made for a large portfolio of thousands of securities.

Individual Return

The Single Index model initiates at single security level by estimating the following relation known as the Characteristic Line.: Ri = αi + βi Rm + ei
Where:
Ri = expected return on security i
αi = intercept of a straight line or alpha coefficient
βi = slope of straight line or beta coefficient
Rm = expected return on index (market)
ei = error term with the mean of zero and a standard deviation which is a constant
This equation estimates the alpha and beta of each security with the market index. In this manner, alphas, and betas of all securities in the portfolio can be estimated.

Estimating Portfolio Return and Portfolio Risk

Portfolio Return: = Rp = αp + βp Rm
Where:
Beta and Alpha of the portfolio having *n* securities and each security with weights *wi*.
$Beta_p$ = βp = ∑βiwi
$Alpla_p$ = αp = ∑ αi wi

Total Risk = Systematic Risk +Unsystematic Risk
The first step to estimate the systematic and unsystematic risk of all securities.
Systematic Risk = SRi = βi^2 * σm^2
Where:
βi = slope of straight line or beta coefficient
σm = standard deviation of index returns
Unsystematic Risk = USRi = σei^2
Where:
σei = standard deviation of error term

The second step to estimate the systematic and unsystematic risk of the portfolio having *n* securities and each security with weights *wi*.
SR_p = βp^2 * σm^2
USR_p = ∑wi^2σei^2
Total Risk of Portfolio = SR_p + USR_p

Let's take a small example to better understand the computation of portfolio return and risk. In order to simplify, we will consider a portfolio comprising two stocks X and Y and we have the following information:
Std Dev X = 8
Std Dev Y = 5
Std Dev market = 6
Wx = 0.6
Wy = 0.4
CorXY = 0.62
CorXM = 0.8
CorYM= 0.74

First let us calculate systematic risk in terms of Beta of the two stocks:
The covariance of Stock X with market and Stock Y with market is not provided. It can be computed using the following relation:
Covariance of Stock X with market = *Correlation X, market * Std. Dev stock X * Std Dev market = 0.8 * 8 * 6 = 38.4*
Covariance of Stock Y with market = *Correlation Y, market * Std. Dev stock Y * Std Dev market = 0.74 * 5 * 6 = 22.2*
β of X = Covariance of Stock X, market/ variance of market = 38.4/36= 1.0667
β of Y = Covariance of Stock Y, market/ variance of market = 22.2/36= 0.6167
Beta of Portfolio = $Beta_p$ = βp = ∑βiwi = 0.6*1.0667+0.4*0.6167 = 0.8867

Unsystematic Risk of X and Y using Variance Approach
Total Risk = Systematic Risk – Unsystematic Risk
Unsystematic Risk = Total Risk - Systematic Risk
Total Risk of StockX = Std Dev^2 = 8^2 = 64
Total Risk of StockY = Std Dev^2 = 5^2 = 25
Systematic RiskX = BetaX^2*Variance of Market = 1.0667^2*6^2 = 40.96
Systematic RiskY = BetaY^2*Variance of Market = 0.6167*^2*6^2 = 13.69
Unsystematic RiskX = 64-40.96 = 23.04
Unsystematic RiskY = 25-13.69 = 11.31.
USR_p = ∑wi^2σei^2
Unsystematic Risk of Portfolio: 0.6^2*23.04+0.4^2*11.31 = 10.104
Systematic Risk of Portfolio = βp^2 * σm^2 = 0.8867^2*36 = 28.30
Total Risk of Portfolio = SR_p + USR_p = 28.30 + 10.104 = 38.944

3. Need for a new model to overcome information intensity of Markowitz Model

The Markowitz Model was based on the premise that investing in multiple securities rather than in a single security, i.e., diversification of the portfolio, reduces the level of risk of investment. However, the model suffers from the limitation of 'information intensity' in compiling the expected returns, standard deviation, variance, covariance of each security to every other security in the portfolio with the number of securities in the portfolio. In order to determine the risk, i.e. the variance of the portfolio, the covariance between each possible pair of securities is required to be computed.
If there are *n* securities, the Markowitz model requires *n* expected returns, *n* variance terms, and *n(n-1) /2* covariance terms. Hence, in case an analyst is considering 50 securities, the Markowitz model will require *50* expected returns, *50* variance terms, and *1225* covariance terms. The computation of a large number of covariance terms is cumbersome especially for institutional investors who have 100 plus securities in their portfolio. This raised the need to simplify the model in terms of computation of data estimates.

4. Compare results and justify

Let's first look at numerical answers:

Total Risk as per Markowitz Model: 0.6^2*64+0.4^2*25+2*0.62*0.6*0.4 = 38.944

Total Risk as per Sharpe Index Model = $SR_p + USR_p$ = 28.30 + 10.104 = 38.944

Return and Risk of Optimal Portfolio using Markowitz Mode for 30 SENSEX stocks:

Risk and Return of the 30 stocks

Variance - Covariance Matrix

Table 3: Variance-covariance matrix

Variance-Covariance Matrix

	AsianPain	AxisBank	BajajFina	BajajFins	BhartiAirt	HCLTechn	HDFCLtd.	HDFCBan	HULLtd.	ICICIBank	IndusIndB	Infosys	ITC	KotakBan
AsianPaints	50.63	15.68	34.30	37.49	12.25	6.94	17.30	8.97	21.73	21.09	35.02	19.64	11.20	21.04
AxisBank	15.68	95.70	78.33	83.71	21.33	26.83	46.14	47.07	3.18	64.26	99.79	25.35	23.04	40.50
BajajFinance	34.30	78.33	169.56	161.27	15.87	30.76	56.67	47.75	25.79	68.59	135.16	44.87	25.79	62.38
BajajFinserve	37.49	83.71	161.27	304.19	15.08	32.66	64.46	58.55	16.57	71.64	135.23	41.95	25.65	66.45
BhartiAirtel	12.25	21.33	15.87	15.08	100.64	3.17	20.43	15.48	11.76	12.26	29.55	35.94	16.95	3.25
HCLTechnologies	6.94	26.83	30.76	32.66	3.17	62.75	18.23	13.76	5.36	23.37	35.99	14.40	13.69	15.23
HDFCLtd.	17.30	46.14	56.67	64.46	20.43	18.23	52.64	44.66	10.39	45.57	73.62	24.72	15.30	35.89
HDFCBank	8.97	47.07	47.75	58.55	15.48	13.76	44.66	72.21	2.98	36.56	78.79	15.51	12.31	26.49
HULLtd.	21.73	3.18	25.79	16.57	11.76	5.36	10.39	2.98	37.67	6.41	3.83	10.07	7.42	12.51
ICICIBank	21.09	64.26	68.59	71.64	12.26	23.37	45.57	36.56	6.41	80.62	87.06	23.61	23.34	40.37
IndusIndBank	35.02	99.79	135.16	135.23	29.55	35.99	73.62	78.79	3.83	87.06	206.55	33.90	37.48	71.34
Infosys	19.64	25.35	44.87	41.95	35.94	14.40	24.72	15.51	10.07	23.61	33.90	90.18	13.40	18.95
ITC	11.20	23.04	25.79	25.65	16.95	13.69	15.30	12.31	7.42	23.34	37.48	13.40	42.76	18.65
KotakBank	21.04	40.50	62.38	66.45	3.25	15.23	35.89	26.49	12.51	40.37	71.34	18.95	18.65	56.45
LandT	16.39	49.34	48.95	58.57	19.35	14.66	32.56	24.87	5.28	45.46	62.71	15.28	30.68	30.75
MandM	7.76	46.56	58.91	67.44	25.58	22.06	33.10	36.24	4.70	42.32	77.45	19.40	31.79	30.29
Maruti	16.95	42.14	42.94	42.01	22.22	22.41	29.81	23.33	10.98	32.44	58.90	20.99	17.00	23.01
Nestle	18.29	11.30	19.39	23.78	7.35	6.12	11.35	2.27	18.21	9.62	6.76	14.91	7.18	12.66
NTPC	4.85	33.07	26.72	30.52	5.51	13.93	16.99	20.50	0.55	28.21	48.44	-3.26	18.78	19.22
PowerGrid	3.21	25.18	33.43	36.68	1.49	8.79	14.88	18.44	4.29	17.95	36.17	7.15	12.51	22.88
RelianceIndustries	21.50	35.19	36.86	64.81	38.36	34.64	28.60	23.14	5.78	30.28	54.29	27.32	21.52	16.53
SBI	17.01	71.40	85.13	84.36	3.55	29.60	46.67	47.52	2.46	73.96	103.03	21.49	22.70	43.53
SunPharma	8.32	22.92	14.81	12.97	38.02	18.00	17.88	14.77	-3.13	17.04	37.56	27.11	16.54	-0.97
TataMotors	3.36	80.52	61.60	85.25	23.17	25.51	48.95	45.41	-8.58	74.41	108.65	22.27	30.82	31.82
TataSteel	-13.84	41.62	36.11	54.68	20.40	27.07	24.58	26.75	-21.28	39.95	54.08	17.76	12.32	24.03
TCS	8.84	19.34	6.67	15.00	43.67	13.14	19.96	12.36	10.64	13.06	8.65	25.54	14.43	3.25
TechMahindra	16.08	30.37	40.22	54.73	42.73	2.30	19.92	13.97	6.24	22.93	30.42	39.58	8.25	8.51
Titan	26.79	40.82	68.89	77.20	24.01	13.09	26.64	19.16	18.74	30.99	58.31	25.30	14.41	24.38
Ultratech	20.07	29.95	37.72	56.94	27.88	24.73	25.61	18.40	8.49	30.18	45.99	24.92	20.74	21.43
Wipro	13.29	10.59	13.75	29.78	36.77	1.37	6.25	3.54	4.37	15.81	-1.32	30.97	-0.62	-3.59

	LandT	MandM	Maruti	Nestle	NTPC	PowerGri	Reliancel	SBI	SunPharm	TataMoto	TataSteel	TCS	TechMahi	Titan	Ultratech	Wipro
AsianPaints	16.39	7.76	16.95	18.29	4.85	3.21	21.50	17.01	8.32	3.36	-13.84	8.84	16.08	26.79	20.07	13.29
AxisBank	49.34	46.56	42.14	11.30	33.07	25.18	35.19	71.40	22.92	80.52	41.62	19.34	30.37	40.82	29.95	10.59
BajajFinance	48.95	58.91	42.94	19.39	26.72	33.43	36.86	85.13	14.81	61.60	36.11	6.67	40.22	68.89	37.72	13.75
BajajFinserve	58.57	67.44	42.01	23.78	30.52	36.68	64.81	84.36	12.97	85.25	54.68	15.00	54.73	77.20	56.94	29.78
BhartiAirtel	19.35	25.58	22.22	7.35	5.51	1.49	38.36	3.55	38.02	23.17	20.40	43.67	42.73	24.01	27.88	36.77
HCLTechnologies	14.66	22.06	22.41	6.12	13.93	8.79	34.64	29.60	18.00	25.51	27.07	13.14	2.30	13.09	24.73	1.37
HDFCLtd.	32.56	33.10	29.81	11.35	16.99	14.88	28.60	46.67	17.88	48.95	24.58	19.96	19.92	26.64	25.61	6.25
HDFCBank	24.87	36.24	23.33	2.27	20.50	18.44	23.14	47.52	14.77	45.41	26.75	12.36	13.97	19.16	18.40	3.54
HULLtd.	5.28	4.70	10.98	18.21	0.55	4.29	5.78	2.46	-3.13	-8.58	-21.28	10.64	6.24	18.74	8.49	4.37
ICICIBank	45.46	42.32	32.44	9.62	28.21	17.95	30.28	73.96	17.04	74.41	39.95	13.06	22.93	30.99	30.18	15.81
IndusIndBank	62.71	77.45	58.90	6.76	48.44	36.17	54.29	103.03	37.56	108.65	54.08	8.65	30.42	58.31	45.99	-1.32
Infosys	15.28	19.40	20.99	14.91	-3.26	7.15	27.32	21.49	27.11	22.27	17.76	25.54	39.58	25.30	24.92	30.97
ITC	30.68	31.79	17.00	7.18	18.78	12.51	21.52	22.70	16.54	30.82	12.32	14.43	8.25	14.41	20.74	-0.62
KotakBank	30.75	30.29	23.01	12.66	19.22	22.88	16.53	43.53	-0.97	31.82	24.03	3.25	8.51	24.38	21.43	-3.59
LandT	71.42	45.04	32.95	11.40	24.05	14.50	11.45	45.87	16.63	54.48	15.33	11.91	23.05	35.77	37.48	5.33
MandM	45.04	128.55	46.00	9.71	33.98	27.97	47.90	52.23	20.66	88.21	30.02	17.63	17.84	28.80	33.74	10.24
Maruti	32.95	46.00	70.27	11.97	25.37	9.90	31.99	29.93	12.98	66.35	18.69	15.64	11.20	31.82	25.97	3.56
Nestle	11.40	9.71	11.97	27.21	4.36	4.64	7.92	3.27	5.53	-3.82	-12.91	8.03	6.11	20.28	8.64	-1.04
NTPC	24.05	33.98	25.37	4.36	52.24	24.51	16.52	40.27	15.77	49.84	21.29	3.37	2.21	16.82	15.81	-3.21
PowerGrid	14.50	27.97	9.90	4.64	24.51	41.48	13.89	32.22	-2.12	33.53	13.58	3.60	-0.94	9.08	6.75	-7.27
RelianceIndustries	11.45	47.90	31.99	7.92	16.52	13.89	116.76	41.37	29.16	40.23	28.02	33.33	15.67	30.18	29.70	19.92
SBI	45.87	52.23	29.93	3.27	40.27	32.22	41.37	122.22	17.45	88.70	45.68	-2.34	11.16	35.22	34.84	5.58
SunPharma	16.63	20.66	12.98	5.53	15.77	-2.12	29.16	17.45	76.48	37.81	19.29	21.80	19.36	7.13	17.56	6.03
TataMotors	54.48	88.21	66.35	-3.82	49.84	33.53	40.23	88.70	37.81	260.87	68.03	39.93	29.62	29.69	39.03	39.14
TataSteel	15.33	30.02	18.69	-12.91	21.29	13.58	28.02	45.68	19.29	68.03	234.95	15.60	30.75	7.76	18.24	28.80
TCS	11.91	17.63	15.64	8.03	3.37	3.60	33.33	-2.34	21.80	39.93	15.60	84.58	24.36	20.11	22.63	36.61
TechMahindra	23.05	17.84	11.20	6.11	2.21	-0.94	15.67	11.16	19.36	29.62	30.75	24.36	75.96	34.88	24.11	43.22
Titan	35.77	28.80	31.82	20.28	16.82	9.08	30.18	35.22	7.13	29.69	7.76	20.11	34.88	84.89	21.69	9.95
Ultratech	37.48	33.74	25.97	8.64	15.81	6.75	29.70	34.84	17.56	39.03	18.24	22.63	24.11	21.69	57.28	22.47
Wipro	5.33	10.24	3.56	-1.04	-3.21	-7.27	19.92	5.58	6.03	39.14	28.80	36.61	43.22	9.95	22.47	113.74

Table 4: Using solver to maximize return

Securities	Weights	Return	Volatility
AsianPaints	0.00	1.673	7.12
AxisBank	0.00	1.364	9.78
BajajFinance	0.00	3.367	13.02
BajajFinserve	0.00	1.705	17.44
BhartiAirtel	0.00	0.978	10.03
HCLTechnolo	0.05	1.381	7.92
HDFCLtd.	0.00	1.139	7.26
HDFCBank	0.03	0.716	8.50
HULLtd.	0.15	1.654	6.14
ICICIBank	0.00	1.992	8.98
IndusIndBank	0.00	1.088	14.37
Infosys	0.00	1.238	9.50
ITC	0.06	0.762	6.54
KotakBank	0.00	1.361	7.51
LandT	0.00	0.920	8.45
MandM	0.00	0.781	11.34
Maruti	0.00	0.895	8.38
Nestle	0.25	1.740	5.22
NTPC	0.03	0.244	7.23
PowerGrid	0.19	0.270	6.44
RelianceIndus	0.00	1.803	10.81
SBI	0.00	1.564	11.06
SunPharma	0.09	0.965	8.75
TataMotors	0.00	0.967	16.15
TataSteel	0.03	0.325	15.33
TCS	0.00	1.092	9.20
TechMahindr	0.02	1.631	8.72
Titan	0.00	3.047	9.21
Ultratech	0.00	1.210	7.57
Wipro	0.09	0.477	10.66
Portfolio	Weight	Return	Volatility
	1.00	12.83119	11.83068

Return and Risk of Optimal Portfolio using Sharpe Single Index Model for 30 SENSEX stocks

Using weight as per output of Markowitz Model:

Table 5: Markowitz-sharpe data

S.No	Stock	Weights	Weights Square	Variance	Alpha	Beta	Beta square	S.R	USR
1	HCLTechnologies	0.05	0.0025	763.49	0.55	0.7	0.49	160.72	602.76
2	HDFCBank	0.03	0.0009	878.57	-0.46	1	0.99	323.69	554.88
3	HULLtd.	0.15	0.0225	458.28	1.31	0.3	0.09	28.4	429.88
4	ITC	0.06	0.0036	520.25	-0.03	0.67	0.44	144.79	375.46
5	Nestle	0.25	0.0625	331.04	1.36	0.32	0.1	33.87	297.17
6	NTPC	0.03	0.0009	635.64	-0.49	0.62	0.39	126.49	509.15
7	PowerGrid	0.19	0.0361	504.69	-0.33	0.51	0.26	83.48	421.22
8	SunPharma	0.09	0.0081	930.51	0.16	0.68	0.47	151.8	778.7
9	TataSteel	0.03	0.0009	2858.5	-0.74	0.9	0.82	266.79	2591.71
10	TechMahindra	0.02	0.0004	924.22	0.71	0.78	0.61	200.13	724.09
11	Wipro	0.09	0.0081	1383.83	-0.05	0.44	0.2	64.2	1319.63
	Portfolio				0.4728	0.4887			
		0.99			1.18	14.16			
					5.21	18.05			
						18.06			
	Portfolio Return				12.59	325.7			
	Systematic Risk of Portfolio				77.9	326.2			
	Unsystematic Risk of Portfolio				66.89				
	Total Risk				144.8				
	S.D				12.03				

Link to Excel Sheet: Markowitz-Sharpe Data

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