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Comparison of dual and primal method in portfolio selection

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Abstract
We studied portfolio selection process in Nigerian stock market. Portfolio selection practically deals with investments and there are varieties of instruments available to an investor at any given time. Financial statements are the major instrument used in the analysis of this work. Due to the fact that Nigerian economy is a developing economy which recently came out from recession, many companies are still striving to remain relevant in the economy and thus, much risk are equally on the business horizon. The burden of borrowed funds, risks, return and liquidity of the stock of various companies are determinant factors for an investor's choice of acceptance or rejection of any stock. In developed economy, major determinant factors are return and risk on stocks. Considering the peculiarity of Nigeria like all other developing countries, we formed portfolio comprising of stocks of selected companies. It is from these stocks in the portfolio that we developed optimization equation to determine investment on stocks applying two simplex methods/approaches and compared the two methods. We found out that dual approach is better to use in selecting a portfolio than primal approach because it was shown that smaller amount of money when invested gets the same profit with that of primal that requires bigger amount.

Keywords: Portfolio selection, primal approach, dual approach, iteration, risk, return

Introduction

Portfolio selection methods and processes are the pillar on which the Modern Portfolio Theory (MPT) propounded by [15] is built on. The model is a system of collecting stocks on investment of different companies and optimization of capital allocation to several securities. Because a portfolio is a collection of securities, the investor must have to take the decision on the portfolio to invest. Thus, the decision of the investor is equivalent to selecting an optimal portfolio from available portfolios; which is often referred to as the portfolio selection.

[10] Described the portfolio selection as a process used to identify the best allocation of securities for an investor with a given savings or consumption behaviour over a given horizon. A good portfolio is more than a long list of good stocks and bonds. It is a balanced whole, providing the investor with protections and opportunities with respect to a wide range of contingencies.

Some types of information concerning securities are needed during portfolio selection such as past performances of individual securities and beliefs of one or more security analysts concerning future performances. Markowitz portfolio theory provides a method to analyze how good a given portfolio is based on only the means and the variance of the returns of the assets contained in the portfolio.

Portfolio diversification is a widely embraced investment strategy that helps to diminish the unpredictability of markets for investors. It has the key benefits of reducing portfolio loss and volatility, especially in a developing economy. Modern Portfolio Theory (MPT), for which Harry Markowitz was awarded a Nobel Prize in 1990, provided the academic foundation for diversifying portfolios. The combination of assets that are perfectly correlated, do not move in lock –step together, the risks contained in a portfolio are lowered and higher risk-adjusted returns can be achieved. The lower the correlation between assets, the greater the reduction in possible risks. Successful portfolio diversification is dependent on combination of asset classes that are not perfectly correlated [3].

The benefits that are associated with diversification includes; reduction in risk of losses and reduction in portfolio volatility. Even when diversifying domestically, significant benefits can be achieved.

Statistical finance emerged because of its importance in having standards and means to manage, monitor, maintain and grow wealth. There were two streams in quantitative financial analysis: fundamental and technical, both of which uses mathematical models for evaluation.

This study is technical financial analysis which used the financial statements of seven companies floated in the Nigerian Stock Exchange over a 5-year period (2012-2016). The analysis in this work is geared towards choosing the proper stock to invest on while forecasting the future performances of these companies. Financial Statement analysis can be referred to as the establishment of relationship between the financial statements of two or more companies to arrive at a conclusion of what to invest on, and thus, emphasizes on the importance of the financial statement in decision making process.

Financial Statement is an investment instrument of analysis which highlights the performances of companies over time within a period in recent past and assists an investor in determining how the company can perform in a near future and as such, to decide how and what to invest on. Financial Statements are historical, because it exposes the operation and outcome of a company businesses tabulated in a way to show the cash flow, profitability, assets and liabilities. On the other hand, financial forecast basically considers the future performances to get an optimal return on investments and in turn minimizes greatly risks taking cognizance of how the output of interested stock has performed in recent past. It is noteworthy that financial statements are a valuable tool that provides guide to business and investment plans for private and corporate investors. The document is a major factor in attracting necessary funds for any investment as investors rely on it to assess the condition and reliability of a business venture.

Modern portfolio theory is based upon Harry max Markowitz studies. Before the Markowitz, modern portfolio, the prize of stock exchanges were being determined by the investment value theory [21]**.** This theory of investment was enough to regard as expected values for determining future stock value that is, when an investor wants to maximize the profit, it was enough considering only one stock. On further study of this method, Markowitz said that risk should not be neglected when considering the portfolio. In Markowitz's article about modern portfolio theory, his suggestions were intended to be practical and implementable. It is however ironic that the primary outgrowth has been normative and theoretical and that modern portfolio theory has rarely been implemented.

While optimizing a portfolio has a major area in finance, the objective of portfolio optimization is to maximize return of portfolio while at the same time minimizing the accompanying risks [2]. uses U.S data to show the significant pricing effects of liquidity as a risk factor.

[11] developed decision rules that can allow one to reach optimal solution to realistic portfolio problems without solving a mathematical programming problem.

[30] provided an overview of different portfolio models with emphasis on the corresponding optimization problems. For the classical Markowitz mean-variance model they presented computational results, and applied dual algorithm for constrained optimization.

[28] developed a multiple criteria linear programming model of the portfolio selection problem. In his research, he found out that the classical linear programming mean risk approaches, turned out to be specific aggregation techniques to apply in their multiple criteria model. The model was based on the preference axioms for the choice under risk. In their finding, they were able to identify solutions of the portfolio selection problem which are optimal with respect to various risk averse preferences. Focusing on the classical and widely known weighting approach to multiple criteria optimization, it results in linear programming problems with large number of constraints.

[31] analyzed the optimal investment strategy in a default able (corporate)bond and a money market account in a continuous time model. Due to jumps in the bond price their market model was incomplete. The treatment of information on the firm's asset value is based on an approach unifying the structural model and the reduced- form model. The optimal investment process were worked out first in their work, for a short time-horizon with a general risk-averse utility function, then a multi-period optimal strategy with logarithmic and power utility were presented using backward induction.

[12] Identified Studied portfolio model based on fuzzy interval numbers under minimizing rule (in brief, PMFM), based on the uncertainty of the expected return of securities. The PMFM model were regarded as a natural generalization of ordinarily portfolio model under the minimize rule. They also studied the PMFM model and arrived at some similar and interesting conclusions in their present paper. These, their conclusion imply that the optimal solution to PMFM is much better than that of the model without fuzzy interval numbers.

[18] investigated a new way of equity portfolio selection that provides maximum diversification along the uncorrelated risk sources inherent in the S&P 500 constituents. This diversified risk parity strategy is distinct from prevailing risk based portfolio construction paradigms. The strategy is characterized by a concentrated allocation that actively adjusts to changes in the underlying risk structure. Also X-raying the risk and the diversification characteristics of traditional risk based like $1/_N$, minimum variance, risk parity, or the most diversified portfolio, they find the diversified risk parity strategy to be superior. While most of these alternatives crucially pick risk based pricing anomalies like the low volatility a normally, they observe the diversified risk parity strategy to more effectively exploits systematic factor tilts.

[17] used fuzzy set theory to solve the unintuitive problem of the Markowitz's mean variance (MV) portfolio model and they extended it to fuzzy investment portfolio selection model. Intervals for expected returns and risk preference were established in their model which took into considerations the investors' different investment appetites and found the optimal resolution for each interval. In their empirical part, their models were in Chinese stocks investment and found that the model can fulfill different kind of investors' objectives. Then, investment risk were minimized when investment limit were added to each stock in the portfolio which indicated that their model is useful in practice.

The correlation between returns was considered in selecting portfolio by [3] and portfolio was also optimized by introducing liquidity constraint into the mean variable constrained optimization problem. The author then worked on the problem as a mean-variance liquidity constrained optimization problem and achieved optimal solution.
[1] in his numerical solution for optimal allocation of investment funds in portfolio selection problem, introduced a procedure for obtaining an optimal solution to the Markowitz mean variance portfolio selection problem using the analytical solution which he developed in his previous research. This led to the emergence of an important model known as the black model.
[25] studied portfolio selection and optimal financial investment in a developing economy. This work was done using simplex method in linear programming problem to obtain the optimal solution.

Methodology

Simplex method in a linear program problem is method used in this work, and data used were sources from secondary data. The data used in this work were obtained from seven Nigeria different companies financial statement, which can also be verified with Nigeria Stock Exchange (NSE).
The companies are Nigeria Breweries, Total Nigeria PLC, Nestle Nigeria PLC, Dangote Cement group of company, Zenith Bank PLC, Guinness Nigeria and united Bank of Africa PLC (UBA). The financial statements of these companies were assessed for five years, from 2012- 2016, to know their performances before one could invest or continue to invest in them or to diversify [9].

These data were collected and used to form the models which were used in solving the prima and dual problem for collecting a portfolio. The two approaches, of simplex method used were Primal and Dual approaches.
Under Prima approach, the data were collected and used to form a model, then put it in simplex table and iterated, the iteration processes were repeated until we got our optimal solution that it feasible.
The same approach was also used in Dual method. After forming the model, it was put in a simplex table and iterated until we got our optimal solution that is feasible.

The Model

Let the primal problem be:

$$\text{Max } Z = \sum_{j=1}^{n} C_j X_j \quad (1)$$

$$Subj\ to\ \sum_{j=1}^{n} a_{ij} X_j \le b_i (i = 1,2, \dots, m) \quad (2)$$

$$x_j \ge 0\ (j = 1,2, \dots, n)$$

The Dual of the primal

$$Min\ W = \sum_{j=1}^{n} B_j Y_i \quad (3)$$

$$Subj\ to\ \sum_{j=1}^{n} a_{ij} y_i \ge c_j (j = 1,2, \dots, n) \quad (4)$$

$$y_i \ge 0\ (i = 1,2, \dots, m)$$

Information from 2016 different companies Financial Statements

Table 1: Companies Financial Statement

Companies	Nigeria breweries (XA)	Guiness (XB)	Nestle (XC)	Total PLC (XD)	Dangote Cement (XE)	Zenith Bank (XF)	U.B.A (XG)
Dividend/Return	4.60	3.20	38.96	17.00	8.50	2.02	0.75
Risk	0.55	0.70	0.82	0.83	0.48	0.85	0.87

In this problem, objective function is in terms of separate companies while the constraint is in form of risks and dividends associated with the companies.
The dividend is summed up to get the total return from the investment. The ratio of each stock to the total return is also calculated which gives us the contribution of each stock to the portfolio following the Markowitz theory on a modern portfolio as stated (see) [9]. We also took the sum of the risk to know the actual or total risk that is being faced by the portfolio. Later, the ratio of each risk to total risk is taken to know how much risk each security is contributing to the portfolio.
Total Return= 4.60 + 3.20 + 38.96 + 17 + 8.50 + 2.02 + 0. = 75.03.
Total Risk Factor= 0.55 + 0.70 + 0.82 + 0.83 + 0.48 + 0.85 + 0.8 = 5.10

Table 2: Simplex Table

CB_i	C_j	0.55	0.29	3.25	1.44	1.22	0.18	0.06	0	0	Solution variable	Ratio
	Basic variable	x_A	x_B	x_C	x_D	x_E	x_F	x_G	s_1	s_2		
0	s_1	0.06	0.04	0.52	0.23	0.11	0.03	0.01	1	0	75.03	144.29
0	s_2	0.11	0.14	0.16	0.16	0.09	0.17	0.17	0	1	5.10	31.88←
	z_j	0	0	0	0	0	0	0	0	0	0	
	$c_j - z_j$	0.55	0.29	3.25	1.44	1.22	0.18	0.06	0	0		

This table is first iteration table for the primal approach

Table 3: Iteration 1: S1 is the leaving variable and xc is the entering variable

CB_i	c_j	0.55	0.29	3.25	1.44	1.22	0.18	0.06	0	0	
	$B.V$	x_A	x_B	x_c	x_D	x_E	x_F	x_G	s_1	s_2	s_v
0	s_1	-0.3	-0.42	0	-0.29	-0.18	-0.52	0.54	1	3.25	58.45
3.25	x_c	0.69	0.88	1	1	0.56	1.06	1.06	0	6.25	31.88

	z_j	2.24	2.86	3.25	3.25	1.82	3.45	3.45	0	20.31	103.61
	$c_j - z_j$	-1.69	-2.57	0	-1.81	-0.6	-3.27	-3.39	0	-20.31	

This table 3 is the result for the primal approach in selecting first stock to invest in, the solution is optimal and feasible. The result showed here is telling us that if an investor invested N31.88 in stock x_C he is going to have a profit of N103.61

Dual approach

Taking the dual of that primal above and use it to form another simplex table.

Table 4: Simplex table for Dual Approach

CBi	c_j	**75.03**	**5.10**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**Solution variable**
	$B.V$	y_A	y_B	s_1	s_2	s_3	s_4	s_5	s_6	s_7	
0	s_1	-0.06	-0.11	1	0	0	0	0	0	0	-0.55
0	s_2	-0.04	-0.14	0	1	0	0	0	0	0	-0.29
0	s_3	-0.52	-0.16	0	0	1	0	0	0	0	-3.25 ←
0	s_4	-0.23	-0.16	0	0	0	1	0	0	0	-1.44
0	s_5	-0.11	-0.09	0	0	0	0	1	0	0	-1.22
0	s_6	-0.03	-0.17	0	0	0	0	0	1	0	-0.18
0	s_7	-0.01	-0.17	0	0	0	0	0	0	1	-0.06
	z_j	0	0	0	0	0	0	0	0	0	0
	$c_j - z_j$	75.03	5.10	0	0	0	0	0	0	0	

Selecting first stock to invest in, by dual approach. This is iteration table for dual approach, looking at this table we found out that we have not gotten our optimal solution that is feasible. Now, we iterate again

Table 5: Iteration 2: S3 is leaving variable and YA entering variable

CB$_i$	**-75.03**	**-5.10**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	
	B.V	y_A	y_B	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_v
0	s_1	0.3	0	1	0	-0.69	0	0	0	0	1.68
0	s_2	0.48	0	0	1	-0.88	0	0	0	0	2.55
5.10	y_B	3.25	1	0	0	-6.25	0	0	0	0	20.31
0	s_4	0.25	0	0	0	-1	1	0	0	0	1.81
0	s_5	0.18	0	0	0	-0.56	0	1	0	0	0.61
0	s_6	0.52	0	0	0	-1.06	0	0	1	0	3.27
0	s_7	0.54	0	0	0	-1.06	0	0	0	1	3.39
	z_j	16.58	5.10	0	0	-31.88	0	0	0	0	103.58
	$c_j - z_j$	58.45	0	0	0	31.88	0	0	0	0	

From table 5 we have

$$W = 103.58 = 103.6$$

Here we see that the solution is optimal and feasible. And from the result we found out that if an investor invested N20.31 that he will get a return of N103.6 when invested in that the same x_C stock with the same profit but different amount

To choose the second-best stock to invest in, removing the selected stock column we have the data below:

Table 6: Companies Financial Statement excluding the selected company

Company	**Nig. Breweries x_A**	**Guiness x_B**	**Total PLC x_D**	**Dangote x_E**	**Zenith bank x_F**	**UBA x_G**
Dividend	4.60	3.20	17.00	8.50	2.02	0.75
Risk	0.55	0.70	0.83	0.48	0.85	0.87

The total dividends and risk were recalculated below with their ratio and form a new table with them.

For The Primal Approach

Table 7: Simplex table of primal for selecting the second stock

C_{BJ}	C_J	**1**	**0.56**	**2.47**	**2.18**	**3**	**0.1**	**0**	**0**	
	B_V	X_A	X_B	X_D	X_E	X_F	X_G	S_1	S_2	S_V
0	S_1	0.13	0.09	0.47	0.24	0.06	0.02	1	0	36.07
0	S_2	0.13	0.16	0.19	0.11	0.20	0.20	0	1	4.28
	Z_J	0	0	0	0	0	0	0	0	
	$C_J - Z_J$	1	0.56	2.47	2.18	3	0.1	0	0	

Table 8: Iteration table 3: S2 is the leaving variable and Xf is the entering variable

C_{BJ}	C_J	1	0.56	2.47	2.18	3	0.1	0	0	
	B_V	X_A	X_B	X_D	X_E	X_F	X_G	S_1	S_2	S_V
0	S_1	0.24	0.05	0.41	0.21	0	-0.04	1	0.3	34.81
3	X_F	0.65	0.8	0.95	0.55	1	1	0	5	21.4
	Z_J	1.95	2.4	2.85	1.65	3	3	0	15	
	$C_J - Z_J$	-0.95	-1.84	-0.38	0.53	0	-2.9	0	-15	

We have not gotten our optimal solution because in $C_J - Z_J$ row, we still have numbers greater than zero.

Table 9: Iteration table, S2 leaving variable and XE entering variable

C_{BJ}	C_J	1	0.56	2.47	2.18	3	0.1	0	0	
	B_V	X_A	X_B	X_D	X_E	X_F	X_G	S_1	S_2	S_V
0	S_1	-0.21	-0.26	0.05	0	-0.38	-0.42	1	-1.61	26.64
2.18	X_E	1.18	1.45	1.72	1	1.82	1.82	0	9	38.91
	Z_J	2.57	3.16	3.75	2.18	3.97	3.97	0	19.62	84.82
	$C_J - Z_J$	-1.57	-2.6	-1.28	0	-0.97	-3.87	0	-19.62	

From table 9, we see that the solution is optimal and feasible, which satisfied the optimal and constraint condition.

$\therefore z = ₦84.82$ and $X_E = ₦38.91$

For the Dual Approach

After recalculating the total risk and return plus their ratio we form another simplex table under dual approach

Table 10: Simplex table of Dual Approach for selecting second stock

CB_i	c_j	36.07	4.28	0	0	0	0	0	0	s_v
	$B.V$	y_A	y_B	s_1	s_2	s_3	s_4	s_5	s_6	
0	s_1	-0.13	-0.13	1	0	0	0	0	0	-1
0	s_2	-0.09	0.16	0	1	0	0	0	0	-0.56
0	s_3	-0.47	-0.19	0	0	1	0	0	0	-2.47
0	s_4	-0.01	-0.11	0	0	0	1	0	0	-2.18
0	s_5	-0.02	-0.20	0	0	0	0	1	0	-3
0	s_6	-0.02	-0.20	0	0	0	0	0	1	-0.1
	z_j	0	0	0	0	0	0	0	0	
	$c_j - z_j$	36.07	4.28	0	0	0	0	0	0	

From table 10 the solution is optimal because, $c_j - z_j \geq 0$ but not feasible, we iterate again until we get an optimal solution that is feasible.

Table 11: Iteration table, S5 is leaving variable and Yb is the entering variable

CB_i	c_j	36.07	36.07	4.28	0	0	0	0	0	0
	$B.V$	y_A	y_B	s_1	s_2	s_3	s_4	s_5	s_6	s_v
0	s_1	0.16	0	1	0	0	-1.18	0	0	1.57
0	s_2	0.26	0	0	1	0	-1.46	0	0	2.61
0	s_3	-0.05	0	0	0	1	-1.73	0	0	1.29
0	S4	0.38	0	0	0	0	-1.82	1	0	0.96
4.28	y_B	2.2	1	0	0	0	-1.84	0	0	19.8
0	s_6	0.42	0	0	0	0	-9.1	0	1	1.06
	z_j	9.42	4.28	0	0	0	-38.95	0	0	84.74
	$c_j - z_j$	26.65	0	0	0	0	38.95	0	0	

$W = 84.74$

The result shows that the investors can invest N19.8 in stock x_E (Dangote) to get a profit ofN84.74

In selecting the third stock to invest in, we follow the same procedures in above tables. We now have this table for Primal Approach and Dual too.

Table 12: Iteration table, of primal approach for selecting the third Stocks

C_{BJ}	C_J	1.21	0.22	2.82	0.32	0.13	0	0	S_V
	B_V	X_A	X_B	X_D	X_F	X_G	S_1	S_2	
0	S_1	-0.23	-0.47	0	-0.55	-1.62	1	-2.82	16.86
2.8	x_D	0.64	0.82	1	1	1.05	0	4.55	17.27
	Z_J	1.80	2.31	2.82	2.82	2.96	0	13.10	48.70
	$C_J - Z_J$	-0.59	-2.09	0	-2.51	-2.83	0	-13.10	

Looking at the table 4.8 we see that the problem has gotten it optimal solution because $C_J - Z_J \leq 0$. The result shows that the investor should invest N12.82 of stock x_D to get a profit of N48.72

The Dual result for selecting the third portfolio

Following the same method used table 4 and table 5, we have this result;

Table 13: Iteration table, of Dual Approach for selecting the third Stocks

CB_i	c_j	27.57	3.8	0	0	0	0	0	0
	$B.V$	y_A	y_B	s_1	s_2	s_3	s_4	s_5	s_v
0	s_1	0.22	0	1	0	-0.64	0	0	0.58
0	s_2	0.39	0	0	1	-0.82	0	0	2.09
3.8	y_B	2.81	1	0	0	-4.55	0	0	12.82
0	s_4	0.55	0	0	0	-1.00	1	0	2.5
0	s_5	0.62	0	0	0	-1.05	0	1	2.82
	z_j	10.68	3.8	0	0	-17.28	0	0	48.72
	$c_j - z_j$	16.89	0	0	0	17.28	0	0	

The result in table 13, is showing us that the solution is optimal and feasible. W=48.72

Shows that the investor should invest N12.82 of stock x_D to get a profit of N48.72.

Out of the seven stocks above, the best three stocks you can invest in are x_C (Nestle), x_E(Dangote cement) and x_D(Total PLC) [9].

Looking at the stocks with the return, one may choosex_C, followed by x_D, beforex_E, without considering the risk involved but the simplex method considered both the risk involved and the return itself.

From the Portfolio diversification theorem, the best method is to combine the stocks instead of investing the whole capital in one stock or same type of goods. This is to ensure profit optimization and reduces risk.

Results

The results from the two approaches used in this study are shown below

Primal Results for three selected stock:

Table 14: Primal Results for the selected Stocks

Stocks	Amount to invest (N)	Return on investment(N)
x_C	31.88	103.6
x_E	38.95	84.74
x_D	17.28	48.72

Dual Results for the three selected stocks:

Table 15: Dual Results for the selected Stocks

Stocks	Amount to invest (N)	Return on investment(N)
x_C	20.31	103.6
x_E	19.8	84.91
x_D	12.82	48.73

Interpretations of the Results

In making the first choice, we saw that investing N31.88 for primal in stock x_C gave us a profit of N103.6 after approximating. Investing N20.31 in the second method gave us the same profit of N103.6.

In making the second choice, the dual result informed us that investing N19.8 in stock x_E, we would get a profit N84.74 and the primal told us that investing N38.95 in stock x_E would give us a profit of N84.91.

In making the third choice, the dual result showed that investing N12.82 to get a profit of N48.72, while the primal method is showing us that investing N17.28 would give us a profit of N48.73.

Then, if the two results where to be approximated to the nearest whole number the same results will be obtained in the returns even though different amounts were invested.

Also, looking at the two results we found out that the first stock selected gave more return than the second and third ones. In addition to this, the second stock that was selected yielded more than that of third one. With this we may conclude that the fourth one to be selected will yield lower than the third one, so also fifth, sixth and so on from the sample stock.

Besides, we choose only three stocks here, because we wanted to invest in only three. We now selected the best three stocks to invest in. An investor that wants to invest in more than three stocks can also go further to check the next stock to invest in or to include in his or her portfolio.

Conclusion

In this work, primal and dual were used to obtain the optimal solution which satisfied the duality second theory called strong duality. The second theorem of duality states that the result of primal should be equal to that of its dual [32]. Meanwhile looking at the results, they are equal when approximated to the nearest whole number, of which the minor difference is due to truncation error.

In the first approach, which is primal, it is easier to solve than using the dual. However, comparing the result from the two approaches, the result from the dual is better than that of the primal.

This is because we invested lower capital to get the same return with that of primal where more capital was invested to earn the same return on investment with that of the dual. At the same time reduces the risk accruable and as such dual helps us to invest lower capital while taking a little risk.

In the selection of second and third stocks to invest in, the choice of selecting them were not dependent on the first choice because the ratios were calculated afresh to choose the next stock to invest in. This means that the dividend and risk ratios of each stock to the total of each of the factors were calculated for available stocks at each stage. The effectiveness function was also established from the available stocks to choose from, with respect to their dividend and risk. This was to enable us to measure the effect of each factor to the other, which shows their contribution to the portfolio.

In the first iteration under primal approach x_C was selected as the first best stock to invest in and how much to allocate to the stock and profit to make from the allocation. The same stock was also selected during dual iteration, and how much to allocate to the same stock and make the same profit. The results from the two approaches are slightly different in money allocation but the same profit. They also gave us the same choice of stocks (best three) to invest in and in the same order. The first choice is x_C, followed by x_E and x_D.

Comparing the result in table 14 and 15 we found out that dual approach is better to use in optimizing portfolio than that of primal because little amount of capital can be invested to get a good return. This will also help in minimizing risk than what is obtainable in the primal approach.

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