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Utilizing the least coast method to solve an assignment problem

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Abstract
In this paper, we took a survey about the assignment problem. An assignment Problem is an excellent area of Linear Programming Problem, behind which the purpose is playing down the complete price tag, time, job scheduling, etc. Here we have the least cost method to solve assignment problems, after which we have compared the obtained outcome with presently used methods, only to understand the rule by which we have solved numerical examples in which the first is balanced and the second is unbalanced. In addition, we have used an excel solver to resolve our assignment problems.

Keywords: Assignment problem, linear programming problem, Hungarian method, least cost method, excel solver

Introduction

We often encounter situations where we need to assign n workers to n jobs. All n workers can run all jobs but at different costs. Our task is thus to find the simple task which provides maximum efficiency and minimum costs. Activities to students, subjects to teachers, pizza delivery drivers to different routes, vendors of various regions, jobs to machines, products to factories, research problems to groups, vehicles, drivers to different Assign to root, etc. This kind of problem is called an assignment problem.

An efficient method was developed by [5] to find the best solution for a problem without directly comparing each solution. This method was called the Hungarian method. This method allowed me to reduce the cost matrix to have at least one zero in every row and column. Therefore, an optimal matching with zero opportunity cost is possible. However, [6] developed a computational approach that exploits the duality effectively. He also found that a mapping problem has optimality only if the mapping problem is complete after all possible transmissions [10]. Examined the attribution of workers to jobs in economies with adjustment frictions. It was the done by matching disparate workers to disparate jobs. He explained that the model is in equilibrium and needs to be extended to a dynamic framework when viewed in quantitative series [8]. Used constraint programming and artificial intelligence to provide a new solution that the bridges the gap between the need for high-quality matches and the need for which timeless matches. In his paper on transport problems [7], he developed a new alternative method for optimally solving the assignment problem. He developed an algorithm for this and used numerical examples to prove that his method yielded results similar to the Hungarian one, but with fewer steps and easier. In their work [11], he used a novel fuzzy assignment problem to model driver characteristics related to route selection [4]. Tackled the geometry of the simplex method and applied it to the assignment problem. In their study, the Hungarian and Gale-Shapely algorithms were implemented in MATLAB and the preference matrix from an Excel file was taken as input, yielding optimal and stable results [13]. Worked on an extension of the parallel auction algorithm proposed by [1]. Her research focused on the availability of local information that always converges to the mapping problem and when utility is to maximized within a linear approximation to the optimum. In [9], the quadratic mapping problem he considered to model the local Problems of the logistics by considering the cost of placing a new item in a particular location and the interactions between all item. He also explained that the Heuristics, branch-and-bound methods and the perfect enumeration methods can be used to solve quadratic imputations [3]. Applied the Hungarian algorithm to baseball team selection. They achieved this by matching the best players for each position on the pitch to create the optimal team [14-20].

Mathematical Formulation of Assignment Problem

To minimize the overall cost or time of a job assignment, consider assigning resources and task sequencing so that each source can be associated with only one job. The effectiveness matrix is given as under:

Table 1: Denote the assignment of facility.

		Activity				Offered
Resource		L_1	L_2	⋯	L_2	1
	K_1	T_{11}	T_{12}	⋯	T_{1n}	1
	K_2	T_{21}	T_{22}	⋯	T_{2n}	1
	⋮	⋮	⋮	⋮	⋮	⋮
	K_3	T_{31}	T_{32}	⋯	Tmn	1
Required		1	1	⋯	1	

Let x_{ij} denote the assignment of facility i to j such that

$$x_{ij} = \begin{cases} 1 & if\ facility\ i\ is\ assigned\ to\ job\ j \\ 0 & otherwise \end{cases}$$

Then, the mathematical model of the assignment problem can be stated as:

$$\text{Min } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} T_{ij}\, x_{ij}$$

Subject to the constraints

$\sum_{j=1}^{n} x_{ij} = 1$ for all i (recourse availability).

$\sum_{i=1}^{m} x_{ij} = 1$ for all j (activity requirement).

and $x_{ij} = 0\ or\ 1$ for all i and j.

Where T_{ij} represents the cost/time of assignment of recourse i to activity j.

Least cost method

Step 1: Firstly, we checked given assignment problem is balanced or not. If problem is unbalanced so firstly, we have change given assignment into balanced assignment problem by adding dummy row/column.

Step 2: Assign unit value for each row and column

Step 3: Select lowest value of entire assignment and allocate assign value one eliminating respective row and column.

Step 4: After adjusting the supply and demand for all uncrossed out rows and column repeat the procedure with next lowest value and repeat this procedure until the entire available supply at various and demand at various destination.

Step 5: Assign u_i and v_j to represent the rows and columns in given matrix.

Step 6: we evaluate $d_{ij} = T_{ij} - (u_i + v_j)$ for vacant cells**.**

Step 7: Testing for d_{ij}

a) If $d_{ij} > 0$, then current cell basic feasible solution is optimal and stop processing.
b) If $d_{ij} = 0$ then alternative solution is existing, with different set allocation and same value.
c) If $d_{ij} < 0$ then the given solution is not an optimal solution and further improvement in the solution is possible.

Select the vacant cell with the largest negative value of d_{ij}, and included in the next solution.

Step 8: Make a closed path from the vacant cell (which was selected in previous step). The right angle turn in this path is allowed only the filled cells and at the original vacant cell. Assign + and – sign alternatively at each corner, starting from the original vacant cell and follows following steps

1. Select the lowest value from cells those are assigned – sign on the closed path.
2. Assign this value to selected vacant cell.
3. Add this value to another filled cell which is marked by + sign.
4. And subtract this value to other filled cells marked by – sign.

Step 9: Repeat steps 4 to Step 7 until optimal solution is not obtained. This processor stops when all $d_{ij} \geq 0$ for unoccupied cells.

Numerical problem

To illustrate our method here, I am going to take following two numerical examples.

Problem 1 (Balanced Assignment Problem): A department has five employees who perform five tasks. The time (in hours) it takes for each person to perform each task is given in the effectiveness matrix.

Table 2: Showing Balanced Assignment

	I	II	III	IV	V
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

Solution 1: By Least Cost Method: After Applying Least Cost Method, we obtained optimal solution as follows:

Table 3: Least Cost Method, we obtained optimal solution

	I	II	III	IV	V	
A	10	**1** 5	13	15	16	**1**
B	**1** 3	9	18	13	6	**1**
C	10	7	2	2	**1** 2	**1**
D	**0** 7	**0** 11	**1** 9	**0** 7	**0** 12	**1**
E	7	9	10	**1** 4	12	**1**
	1	**1**	**1**	**1**	**1**	

From Table 2. We obtained the optimal solution i.e. employee take minimum time duration 23 hours to finish their task with the following allocation of work:

I = 3 II = 5 III = 9 IV = 4 V = 2

By Hungarian method: After applying Hungarian Method, we obtained the optimal solution as follows:

Table 4: Showing Hungarian Method, we obtained the optimal solution

	I	II	III	IV	V
A	7	0	8	12	11
B	0	4	13	10	1
C	10	5	0	2	0
D	0	2	0	0	3
E	3	3	4	0	6

From Table 3. We obtained the optimal solution i.e. employee take minimum time duration 23 hours to finish their task with the following allocations work:

I = 3 II = 5 III = 9 IV = 4 V = 2

By Excel Solver: After applying Excel Solver, we obtained the optimal solution as follows:

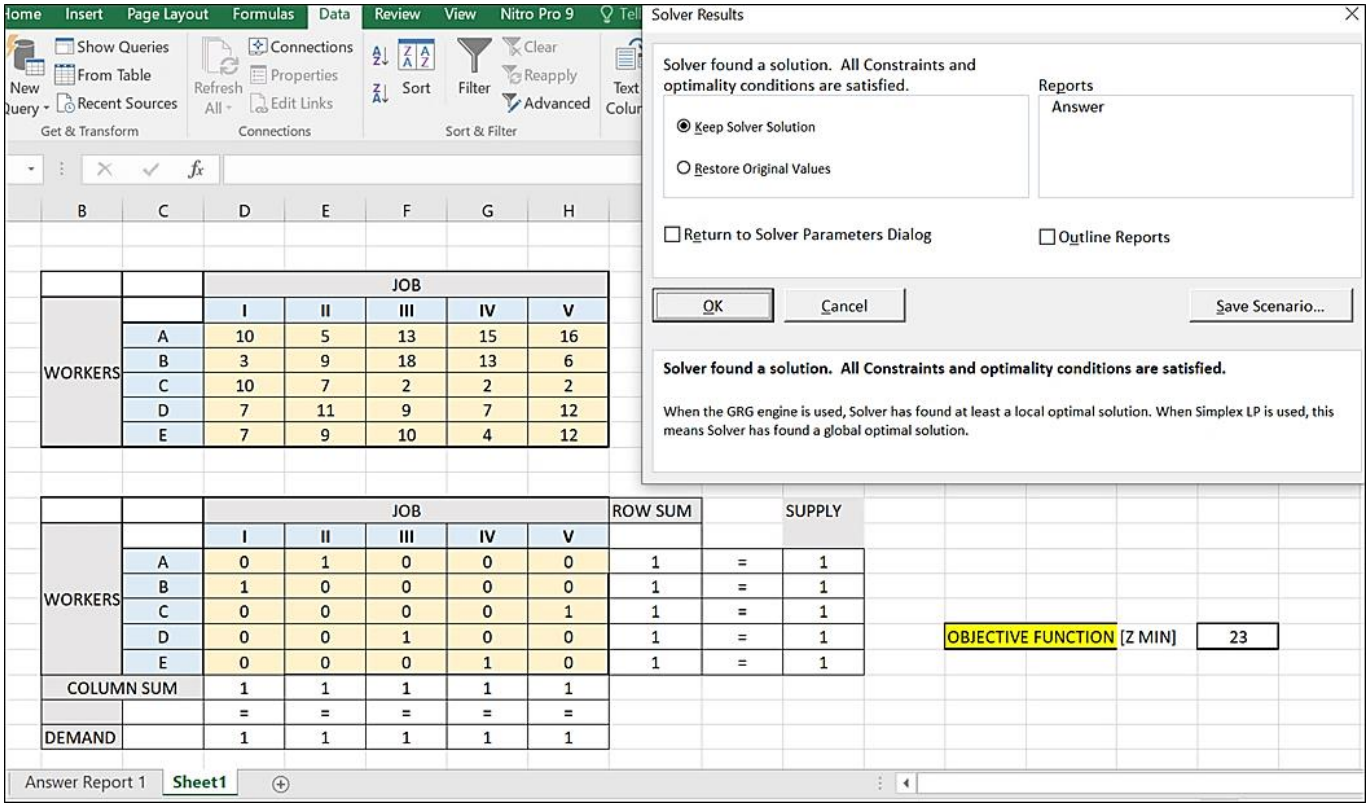

Image 1: Showing optimal solution i.e. employee take minimum time duration

From Image 1. We obtained the optimal solution i.e. employee take minimum time duration 23 hours to finish their task with the following allocations work:

I = 3 II = 5 III = 9 IV = 4 V = 2

After Least Cost Method, Hungarian method and Excel Solver, we obtained the best time for the work assigned to the employee with the appropriate time (in hours) as follows:

Table 5: Least Cost Method, Hungarian method and Excel Solver

S. No.	Name of Method	Obtained Allocation	Obtained Result
1.	Least Cost Method	I = 3; II = 5; III = 9; IV = 4; V = 2	23
2.	Hungarian Method	I = 3; II = 5; III = 9; IV = 4; V = 2	23
3.	Excel Solver	I = 3; II = 5; III = 9; IV = 4; V = 2	23

Problem 2 (Unbalanced Assignment Problem): There are five typists in a company who are employed on an hourly rate basis. Their speeds and charges vary. It was earlier understood that each typist was assigned only one job, and he was paid for a full hour regardless of how much time he spent working.

Table 6: Showing optimal solution i.e. firm pay

	P	Q	R	S	T
A	9	11	15	10	11
B	12	9	10	10	9
C	4	11	14	11	7
D	14	8	12	7	8

Solution 2: By Least Cost Method: After Applying Least Cost Method, we obtained optimal solution as follows:

Table 7: After Applying Least Cost Method.

	P	Q	R	S	T	
A	9	**0** 11	15	**0** 10	**1** 11	**1**
B	12	**1** 9	**0** 10	10	9	**1**
C	**1** 4	11	14	11	**0** 7	**1**
D	14	8	12	**1** 7	8	**1**
E	0	0	**1** 0	0	0	**1**
	1	**1**	**1**	**1**	**1**	

From Table 5. We obtained the optimal solution i.e. firm pay Rs 31 minimum cost (In hundred) with the following work to their employee allocation of work:

P = 4 Q = 9 R = 0 S = 7 T = 11

By Hungarian Method: After applying Hungarian Method, we obtained the optimal solution as follows:

Table 8: Showing Hungarian Method, we obtained the optimal solution.

	P	Q	R	S	T
A	0	0	4	0	0
B	5	0	1	2	0
C	0	5	8	6	1
D	8	0	4	0	0
E	2	0	0	1	0

From Table 6. We obtained the optimal solution i.e. firm pay Rs 31 minimum cost (In hundred) with the following work to their employee allocation of work:

P = 4 Q = 9 R = 0 S = 7 T = 11

By Excel Solver: After applying Excel Solver, we obtained the optimal solution as follows:

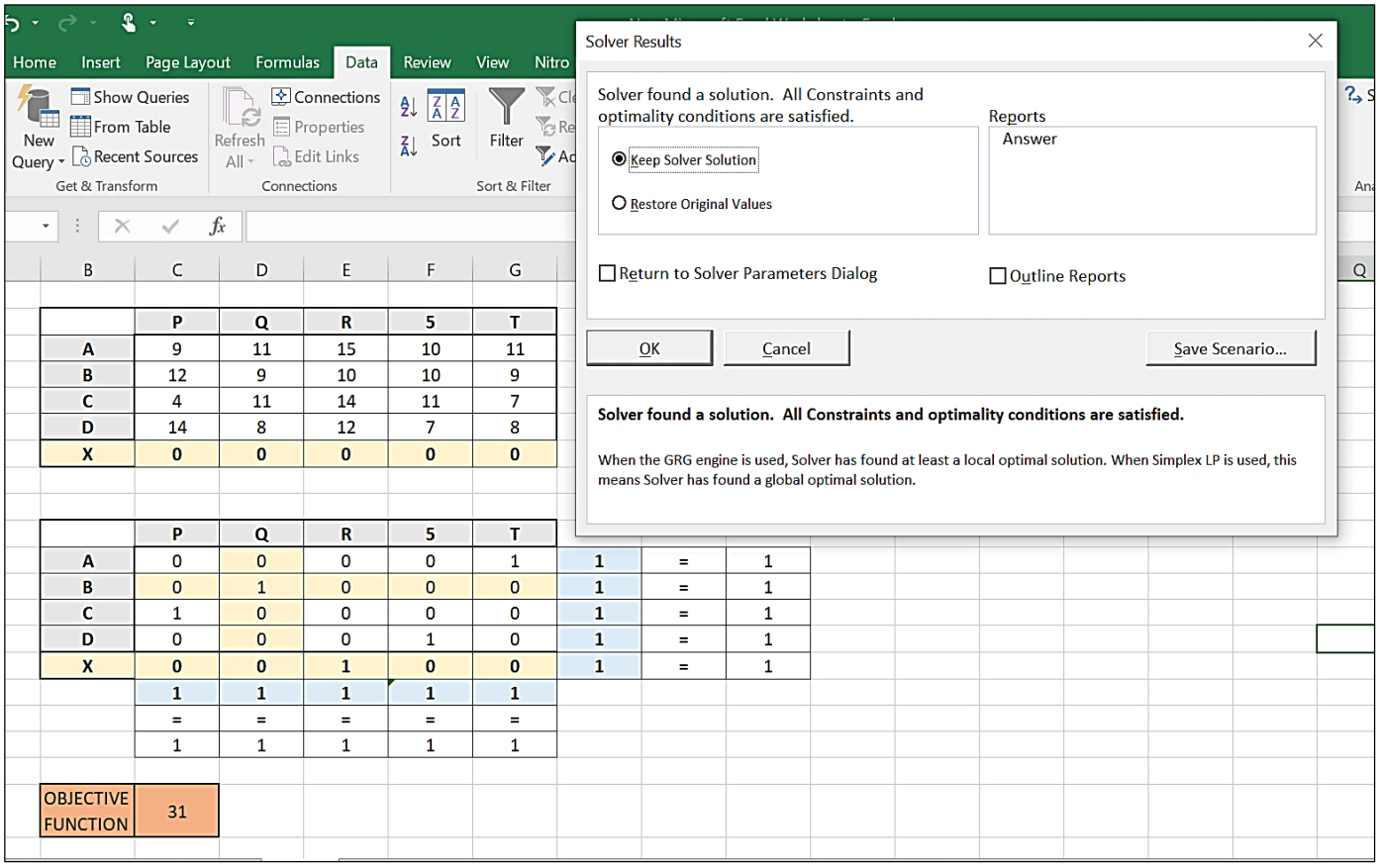

Image 2: Showing the Excel Solver, we obtained the optimal solution

From Image 2 We obtained the optimal solution i.e. firm pay Rs 31 minimum cost (In hundred) with the following work to their employee allocation of work:

P = 4 Q = 9 R = 0 S = 7 T = 11

After Least Cost Method, Hungarian method and Excel Solver, we obtained the best cost for the work assigned to the employee with the labor cost as follows:

Table 9: Least Cost Method and again compared the optimal result

S. No.	Name of Method	Obtained Allocation	Obtained Result
1.	Least Cost Method	P = 4; Q = 9; R = 0; S = 7; T = 11	31
2.	Hungarian Method	P = 4; Q = 9; R = 0; S = 7; T = 11	31
3.	Excel Solver	P = 4; Q = 9; R = 0; S = 7; T = 11	31

Conclusion

In this article, we have studied the assignment problem using the Least Cost Method and again compared the optimal result with an existing method, namely the Hungarian method and excel solver. After these studies, we found that the first and second numerical example yielded the same optimal solution, with the same task assignment. In other words, even the Least Cost Method will generate more than one job assignment for the same problem. On the other hand, we concluded that while solving a balanced or unbalance assignment problem, solve manually or by using excel solver in both the cases we obtained the same optimal solution and with same jobs allocation.

References

1. Bertsekas DP, Castanon DA. Parallel synchronous and Asynchronous implementations of the auction algorithm Alphatech Report; c1989. www.mit.edu/~dimitrib/parauction.pdf

2. Bogomolnaia A, Moulin H. New Solution to the Random Assignment Problem, Journal of Economic Theory. 2001;100:295-328.
3. Britz SS, Maltitz MJ. Application of the Hungarian Algorithm in Baseball team selection and Assignment; c2010. http://pdfmanualinfo.blogspot.com/2012/10/application-of-hungarianalgorithm-in.html?m=1
4. Cheung R, Jesús AD. The Geometry of the Simplex Method and Applications to the Assignment Problems; c2011. www.math.ucdavis.edu/index.php/download-file/view/22/192
5. Konig D. Graphok es matrixok [in Hungarian: Graphs and matrices]. Mat. Fiz. Lapok. 1931;38:116-119.
6. Kuhn HW. The Hungarian Method for the assignment problem, naval Research Logistics Quarterly, Kuhn's original publication. 1955;2:83-97.
7. Mansi SG. A study on Transportation Problem, Transshipment problem, Assignment problem and Supply chain management, a Ph.D. Thesis at Saurashtra University; c2011. http://ethesis.saurashtrauniversity.edu/id/916
8. Naveh Y, Richter Y, Altshuler Y, Gresh DL, Connors DP. Workforce optimization: Identification and assignment of professional workers using constraint programming. IBM Journal of Research and Development. 2007;51(3.4):263-279.
9. Povh J. Assignment Problems in Logistics. Logistics & Sustainable Transport; c2008, 1(3).
10. Robert S. The Assignment of Workers to Jobs in an Economy with Coordination Frictions. Journal of political Economy; c2005, 113(5).
11. Shafahi Y, Ramezani H. Application of fuzzy theory of traffic assignment. Department of Civil Engineering, Sharif University of Technology: MOAS'07; Proceedings of the 18th conference on Proceedings on the 18th IASTED International Conference: Modeling and Simulation; c2007. p. 604-608. http://dl.acm.org/citation.cfm?id=12954749
12. Sharma JK. Operations Research theory and Application, Macmillian publishers, Indian limited; c2013.
13. Zavlanos MM, Spesivtsev L, Pappas GJ. A distributed auction algorithm for the assignment problem, In Decision and Control. CDC. 47th IEEE Conference on. IEEE; c2008. p. 1212-1217.
14. Adukia KL, Agrawal B. Optimal Solution of Assignment Problem with the Help of Least Value Algorithm. International Journal of Research and Analytical Reviews. 2022;9(4):335-341.
15. http://ijrar.org/viewfull.php?&p_id=IJRAR22D2981
16. DOI (Digital Object Identifier)- http://doi.one/10.1729/Journal.32536
17. Adukia KL, Agrawal B. Optimal solution of fuzzy linear programming problems for trapezoidal number by using method of matrix inversion. ©IJEDR. 2022;10(4):80-84. https://www.ijedr.org/archive.php?vol=10&issue=4#IJEDR2204012
18. Agrawal B, Kumar S, Sharma G. Optimum Solution of a Transportation Problem with an Excel Solver. International Journal of Research and Analytical Reviews. 2022;9(3):892-901. http://ijrar.org/viewfull.php?&p_id=IJRAR22C1598 DOI (Digital Object Identifier)- http://doi.one/10.1729/Journal.33184
19. https://www.researchgate.net/publication/362344463_optimum_solution_of_a_transportation_problem_with_an_excel_solver
20. Agrawal B, Kumar P. An Approach of L.P.P Method-Game Problem using Simplex Method. International Journal of Research and Analytical Reviews. 2022;9(4):927-935. http://ijrar.org/viewfull.php?&p_id=IJRAR22D2939 DOI (Digital Object Identifier)-: http://doi.one/10.1729/Journal.33185
21. Agrawal B, Singh P. Linear Programming in an Operation Research for Optimum Best Solution in Real World Problem using Excel Solver. International Journal of Research and Analytical Reviews. 2022;9(4):34-40. http://ijrar.org/viewfull.php?&p_id=IJRAR22D1602 DOI (Digital Object Identifier)- http://doi.one/10.1729/Journal.33183
22. Gangrade A, Agrawal B, Kumar S, Mansuri A. A study of applications of graph colouring in various fields. International Journal of Statistics and Applied Mathematics. 2022;7(2):51-53. DOI: 10.22271/maths.2022.v7.i2a.795.
23. Shrivastava B, Agrawal B, Kumar S. Fuzzy linear programming problem with α-cut and roubast ranking methods. International Journal of Statistics and Applied Mathematics. 2022;7(2):57-62. DOI: 10.22271/maths.2022.v7.i2a.797.
24. Tewari P, Agrawal B. A study of linear programming technique. International Journal of Statistics and Applied Mathematics. 2022;7(2):54-56. DOI: 10.22271/maths.2022.v7.i2a.796