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New forms of connectedness in micro ideal topological spaces

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Abstract
Micro topological spaces are the extension of nano topological spaces. The intention of this paper is to introduce connectedness in Micro topological spaces together with an ideal. We introduced Micro connectedness, MI-connectedness, MI-Cl-connectedness and MI-Cl*-connectedness in Micro ideal topological spaces. We investigated the properties of them and the relationship between them and derived the related theorems. Also we introduced M-Component, MI- Component, MI-Cl-Component, MI-Cl*-Component and discussed their maximality.
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Keywords: Micro topological spaces, Micro connectedness, MI-connectedness, MI-Cl –connectedness, MI-Cl*-connectedness, Micro ideal topological spaces

1. Introduction

The concept of nano topology was first introduced by M. Lellis Thivagar *et al.* [15], which is defined in terms of lower and upper approximations and the boundary region of a subset of a universe. The notion of approximations and boundary region of a set was proposed by Z. Pawlak [23] in order to introduce the concept of rough set theory. M. Parimala *et al.* [22] introduced the concept of nano ideal topological spaces. In 2016, M. Lellis Thivagar and V. Sutha Devi introduced some new sort of operators in nano ideal topological spaces. In 2019, S. Chandrasekar [3] introduced the concept of micro topology which is an extension of nano topology. The set of elements of $(U, \tau_R(X), I)$ that satisfies $A \subseteq n\operatorname{int}(A_n^*)$ is called the set of Nano ideal open sets [20]. In a nano topological space, for any $\mu \notin \tau_R(X)$, the collection $\mu_R(X) = \{N \cup (N' \cap \mu) : N, N' \in \tau_R(X)\}$ is called the micro topology on U. The triplet $(U, \tau_R(X), \mu_R(X))$ is called the micro topological space. The elements of $\mu_R(X)$ are called micro open sets and their complements are micro closed sets [3].

Ideal topology is a topological space endowed with an additional structure namely the ideal. Kuratowski [13, 14] introduced the concept of local functions in ideal topological spaces. The notion of Kuratowski operator plays a vital role in defining ideal topological space which has its application in localization theory in set topology by Vaidyanathaswamy [27]. In 1990, Jankovic and Hamlett [8, 9] developed new topologies from old via ideals and introduced I-open sets with respect to an ideal I in 1992. The properties like continuity, separation axioms, connectedness, compactness and resolvability have been generalized using the concept of ideals in topological spaces. An ideal I as we know is a nonempty collection of subsets of X closed with respect to finite union and heredity. For a subset A of X, the local function of A is defined as follows: $A^* = \{x \in X: U \cap A \notin I \text{ for every } U \in \tau(x)\}$, where $\tau(x)$ is the collection of all nonempty open sets containing x. In this respect the study of $*$-topology is interesting which had been studied by Jankovic and Hamlett [8, 9], Modak and Bandyopadhyay [18, 19] and many other in detail and its one of the powerful base is $\beta(I, \tau) = \{V - A: V \in \tau, A \in I\}$ [4]. It is also denoted as $\tau * (I)$ [8, 9] and its closure operator is defined as $Cl*(A) = A \cup A*$. Again it is happened that $\tau \subset \tau * (I)$. The theory of ideals gets a new dimension in the case it satisfies $I \cap \tau = \{\emptyset\}$.

Such ideals are termed as codense ideals by Dontchev, Ganster and Rose [5]. The study of connectedness in an ideal topological space was introduced by Ekici and Noiri in [6]. The authors Sathiyasundari and Renukadevi [24] studied it further in detail. We in this paper introduce and study some different types of connectedness with the help of the ideals in Micro topological spaces. We also characterize these connectedness and interrelate with earlier connectedness.

2. Preliminaries

Definition: 2.1 A micro topological space $(U,\tau_R(X),\mu_R(X))$ with an ideal I on U is called a micro ideal topological space and is denoted by $(U,\tau_R(X),\mu_R(X),I)$.

Definition: 2.2 Let $(U,\tau_R(X),\mu_R(X),I)$ be a micro ideal topological space. A set operator $(A)^{*M}:P(U)\to P(U)$ is called the micro local function of I on U, is defined as $(A)^{*M}:\{u\in U:G_m(u)\cap A\notin I, where\ G_m(u)\in \mu_R(X)\}$. The $Mcl^*(A)=A\cup(A)^{*M}$ is the micro closure operator on U.

Definition: 2.3 A subset A of a micro ideal topological space $(U,\tau_R(X),\mu_R(X),I)$ is said to be micro ideal open if $A\subseteq M\operatorname{int}(A)^{*M}$. We denote $MIO(U)=\{A\subseteq U: A\subseteq M\operatorname{int}(A)^{*M}\}$.

3. Micro Ideal Connected Spaces

Connectedness is a topological property, since it is formulated entirely in terms of the collection of open sets in U.

Definition: 3.1 A Micro topological space $(U,\tau_R(X),\mu_R(X))$ is called M - Connected if U cannot be written as the disjoint union of two non empty micro open sets.

Definition: 3.2 A Micro ideal topological space $(U,\tau_R(X),\mu_R(X),I)$ is called MI - Connected if U cannot be written as the disjoint union of two non empty MI - open sets D and E such that $\overline{D}\cap E=D\cap\overline{E}=\phi$. If U is not MI - Connected, it is said to be MI - Disconnected.

Definition: 3.3 Let U be a micro ideal topological space. A separation of U is a pair A, B of disjoint nonempty MI - open sets of U, whose union is U. The space U is MI - connected if there does not exist a separation of U.

Theorem: 3.4 Every MI - Connected space is M – Connected.

Proof. Let U be MI – Connected. Suppose, if U is not M - Connected, then U can be written as the disjoint union of two non-empty M – open set A and B. Then $A=M\operatorname{int}A$ and $B=M\operatorname{int}B$. $A=M\operatorname{int}A\subseteq M\operatorname{int}A^{*M}$ and $B=M\operatorname{int}B\subseteq M\operatorname{int}B^{*M}$. Hence A and B are MI -open in U. That is, U can be written as the disjoint union of MI - open sets A and B, which is a contradiction. Thus U is M - Connected

Remark: 3.5 Converse of the above theorem need not be true. If U is M - Connected, then U need not be MI – Connected. For example, let $U=\{1,2,3,4,5\}$, $U/R=\{\{1,2\},\{3\},\{4,5\}\}$, $X=\{1,2,3\}$.This gives $\tau_R(X)=\{\phi,U,\{1,2,3\}\}$. Let $\mu=\{4\}$, then $\mu_R(X)=\{\phi,\{4\},\{1,2,3\},\{1,2,3,4\},U\}$. If $I=\{\phi,\{1,2,3\}\}$, then micro ideal open sets are $\{\phi,\{1\},\{2\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{1,5\},\{2,3\}\{2,4\},\{2,5\},\{3,4\},\{3,5\},\{1,2,4\},\{1,2,5\},\{1,3,4\},\{1,3,5\},\{1,4,5\},\{2,3,4\},\{2,3,5\},$ $\{2,4,5\},\{3,4,5\},\{1,2,4,5\},\{1,3,4,5\},\{2,3,4,5\}\}$. Here U is M-Connected, but U is not MI – Connected.

The following is an alternate definition of connectedness:

Theorem: 3.6 A Micro ideal space $(U,\tau_R(X),\mu_R(X),I)$ is MI - connected if and only if the only subsets of U that are both MI-open and MI-closed in U are empty set and U itself.

Proof. Let U be not MI -connected. Then, there exists MI -open sets A and B which form a separation of U. Then by definition of separability, A is an MI - open set which is neither empty nor equal to U. Also since $A=U\setminus B$, A is MI - closed as well. Likewise, B is also both MI - open and MI - closed. Conversely, let us assume that there exists a set A which is neither empty nor equal to U, which is both MI - open and MI - closed in U. Then set $U\setminus A$ is an MI -open subset of U. Also A and $U\setminus A$ together form a separation for U and hence set U is not MI - connected. Thus, we have proved the contrapositive of the reverse statement.

Theorem: 3.7 Let $(U,\tau_R(X),\mu_R(X),I)$ be a micro ideal topological space. If U is MI - connected, then U cannot be written as the union of two disjoint non-empty MI - closed sets.

Proof. Suppose not, if U can be written as the disjoint union of two non-empty MI – closed sets A and B, then $U = A \cup B$ and $A \cap B = \phi$. Then $A = B^C$ and $B = A^C$. Since A and B are MI – closed sets, which implies that A and B are MI – open sets. Therefore U is not MI - Connected, which is a contradiction.

Theorem: 3.8 V is a subspace of U, a separation of V is a pair of disjoint nonempty MI – open sets A and B whose union is V iff neither of which contains a limit point of the other. The space V is MI - connected if there exists no separation of V.
Proof. Let A and B form the separation of V. We need to show that A and B do not contain each other's limit points. We first show that B does not contain any limit points of A. Since $\overline{A} \cap V$ is the MI - closure of A in V, we need to show that its intersection with B is an empty set. Here, $\overline{A}$ is the MI - closure of A in U. Since A is also MI - closed in V, we have $A = \overline{A} \cap V$. But since A and B are disjoint by hypothesis, $\overline{A} \cap V$ is also disjoint with B. Hence, B does not contain any limit points of A. Similarly, we can show that A does not contain any limit points of B. Conversely, suppose that A and B are disjoint nonempty sets whose union is V, neither of which contains a limit point of the other, that is $\overline{A} \cap B = \phi = A \cap \overline{B}$. Alongwith the facts that $A \cap B = \phi$ and $A \cap B \subset \overline{A} \cap B$, we conclude that $A = \overline{A}$. i.e., A is MI - closed. Likewise, we can show that B is also MI - closed. Since $B = V \setminus A$ and $A = V \setminus B$, both A and B are MI - open in V as well.

4. Micro Ideal Closure and Closure* Connected sets

Definition: 4.1 Non empty subsets A, B of an MI-space $(U, \tau_R(X), \mu_R(X), I)$ are called MI-separated (resp. M-separated, MI*-separated) if $MCl*(A) \cap B = A \cap MCl*(B) = A \cap B = \phi$ (resp. $MCl(A) \cap B = A \cap MCl(B) = A \cap B = \phi$, $A^{*M} \cap B = A \cap B^{*M} = A \cap B = \phi$).

Definition: 4.2 Non empty subsets A, B of an MI-space $(U, \tau_R(X), \mu_R(X), I)$ are called MI-Cl*-separated (resp. MI-Cl-separated) if $MCl*(A) \cap B^{*M} = A^{*M} \cap MCl*(B) = A \cap B = \phi$ (resp. $MCl(A) \cap B^{*M} = A^{*M} \cap MCl(B) = A \cap B = \phi$).

Theorem: 4.3 Let E and F are subsets of an MI- space $(U, \tau_R(X), \mu_R(X), I)$. E and F are MI*-separated iff E and F are MI-separated.

Proof. Let E and F are MI*-separated. Then $E^{*M} \cap F = E \cap F^{*M} = E \cap F = \phi$. Consider $MCl*(E) \cap F = (E \cup E^{*M}) \cap F = (E \cap F) \cup (E^{*M} \cap F) = \phi$ $E \cap MCl*(F) = E \cap (F \cup F^{*M}) = (E \cap F) \cup (E \cap F^{*M}) = \phi$. Hence E and F are MI-separated.

Conversely, if E and F are MI-separated, then $MCl*(E) \cap F = E \cap MCl*(F) = E \cap F = \phi$. $\phi = MCl*(E) \cap F = (E \cup E^{*M}) \cap F = (E \cap F) \cup (E^{*M} \cap F) = \phi \cup (E^{*M} \cap F) = E^{*M} \cap F$ and $\phi = E \cap MCl*(F) = E \cap (F \cup F^{*M}) = (E \cap F) \cup (E \cap F^{*M}) = \phi \cup (E \cap F^{*M}) = E \cap F^{*M}$. Hence E and F are MI*-separated.

Theorem: 4.4 Let E and F are subsets of an MI- space $(U, \tau_R(X), \mu_R(X), I)$. If E and F are M-separated, then those are MI-separated.

Proof. Let E and F are M-separated. Then $MCl(E) \cap F = E \cap MCl(F) = E \cap F = \phi$. $MCl*(E) \cap F \subseteq MCl(E) \cap F = \phi$ and $E \cap MCl*(F) \subseteq E \cap MCl(F) = \phi$. Hence E and F are MI-separated.

Theorem: 4.5 Let E and F are subsets of an MI- space $(U, \tau_R(X), \mu_R(X), I)$. If E and F are MI-Cl-separated, then those are MI-Cl*-separated.

Proof. Let E and F are MI-Cl-separated. Then $MCl(E) \cap F^{*M} = E^{*M} \cap MCl(F) = E \cap F = \phi$. $MCl*(E) \cap F^{*M} \subseteq MCl(E) \cap F^{*M} = \phi$ and $E^{*M} \cap MCl*(F) \subseteq E^{*M} \cap MCl(F) = \phi$. Hence E and F are MI-Cl*-separated.

Theorem: 4.6 Let E and F are subsets of an MI- space $(U, \tau_R(X), \mu_R(X), I)$. If E and F are MI-Cl*-separated, then those are MI*-separated.

Proof. Let E and F are MI-Cl*-separated. Then $MCl*(E) \cap F^{*M} = E^{*M} \cap MCl*(F) = E \cap F = \phi$. $E \cap F^{*M} \subseteq MCl*(E) \cap F^{*M} = \phi$ and $E^{*M} \cap F \subseteq E^{*M} \cap MCl*(F) = \phi$. Hence E and F are MI*-separated.

From the above theorems, we have the following implications
$MI-Cl-separated \Rightarrow MI-Cl*-separated \Rightarrow MI*-separated \Leftrightarrow MI-separated \Leftarrow M-separated$

Definition: 4.7 A subset E of an M-space $(U, \tau_R(X), \mu_R(X),)$ is called M-connected if E cannot be written as the union of two M-separated sets in U.

Definition: 4.8 A subset E of an MI-space $(U, \tau_R(X), \mu_R(X), I)$ called
(i) MI*-connected if E cannot be written as the union of two MI*-separated sets in U.
(ii) MI-connected if E cannot be written as the union of two MI-separated sets in U.
(iii) MI-Cl*-connected if E cannot be written as the union of two MI-Cl*-separated sets in U.
(iv) MI-Cl-connected if Ecannot be written as the union of two MI-Cl-separated sets in U.

From the above definitions, we have the following implications
$M-connected \Leftarrow MI-connected \Leftrightarrow MI*-connected \Rightarrow MI-Cl*-connected \Rightarrow MI-Cl-connected$

Theorem: 4.9 Let A_1 and A_2 be two MI - connected sets in $(U, \tau_R(X), \mu_R(X), I)$ with $A_1 \cap A_2 \notin I$. Then $A_1 \cup A_2$ is MI - connected.

Proof. Suppose $A_1 \cup A_2$ is not MI -connected. Then $A_1 \cup A_2 = D \cup E$, where $D, E \notin I$ and $(A_1 \cup A_2) \cap (D \cap E) = \phi = (D \cap E) \cap (A_1 \cup A_2)$, we have $A_1 \cap A_2 = (A_1 \cap A_2 \cap D) \cup (A_1 \cap A_2 \cap E) \notin I$. So either $D \cap A_1 \cap A_2 \notin I$ or $E \cap A_1 \cap A_2 \notin I$. Suppose $D \cap A_1 \cap A_2 \notin I$, then $D \cap A_1 \notin I$ and $D \cap A_2 \notin I$. Since $A_1 = (D \cap A_1) \cup (E \cap A_1)$ is MI - connected, either $D \cap A_1 \in I$ or $E \cap A_1 \in I$. As $D \cap A_1 \notin I$, we have $E \cap A_1 \in I$. Similarly, we have $E \cap A_2 \in I$. So $E = (E \cap A_1) \cup (E \cap A_2) \in I \Rightarrow E \in I$, which is a contradiction. Hence $A_1 \cup A_2$ is MI - connected.

Theorem: 4.10 Let $(U, \tau_R(X), \mu_R(X), I)$ be an MI-space. If E is an MI-Cl-connected subsets of U and H and G are MI-Cl-separated sets of U with $E \subset H \cup G$, then either $E \subset H$ or $E \subset G$.

Proof. Let H and G be MI-Cl-separated sets and hence $H^{*M} \cap MCl(G) = MCl(H) \cap G^{*M} = H \cap G = \phi$. Let $E \subset H \cup G$. Since $E = (E \cap H) \cup (E \cap G)$ and $(E \cap G)^{*M} \cap MCl(E \cap H) \subset G^{*M} \cap MCl(H) = \phi$. In the similar way, we have $(E \cap H)^{*M} \cap MCl(E \cap G) = \phi$. Moreover $(E \cap H) \cap (E \cap G) \subset H \cap G = \phi$.Suppose that $E \cap H$ and $E \cap G$ are non-empty, then E is not an MI-Cl-connected set. This is a contradiction. Thus either $E \cap H = \phi$ or $E \cap G = \phi$. This implies that $E \subset H$ or $E \subset G$.

Theorem: 4.11 Let $(U, \tau_R(X), \mu_R(X), I)$ be an MI-space. If E is an MI-Cl*-connected subsets of U and H and G are MI-Cl*-separated sets of U with $E \subset H \cup G$, then either $E \subset H$ or $E \subset G$.
Proof. The proof is similar to the above theorem.

Theorem: 4.12 If E is an MI-Cl-connected subset of $(U, \tau_R(X), \mu_R(X), I)$ and $E \subset F \subset E^{*M}$, then F is also an MI-Cl-connected subset of U.

Proof. Suppose F is not an MI-Cl-connected subset of $(U, \tau_R(X), \mu_R(X), I)$, then there exist MI-Cl-separated sets H and G such that $F = H \cup G$.This gives that H and G are non-empty and $G^{*M} \cap MCl(H) = \phi = MCl(G) \cap H^{*M}$. By the above theorem, we have that either $E \subset H$ or $E \subset G$. Suppose that $E \subset H$. Then $E^{*M} \subset H^{*M}$. This implies that $G \subset F \subset E^{*M}$ and $MCl(G) = E^{*M} \cap MCl(G) = H^{*M} \cap MCl(G) = \phi$. Thus G is an empty set. Since G is nonempty, this is a contradiction. Hence F is MI-Cl-connected.

Theorem: 4.13 If E is an MI-Cl*-connected subset of $(U, \tau_R(X), \mu_R(X), I)$ and $E \subset F \subset E^{*M}$, then F is also an MI-Cl*-connected subset of U.

Proof. Suppose F is not an MI-Cl*-connected subset of $(U, \tau_R(X), \mu_R(X), I)$, then there exist MI-Cl*-separated sets H and G such that $F = H \cup G$.This gives that H and G are non-empty and $G^{*M} \cap MCl*(H) = \phi = MCl*(G) \cap H^{*M}$. By theorem, we have that either $E \subset H$ or $E \subset G$.Suppose that $E \subset H$. Then $E^{*M} \subset H^{*M}$. This implies that, $G \subset F \subset E^{*M}$ and $MCl*(G) = MCl*(E^{*M}) \cap MCl*(G) \subset MCl(E^{*M}) \cap MCl*(G) \subset E^{*M} \cap MCl*(G) = H^{*M} \cap MCl*(G) = \phi$. Thus G is an empty set. Since G is nonempty, this is a contradiction. Hence F is MI-Cl*-connected.

Corollary: 4.14 Let $(U, \tau_R(X), \mu_R(X), I)$ be an MI-space
a) If E is an MI-Cl-connected set in U, then E^{*M} is MI-Cl-connected.

b) If E is an MI-Cl*-connected set in U, then E^{*M} is MI-Cl*-connected.

Proof. a) Let E be an MI-Cl-connected set, then $F = H \cup G$, H and G are nonempty disjoint sets and $H^{*M} \cap MCl(G) \neq \phi \neq G^{*M} \cap MCl(H)$. Since $Cl(E^{*M}) \subseteq Cl(E)$ and $(E^{*M})^{*M} \subseteq E^{*M}$ for any set E, $(H^{*M})^{*M} \cap MCl(G^{*M}) \subseteq H^{*M} \cap MCl(G) \neq \phi$. Similarly, $(G^{*M})^{*M} \cap MCl(H^{*M}) \neq \phi$ and $G^{*M} \cap H^{*M} \neq \phi$. $\therefore$ E^{*M} is MI-Cl-connected.

b) In the similar manner we can prove the theorem using the definition of MI-Cl*-connected set.

Corollary: 4.15 Let $(U, \tau_R(X), \mu_R(X), I)$ be an MI-space

a) If $I \cap \mu = \{\phi\}$, then for any nonempty MI-open, MI-Cl-connected set V, MCl(V) is also MI-Cl-connected set in U.

b) If $I \cap \mu = \{\phi\}$, then for any nonempty MI-open, MI-Cl*-connected set V, MCl (V) is also MI-Cl*-connected set in U.

Proof. a) Let V be any nonempty, MI-open, MI-Cl-connected set and $I \cap \mu = \{\phi\}$. Then there exist non disjoint sets H and G such that $V = H \cup G$ and $H^{*M} \cap MCl(G) \neq \phi \neq G^{*M} \cap MCl(H)$. $MCl(V) = MCl(H \cup G) = MCl(H) \cup MCl(G)$. $MCl(H) \cap MCl(G) \supseteq MCl(H \cap G) \neq MCl(\phi) \neq \phi$.

$[MCl(H)]^{*M} \cap MCl(MCl(G)) \subseteq MCl(MCl(H)) \cap MCl(MCl(G)) = MCl(H) \cap MCl(G) \neq \phi$. Similarly $MCl(MCl(H)) \cap [MCl(G)]^{*M} \neq \phi$. $\therefore MCl(V)$ is MI-Cl-connected.

b) Let V be any nonempty, MI-open, MI-Cl-connected set and $I \cap \mu = \{\phi\}$. Then there exist non disjoint sets H and G such that $V = H \cup G$ and $H^{*M} \cap MCl*(G) \neq \phi \neq G^{*M} \cap MCl*(H)$. $MCl(V) = MCl(H \cup G) = MCl(H) \cup MCl(G)$. $MCl(H) \cap MCl(G) \supseteq MCl(H \cap G) \neq MCl(\phi) \neq \phi$.

$[MCl(H)]^{*M} \cap MCl*(MCl(G)) \subseteq MCl(MCl(H)) \cap MCl*(MCl(G)) \subseteq MCl(MCl(H)) \cap MCl(MCl(G)) MCl(H) \cap$. $MCl(G) \neq \phi$. Similarly $MCl*(MCl(H)) \cap [MCl(G)]^{*M} \neq \phi$. $\therefore MCl(V)$ is MI-Cl*-connected.

Theorem: 4.16 If $\{M_i : i \in N\}$ is a nonempty family of MI-Cl-connected sets of an MI-space $(U, \tau_R(X), \mu_R(X), I)$ with $\bigcap_{i \in N} M_i \neq \phi$, then $\bigcup_{i \in N} M_i$ is MI-Cl-connected.

Proof. Suppose $\bigcup_{i \in N} M_i$ is not MI-Cl-connected. Then $\bigcup_{i \in N} M_i = H \cup G$, where H and G are MI-Cl-separated sets in U. Since $\bigcap_{i \in N} M_i \neq \phi$, there exist a point $x \in \bigcap_{i \in N} M_i$. Since $x \in \bigcup_{i \in N} M_i$, either $x \in H$ or $x \in G$. Suppose that $x \in H$. Since $x \in M_i$, for each $i \in N$, then M_i intersect H for each $i \in N$. By theorem, $M_i \subset H$ or $M_i \subset G$. Suppose $M_i \subset H$. Since H and G are disjoint, $M_i \subset H$ for all $i \in N$ and hence $\bigcup_{i \in N} M_i \subseteq H$. This implies that, G is empty. This is a contradiction. Suppose that $x \in G$, by the similar way we can prove that H is empty, which gives a contradiction. Hence $\bigcup_{i \in N} M_i$ is MI-Cl-connected.

Theorem: 4.17 If $\{M_i : i \in N\}$ is a nonempty family of MI-Cl*-connected sets of an MI-space $(U, \tau_R(X), \mu_R(X), I)$ with $\bigcap_{i \in N} M_i \neq \phi$, then $\bigcup_{i \in N} M_i$ is MI-Cl*-connected.

Proof. The proof is similar to the above theorem.

Corollary: 4.18 Let $(U, \tau_R(X), \mu_R(X), I)$ be an`MI-space.

a) If E is an MI-Cl-connected subset of U and $E \cap E^{*M} \neq \phi$, then $MCl*(E)$ is an MI-Cl- connected set.

b) If E is an MI-Cl*-connected subset of U and $E \cap E^{*M} \neq \phi$, then $MCl*(E)$ is an MI-Cl*- connected set.

Proof. Since $E \cap E^{*M} \neq \phi$, then by theorem, $E \cup E^{*M}$ is MI-Cl-connected. $MCl*(E) = E \cup E^{*M}$ is MI-Cl-connected. Similarly, if E is MI-Cl*-connected and $E \cap E^{*M} \neq \phi$, then $MCl*(E)$ is an MI-Cl*- connected set.

Theorem: 4.19 Let $(U, \tau_R(X), \mu_R(X), I)$ be an MI-space. Let $\{E_\beta : \beta \in \Omega\}$ be a family of MI-Cl-connected subsets of U and E be an MI-Cl-connected subset of U. If $E \cap E_\beta \neq \phi$ for every β, then $E \cup (\cup E_\beta)$ is an MI-Cl-connected set.

Proof. Since $E \cap E_\beta \neq \phi$ for each β, by theorem, $E \cup E_\beta$ is MI-Cl-connected for each β. Moreover $E \cup (\cup E_\beta) = \cup (E \cup E_\beta)$ and $\cap (E \cup E_\beta) \supset E \neq \phi$. Thus by theorem, $E \cup (\cup E_\beta)$ is MI-Cl-connected.

Theorem: 4.20 Let $(U, \tau_R(X), \mu_R(X), I)$ be an`MI-space. Let $\{E_\beta : \beta \in \Omega\}$ be a family of MI-Cl*-connected subsets of U and E be an MI-Cl*-connected subset of U. If $E \cap E_\beta \neq \phi$ for every β, then $E \cup (\cup E_\beta)$ is an MI-Cl*-connected set.
Proof. The proof is similar to the above theorem.

Definition: 4.21 A subset E of U is called MI-dense in itself if $E = E^{*M}$.

Theorem: 4.22 Let $(U, \tau_R(X), \mu_R(X), I)$ be an MI-space. If E and F, subsets of U are MI-Cl-separated and MI-dense in itself and $E \cup F \in \mu_R(X)$, then E and F are M-open and hence MI-open.

Proof. Since E and F are MI-Cl-separated in U, then $E = (E \cup F) \cap (U - MCl(F))$. Since $E \cup F \in \mu_R(X)$ and $MCl(F)$ is M-closed in U, then E is M-open in U. By the similar way, we obtain that F is M-open. Since E and F are MI-dense in itself, then E and F are MI-open.

Theorem: 4.23 Let $(U, \tau_R(X), \mu_R(X), I)$ be an MI-space. If E and F are MI-Cl*-separated and MI-dense in itself subsets of U and $E \cup F \in \mu_R(X)$, then E and F are M-open and hence MI-open.
Proof. The proof is similar to the above theorem.

Definition: 4.24 Let $(U, \tau_R(X), \mu_R(X), I)$ be an MI-space and $u \in U$. The union of all MI-Cl-connected (resp. MI-Cl*-connected) subsets of U containing u is called the MI-Cl –component (resp. MI-Cl*-component) of U containing u.

Definition: 4.25 a) Each MI-Cl-component of an MI-space $(U, \tau_R(X), \mu_R(X), I)$ is a maximal MI-Cl-connected set of U.
b) Each MI-Cl*-component of an MI-space $(U, \tau_R(X), \mu_R(X), I)$ is a maximal MI-Cl*-connected set of U.

Theorem: 4.26 The set of all distinct MI-Cl-component of an MI-space $(U, \tau_R(X), \mu_R(X), I)$ forms a partition of U.
Proof. Let E and F be two distinct MI-Cl-component of U. Suppose E and F intersect, then by theorem, $E \cup F$ is MI-Cl-connected in U. Since $E \subset E \cup F$, then E is not maximal. Thus E and F are disjoint and hence form a partition of U.

Theorem: 4.27 The set of all distinct MI-Cl*-component of an MI-space $(U, \tau_R(X), \mu_R(X), I)$ forms a partition of U.

Theorem: 4.28 Let $(U, \tau_R(X), \mu_R(X), I)$ be an MI-space, I is condense. Then each MI-Cl-connected subset of U which is both M-open and MI-closed is MI-Cl-component of U.

Proof. Let E be an MI-Cl-connected subset of U such that E is both M-open and MI-closed. Let $u \in E$. Since E is an MI-Cl-connected subset of U containing u, if D is the MI-Cl-component containing u, then $E \subseteq D$. Let E be a proper subset of D. Then D is non-empty and $D \cap (U - E) \neq \phi$. Since E is M-open and MI-closed, U-E is M-closed and MI-open and $(E \cap D) \cap ((U - E) \cap D) = \phi$. Also $(E \cap D) \cup ((U - E) \cap D) = (E \cup (U - E)) \cap D = D$. Again E and U-E are two nonempty disjoint M-open set and MI-open set respectively, such that $E \cap MCl(U - E) = \phi = MCl*(E) \cap (U - E)$. This implies that $E^{*M} \cap MCl(U - E) = \phi = MCl(E) \cap (U - E)^{*M}$, since I is condense and $(U - E)^{*M} \subset MCl(U - E)$. This shows that $E \cap D$ and $(U - E) \cap D$ are MI-Cl-separated sets. This is a contradiction. Hence E is not a proper subset of D and E=D. This completes the proof.

Theorem: 4.29 Let $(U, \tau_R(X), \mu_R(X), I)$ be an MI-space, I is condense. Then each MI-Cl*-connected subset of U which is both M-open and MI-closed is MI-Cl*-component of U.

Proof. Let E be an MI-Cl*-connected subset of U such that E is both M-open and MI-closed. Let $u \in E$. Since E is an MI-Cl*-connected subset of U containing u, if D is the MI-Cl*-component containing u, then $E \subseteq D$. Let E be a proper subset of D. Then D is non-empty and $D \cap (U - E) \neq \phi$. Since E is M-open and MI-closed, U-E is M-closed and MI-open and $(E \cap D) \cap ((U - E) \cap D) = \phi$. Also $(E \cap D) \cup ((U - E) \cap D) = (E \cup (U - E)) \cap D = D$. Again E and U-E are two nonempty disjoint M-open set and MI-open set respectively, such that $E \cap MCl*(U - E) = \phi = MCl*(E) \cap (U - E)$.

This implies that $E^{*M} \cap MCl*(U-E) \subset MCl*(E) \cap MCl(U-E) = MCl*(E) \cap (U-E) = \phi$ and $MCl*(E) \cap (U-E)^{*M} \subset E \cap MCl(U-E) = \phi$. This shows that $E \cap D$ and $(U-E) \cap D$ are MI-Cl*-separated sets. This is a contradiction. Hence E is not a proper subset of D and E=D. This completes the proof.

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