ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 8.4
IJAR 2023; 9(5): 253-255
www.allresearchjournal.com
Received: 25-02-2023
Accepted: 18-04-2023

Dr. Sandeep Kumar Tiwari
Associate Professor,
Department of Mathematics,
Motherhood University,
Roorkee, Uttarakhand, India

Corresponding Author:
Dr. Sandeep Kumar Tiwari
Associate Professor,
Department of Mathematics,
Motherhood University,
Roorkee, Uttarakhand, India

Irrotational motion of an oscillating flat plate of incompressible viscous fluid through porous medium under the variation of magenetic field

Dr. Sandeep Kumar Tiwari

Abstract
In the present paper, we have investigated irrotational motion of an oscillating flat plate of incompressible viscous fluid through porous medium under the variation of magnetic field. We have investigated path line, stream line, velocity potential, stream function complex potential and motion.

Keywords: Stream function, Complex potential, porous medium

Introduction
The study of the magnetic properties of electrically conducting fluids. There are various type of such Magneto fluids include plasmas, liquid metals, salt water, and electrolytes. The word "Magne to hydrodynamics" is derived from magneto-meaning magnetic field, hydro-meaning water and dynamics meaning movement. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself. The set of equations that describe MHD are a combination of the Navier–Stokes equations of fluid dynamics. These differential equations must be solved simultaneously, either analytically or numerically. Oscillating flat plate, refers to the boundary layer close to a solid wall in oscillatory flow of a viscous fluid. It refers to the similar case of an oscillating plate in a viscous fluid at rest, with the oscillation direction (s) parallel to the plate. For the case of laminar flow at low Reynolds numbers over a smooth solid wall, George Gabriel Stokes – after whom this boundary layer is called – derived an analytic solution, one of the few exact solutions for the Navier–Stokes equations. In turbulent flow, this is still named a Stokes boundary layer, but now one has to rely on experiments, numerical simulations or approximate methods in order to obtain useful information on the flow. The problem was based on how the motion is irrotational whenever the magnetic field varied in incompressible viscous fluid, also the plate is porous the fluid flow easily due to pores.

Literature Review
In the present paper, we have investigated irrotational motion of an oscillating flat plate of incompressible viscous fluid through porous medium under the variation of magnetic field. Attempts have made by several researchers. i.e. A.C. Srivastava [1] investigated flow in a porous medium induced by torsional oscillation of a disk near its surface. A.K. Singh, Ajay Kumar Singh and N.P. Singh [2] investigated heat and mass transfer in MHD flow of a viscous plate under oscillatory suction velocity. B.S. Hundal and Rajneesh Kumar [3] investigated wave propagation in a fluid saturated incompressible porous medium. G.D. Gupta and Rajesh Johari [4] investigated MHD three dimensional flow past a porous plate. H.B. Lofgren [5] investigated ideal solidification of a liquid meatel boundary layer flow over a conveying substrate. H.K. Mondal and T.K. Choudhary [6] investigated unsteady cinvective flow of a viscous thermally stratified fluid through a vertical channel. J.D. Gibbon and C.R. Doering [7] investigated intermittency in solutions of three dimensional Navier-stokes equation. We have investigated in such type fluid, the stream line, path line, velocity potential, complex potential is possible and motion is rotational or irrotational.

Formulation of the problem

Let there be an infinite flat plate oscillating in its own plane with velocity $u_0\ (\eta)^{\sin\xi t}$, $v_0\ (\eta)^{\sin\xi t}$, $w_0\ (\eta)^{\sin\xi t}$ in infinite incompressible viscous fluid through porous medium under the variation of magnetic field of permeability κ.
With the boundary condition

$$\left.\begin{aligned} u &= u_0 \text{ at } x = 0\\ u &= 0 \text{ at } x = \infty\\ v &= v_0 \text{ at } x = 0\\ v &= 0 \text{ at } x = \infty\\ w &= w_0 \text{ at } x = 0\\ w &= 0 \text{ at } x = \infty \end{aligned}\right\} \quad (1)$$

Governing Equation

The governing equation of motion of an incompressible viscous fluid through porous medium under the variation of magnetic field of permeability κ.

$$\nabla.\bar{q} = 0 \quad (2)$$

$$\rho\frac{\partial\bar{q}}{\partial t} = -\nabla p - \frac{\mu}{k}\bar{q} + \mu\nabla^2\bar{q} + \frac{\sigma\mu^2{B_0}^2\bar{q}}{\rho} \quad (3)$$

Solution of the problem

The equation (1) is satisfied identically and equation (2) becomes

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} - \frac{\mu}{k}u + \mu\frac{\partial^2 u}{\partial x^2} + \frac{\sigma\mu^2{B_0}^2 u}{\rho} \quad (4)$$

and $$\rho\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} - \frac{\mu}{k}v + \mu\frac{\partial^2 v}{\partial x^2} + \frac{\sigma\mu^2{B_0}^2 v}{\rho} \quad (5)$$

$$\rho\frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} - \frac{\mu}{k}w + \mu\frac{\partial^2 w}{\partial x^2} + \frac{\sigma\mu^2{B_0}^2 w}{\rho} \quad (6)$$

Substituting u, v, w and solving we get

$$\frac{d^2u}{dx^2} - \left[\frac{1}{k} + \frac{\xi\cos\xi t\log\eta}{\vartheta} - \sigma\vartheta{B_0}^2\right]u(x) = 0 \quad (7)$$

$$u = u_0\exp\left[-x\sqrt{\frac{1}{k} + \frac{\xi\cos\xi t\log\eta}{\vartheta} - \sigma\vartheta{B_0}^2}\right](\eta)^{\sin\xi t} \quad (8)$$

$$v = v_0\exp\left[-y\sqrt{\frac{1}{k} + \frac{\xi\cos\xi t\log\eta}{\vartheta} - \sigma\vartheta{B_0}^2}\right](\eta)^{\sin\xi t} \quad (9)$$

$$w = w_0\exp\left[-z\sqrt{\frac{1}{k} + \frac{\xi\cos\xi t\log\eta}{\vartheta} - \sigma\vartheta{B_0}^2}\right](\eta)^{\sin\xi t} \quad (10)$$

The path line is

$$\frac{1}{\sqrt{\frac{1}{k} + \frac{\xi\cos\xi t\log\eta}{\vartheta} - \sigma\vartheta{B_0}^2}}\left[\exp\left\{x\sqrt{\frac{1}{k} + \frac{\xi\cos\xi t\log\eta}{\vartheta} - \sigma\vartheta{B_0}^2}\right\} - 1\right] = \frac{\mu_0}{\xi\{1-(\log\eta)^2\}}\left[\frac{(\eta)^{\sin\xi t}}{\cos\xi t}(t\cos t - \log\eta) + u_0\log\eta\right] \quad (11)$$

$$\frac{1}{\sqrt{\frac{1}{k} + \frac{\xi\cos\xi t\log\eta}{\vartheta} - \sigma\vartheta{B_0}^2}}\left[\exp\left\{y\sqrt{\frac{1}{k} + \frac{\xi\cos\xi t\log\eta}{\vartheta} - \sigma\vartheta{B_0}^2}\right\} - 1\right] = \frac{\mu_0}{\xi\{1-(\log\eta)^2\}}\left[\frac{(\eta)^{\sin\xi t}}{\cos\xi t}(t\cos t - \log\eta) + v_0\log\eta\right] \quad (12)$$

$$\frac{1}{\sqrt{\frac{1}{k} + \frac{\xi\cos\xi t\log\eta}{\vartheta} - \sigma\vartheta{B_0}^2}}\left[\exp\left\{z\sqrt{\frac{1}{k} + \frac{\xi\cos\xi t\log\eta}{\vartheta} - \sigma\vartheta{B_0}^2}\right\} - 1\right] = \frac{\mu_0}{\xi\{1-(\log\eta)^2\}}\left[\frac{(\eta)^{\sin\xi t}}{\cos\xi t}(t\cos t - \log\eta) + w_0\log\eta\right] \quad (13)$$

The stream line is

$$v_0\left[\exp\left\{x\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}\right\}-1\right]=u_0\left[\exp\left\{y\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}\right\}-1\right] \quad (14)$$

$$w_0\left[\exp\left\{y\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}\right\}-1\right]=v_0\left[\exp\left\{z\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}\right\}-1\right] \quad (15)$$

$$u_0\left[\exp\left\{z\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}\right\}-1\right]=w_0\left[\exp\left\{x\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}\right\}-1\right] \quad (16)$$

The velocity potential is

$$(\varphi)_a=\frac{u_0\left[-x\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}\right]}{\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}}(\eta)^{\sin\xi t} \quad (17)$$

$$(\varphi)_b=\frac{v_0\left[-y\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}\right]}{\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}}(\eta)^{\sin\xi t} \quad (18)$$

$$(\varphi)_c=\frac{w_0\left[-z\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}\right]}{\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}}(\eta)^{\sin\xi t} \quad (19)$$

The Stream function is

$$\Psi=u_0\, y\exp\left[-x\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}\right](\eta)^{\sin\xi t} \quad (20)$$

The Complex potential is

$$W=u_0\exp\left[-x\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}\right](\eta)^{\sin\xi t}\left[\frac{1}{\sqrt{\frac{1}{k}+\frac{\xi\cos\xi t\log\eta}{\vartheta}-\sigma\vartheta {B_0}^2}}+iy\right] \quad (21)$$

Also curl q = 0 …………… (22)

Result and Discussion

In the present paper, we have investigated velocity components, path line, stream line, velocity potential, stream function, complex potential and irrotational motion of an oscillating flat plate of incompressible viscous fluid through porous medium under the variation of magnetic field. Given by the equation (8-22).

Nomenclature: $\overline{q}$ = velocity vector, u = Velocity along x axis, v = Velocity along y axis, w = velocity along z axis, φ = Velocity potential, ψ = Stream function, W = Complex potential, ρ = Density of fluid. μ = coefficients of viscosity, K = permeability of porous medium.

Reference

1. Srivastava AC, Angew Z. Math. Phys. 1999;50:529 -545.
2. Singh AK, Ajay Kumar Singh, Singh NP. Ind. J Pure and Appl. Math. 2003;34(3):429-442.
3. Hundal S, Rajneesh Kumar. Ind. J Pure and Appl. Math. 2003;34(4):651-665.
4. Gupta GD, Rajesh Johari. Ind. J Pure and Appl. Math. 2001;32(3):377-386.
5. Lofgren HB. J fluid mech. 2001;446:121-131.
6. Mondal HK, Choudhary TK. *Acta ciencia indica*. 1993;19(1):15-18.
7. Gibbon JD, Douering CR. J Fluid Mech; c2003. p. 227-235.