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Heptagonal graceful labeling of caterpillar and path related graphs

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Abstract
Numbers of the form $\frac{5n^2-3n}{2}$ for all $n \geq 1$ are called heptagonal numbers. Let G be a graph with p vertices and q edges. Let $f : V(G) \to \{0, 1, 2, \dots, N_q\}$ where N_q is the q^{th} heptagonal number be an injective function. Define the function $f^* : (E(G)) \to \{N_1, N_2, N_3, \dots, N_q\}$ such that $f^*(uv) = |f(u) - f(v)|$ for all edges $uv \in E(G)$. If $f^*(E(G))$ is a sequence of distinct consecutive numbers $\{N_1, N_2, N_3, \dots, N_q\}$ then the function f is said to be heptagonal graceful labeling and the graph which admits such a labeling is called a heptagonal graceful graph.

Keywords: Heptagonal graceful labeling, heptagonal graceful of some graphs, heptagonal graceful labeling of path graphs, heptagonal graceful labeling of caterpillar graphs

1. Introduction
Numbers of the form $\frac{n(n+1)}{2}$ for all $n \geq 1$ are called triangular numbers. J. Devaraj & T. A. Thankam studied some classes of triangular graceful graphs [1]. Numbers of the form $\frac{n(3n-1)}{2}$ for all $n \geq 1$ are called pentagonal numbers and S. Mahendran & K. Murugan introduced pentagonal graceful labeling of some graphs [4] and it is defined as follows:
Let G be a graph with p vertices and q edges. Let $f: V(G) \to \{0, 1, 2, \dots, P_q\}$ where P_q is the q^{th} pentagonal number be an injective function. Define the function $f^*: E(G) \to \{1, 5, \dots, P_q\}$ such that $f^*(uv) = |f(u) - f(v)|$ for all edges $uv \in E(G)$. If $f^*(E(G))$ is a sequence of distinct consecutive pentagonal numbers $\{P_1, P_2, \dots, P_q\}$, then the function f is said to be pentagonal graceful labeling [4] and the graph which admits such a labeling is called a pentagonal graceful graph. Inspired from the above works in this chapter we study the heptagonal graceful labeling of path related graphs.

2. Preliminaries
Definition 2.1: Let $v_1, v_2, v_3, \dots, v_m$ be the m vertices of P_m. From each vertex $v_i, i = 1,2, \dots, m$, there are $n_i, i = 1,2, \dots, m$ pendant vertices say $v_{i1}, v_{i2}, \dots, v_{in_i}$. The resultant graph is a caterpillar and is denoted as $B(n_1, n_2, \dots, n_m)$ [6].
The caterpillar can also be defined in the following way:
G is called a caterpillar if G is a tree such that the removal of the vertices with degree 1 results in a path, and that path is called the spine of the caterpillar [6].

Definition 2.2: $P_n(1, 2, \dots, n)$ [4] is a graph obtained from a path of vertices $v_1, v_2, v_3, \dots, v_n$ having the path length n by joining i pendant vertices at each of its i^{th} vertex.

Definition 2.3: The Corona $G_1 \odot G_2$ [4] of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p vertices) and p copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

3. Results and Discussions

Theorem 3.1: The caterpillar $B(n_1, 0, n_2)$ [6] is heptagonal graceful for all $n_1, n_2 \geq 1$.

Proof.

Let v_1, v_2, v_3 be three vertices of path P_3. From v_1, there are n_1 pendant vertices say $u_1, u_2, \dots, u_{n_1}$ and from v_3 , there are n_2 pendant vertices say $w_1, w_2, \dots, w_{n_2}$. The resulting graph is defined as $B(n_1, 0, n_2)$. Let it be $G = (V, E)$. Then G has $n_1 + n_2 + 3$ vertices and $n_1 + n_2 + 2 = m$ edges.

Let $n_1 + n_2 + 2 = q\ (say)$.

Define $f : V\ (B(n_1, 0, n_2)) \rightarrow \{0, 1, 2, \dots, N_q\}$ be defined as follows:

$f(v_1) = N_q$

$f(v_2) = 0$

$f(v_3) = N_{q-n_1-1}$

$f(u_i) = N_q - N_{q-i}, 1 \leq i \leq n_1$

$f(w_j) = N_{q-n_1-1} + N_j, 1 \leq j \leq n_2$

We shall prove that G admits heptagonal graceful labeling. From the definition, it is clear that $\max_{v \epsilon V(G)} f(v)$ is N_q and $f(v) \in \{0,1,2, \dots, N_q\}$.

Also from the definition, all the vertices of G have different labeling.

Hence f is one to one.

It remains to show that the edge values are of the form $\{N_1, N_2, \dots, N_q\}$.

The induced edge function $f^*: E(G) \rightarrow \{1,2, \dots, N_q\}$ is defined as follows:

$f^*(v_1v_2) = N_q$

$f^*(v_2v_3) = N_{q-n_1-1}$

$f^*(v_1u_i) = N_{q-i}, 1 \leq i \leq n_1$

$f^*(v_3w_j) = N_j, 1 \leq j \leq n_2$

Clearly f^* is one to one and $f^*(E(G) = \{N_1, N_2, \dots, N_q\}$. Therefore G admits heptagonal graceful labeling.

Hence the graph $B(n_1, 0, n_2)$ is heptagonal graceful.

Example: 3.2: The heptagonal graceful labeling of $B(5,0,4)$

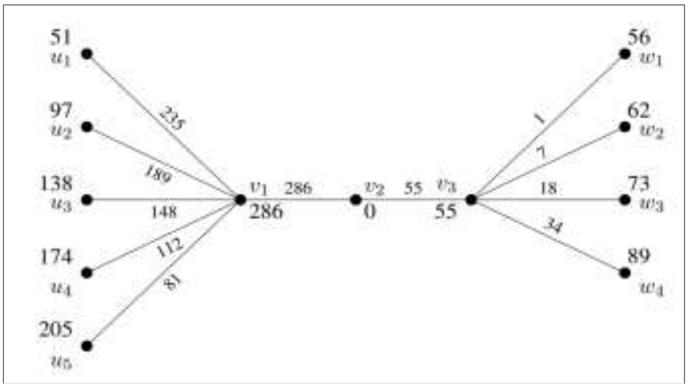

Fig 1: Heptagonal graceful labeling of $B(5,0,4)$

Lemma: 3.3: The caterpillar $B(n_1, 1, n_2)$ [6] is heptagonal graceful for all $n_1, n_2 \geq 1$.

Example: 3.4: The heptagonal graceful labeling of $B(5,1,4)$ is given below.

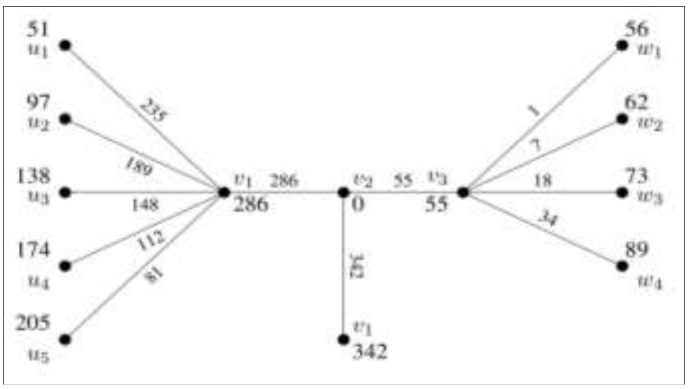

Fig 2: Heptagonal graceful labeling of $B(5,1,4)$

Theorem 3.1: Caterpillars $B(n_1, n_2, n_3, \dots, n_m)$ [4] are heptagonal graceful.

Proof. Let G be the caterpillar graph $B(n_1, n_2, n_3, \dots, n_m)$
Let $v_1, v_2, v_3, \dots, v_m$ be the m vertices of the path P_m .
From each vertexv_i,$i = 1, 2, \dots, m$ there are $n_i, i = 1,2, \dots, m$pendant vertices say $v_{i1}, v_{i2}, v_{i3}, \dots, v_{in_i}$.
The resultant graph is a caterpillar and is denoted as $B(n_1, n_2, n_3, \dots, n_m)$
Assume ≥ 3 .
Clearly $B(n_1, n_2, n_3, \dots, n_m)$ has $n_1 + n_2 + n_3 + \cdots + n_m + (m-1)$ edges
Let $q = n_1 + n_2 + n_3 + \cdots + n_m + (m-1)$
Define $f : V(B(n_1, n_2, n_3, \dots, n_m)) \to \{0, 1, 2, \dots, N_q\}$ be defined as follows:
$f(v_1) = 0$
$f(v_{1i}) = N_{q-(i-1)}\, for\ 1 \leq i \leq n_1$
$f(v_2) = f(v_1) + N_{q-n_1}$
$f(v_{2i}) = f(v_2) - N_{q-n_1-i}\, where\ i = 1, 2, \dots, n_2$
$f(v_3) = f(v_2) - N_{q-n_1-n_2-1}$
$f(v_{3i}) = f(v_3) + N_{q-n_1-n_2-i-1}, i = 1, 2, \dots, n_3$
$f(v_4) = f(v_3) + N_{q-n_1-n_2-n_3-2}$
$f(v_{4i}) = f(v_4) - N_{q-n_1-n_2-n_3-2-i}, i = 1, 2, \dots, n_4$
and so on In general,

$$f(v_m) = \begin{cases} f(v_{m-1}) - N_{q-n_1-n_2-\cdots-n_{m-1}-(m-2)}, for\ m > 1\ and\ if\ m\ is\ odd \\ f(v_{m-1}) + N_{q-n_1-n_2-\cdots-n_{m-1}-(m-2)}, for\ m > 1\ and\ if\ m\ is\ even \end{cases}$$

Also we have,

$$f(v_{m_i}) = \begin{cases} f(v_m) - N_{q-n_1-n_2-\cdots-n_{m-1}-(m-2)-i}, for\ m > 1\ and\ if\ m\ is\ even, 1 \leq i \leq n_m \\ f(v_m) + N_{q-n_1-n_2-\cdots-n_{m-1}-(m-2)-i}, for\ m > 1\ and\ \text{if}\ m\ is\ odd,\ 1 \leq i \leq n_m \end{cases}$$

For $i = n_m, f(v_{mn_m}) = f(v_m) \pm N_{q-n_1-\dots-n_{m-1}-(m-2)-n_m}$
$i.e) f(v_{mn_m}) = f(v_m) \pm N_{q-n_1-\dots-n_{m-1}-n_m-m+2}$
$= f(v_m) \pm N_1$

Clearly the vertex labels are distinct and the resulting edge labels are of the form
$\{N_1, N_2, \dots, N_q\}$.
Hence the caterpillar graph $B(n_1, n_2, n_3, \dots, n_m)$ is heptagonal graceful.

Example 3.2: Heptagonal graceful labeling of Caterpillar graph B (3, 2, 3, 5) [4] is given below

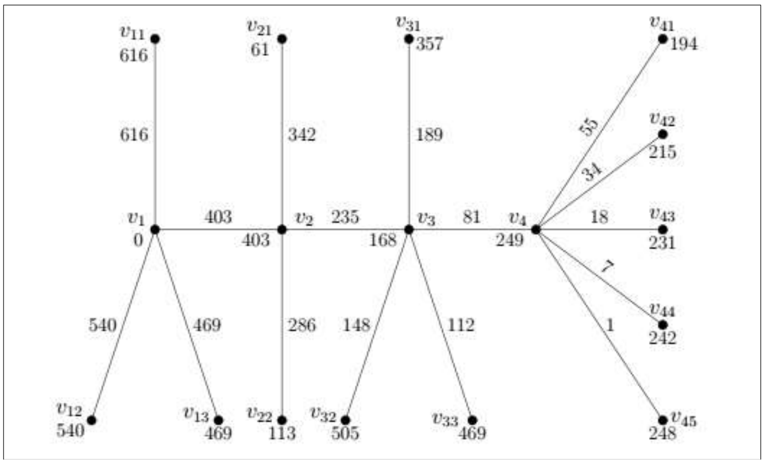

Fig 3: Heptagonal graceful labeling of Caterpillar graph B (3, 2, 3, 5)

Corollary 3.3: When $n_i = m$, $1 \leq i \leq m$, the graph $P_n \odot \overline{K_m}$ [4] is heptagonal graceful for all $n \geq 2$ and $m \geq 1$.

Example 3.4: Heptagonal graceful labeling of $P_3 \odot \overline{K_3}$ [4] is shown below

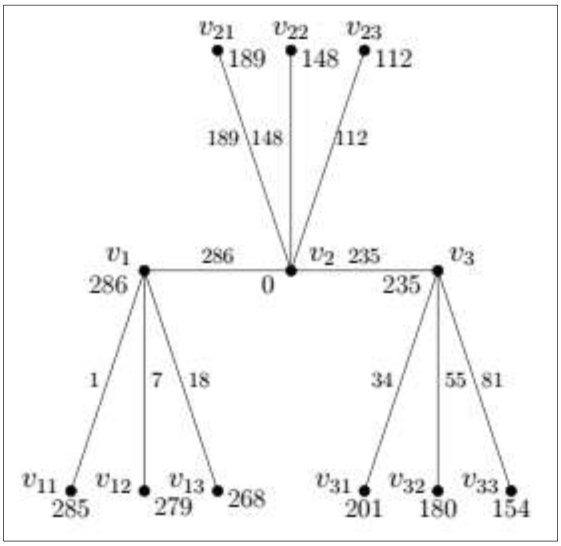

Fig 4: Heptagonal graceful labeling of $P_3 \odot \overline{K_3}$

Corollary 3.5: When $m = 1$, the graph $P_n \odot K_1$ [4] is called a comb. Comb is heptagonal graceful.

Example 3.6: Heptagonal graceful labeling of $P_5 \odot K_1$ [4] is shown below

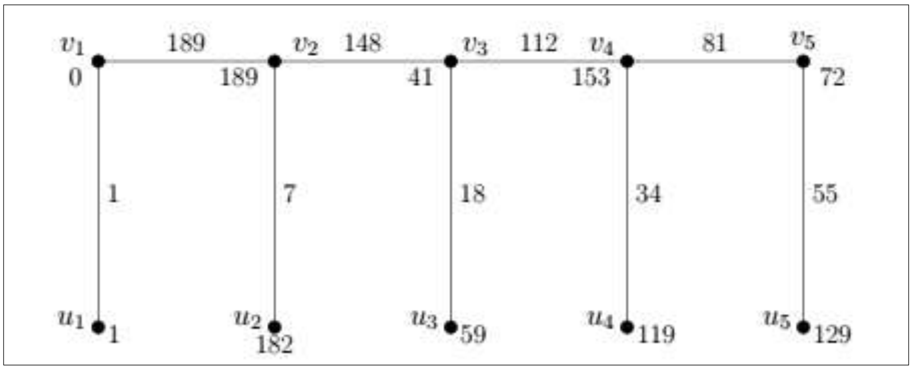

Fig 5: Heptagonal graceful labeling of $P_5 \odot K_1$

Corollary 3.7: $P_n(1, 2, \dots, n)$ [4] is heptagonal graceful.

Example 3.8: Heptagonal graceful labeling of $P_n(1, 2, \dots, n)$ [4] is shown below

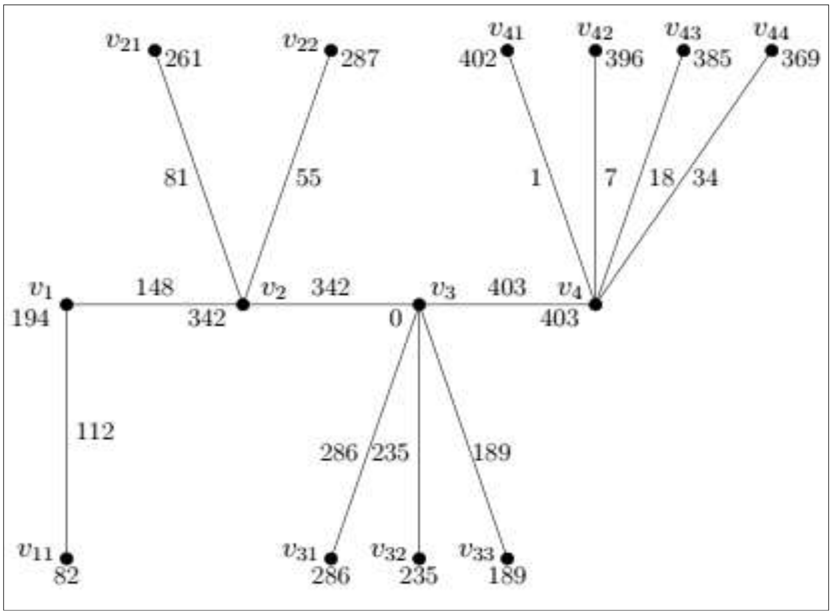

Fig 6: Heptagonal graceful labeling of $P_4(1, 2, \dots, n)$

Conclusion

Here we investigate four results corresponding to heptagonal graceful labeling. Analogous work can be carried out for other families and in the context of different types of graph labeling techniques.

References

1. Devaraj J, Thankam TA. On Triangular Graceful Graphs. Bull Pure Appl Sci Math Stat. 2011;30(2):279-285.
2. Gallian JA. A Dynamic Survey of Graph Labeling. Electron J Combin. 2018;1(Dynamic Surveys)
3. Lina SS, Asha. Odd Triangular Graceful Labeling on Simple Graphs. Malaya J Matematik. 2020;8(4):1574-1576.
4. Mahendran S, Murugan K. Pentagonal Labeling of Some Graphs. World Sci News. 2021;155:98-112.
5. Mangaleswaran S, Ganapathyraman R. Odd Triangular Graceful Labeling and Odd Triangular Graceful Number of Special Trees and Some Graphs. Math Stat Eng Appl. 2022;71(4):10982-10995.
6. Nisaya MP, Ramesh DST. Pentagonal Graceful Labeling of Caterpillar Graphs. Int J Eng Dev Res. 2018;6(4):150-154.