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Mathematical formulation of the steady state -flow equation for isotropic porous media, based on combining three independent equations, continuity equation, motion equation and state equation

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Abstract
Oil and Gas accumulations located in the porous spaces of the formation exist as a single hydraulically connected system. During the life of an oil field the production tends to pass through a number of stages, based on this it is very important to know as much about types of flow systems in reservoir rock in order to use the appropriate model to describe the relationship between the pressure and flow rate. Types of reservoir fluids with their representative equations and the combination of the three equations; the equation of continuity, the equation of motion and the equation of state treated not only in the physical concept but also in the mathematical one step by step deriving the basic equations applicable to each of the types of fluids in the steady state and in isotropic porous media, are the object of this paper.

Keywords: Compressibility, steady-state flow, reservoir, isotropic, equation

1. Introduction
Fluid flow in the porous medium represents one of the basic disciplines of the reservoir engineering. The majore influences on fluid flow, number of phases present, flow regimes, compresibility of the fluid and geometry of flow systems plays a key role for the determination of different hydrodinamic parameters.
All particle movements and the physico-chemical processes that characterize them have long received considerable attention in many fields such as colloidal, chemical, biomedical and environmental engineering [1]. The flow in porous media occurs in many natural and industrial processes, starting from the processes of underground filtration including here (The flow of fluids in the layer whether single-phase, two-phase or three-phase) as well as the flow of fluids or the process of penetration of different fluids, for example through the soil, or in different asphalt or textile layers (Filters), etc. [6]. The space where the fluid moves in the oil bearing rock is made up of a very large number of gaps and very fine channels, which form a tangled network spread in a different way and with cross-sections that change according to the geometric shape. Therefore, starting from the practical meaning of the word, by agreement, we will call the movement of fluids in porous media and fractures filtration [10, 14, 15]. The filtration process in porous media is affected by various forces, and its essence is to consume energy and produce fluid through the wellbore [2, 5, 8]. Mathematical models of filtration are generally based on Darcy's law [5]. In the laminar flow region where the fluid velocity is low compared to the turbulent flow where the fluid viscosity is high and the pressure gradient is high which increases at a greater rate than the flow rate, we will say that Darcy's law applies only in the laminar flow region [3]. During the application of Darcy's law, that part of the rock or the structure of the porous space, whose dimensions are quite large compared to the size of the pore channels, is taken into consideration. In other words, it is a statistical law that averages the behavior of many pore channels [3]. PVT laboratory studies for the determination of fluid properties, pressure gradients during the filtration process, physical properties of the rock (Matrix) included here; permeability, porosity and saturation, as well as the geometry and type of flow condition, make possible the full characterization of the mechanism of fluid flow through a porous medium [7].

Mechanical and physical properties of fluids, the main characteristics of the tank such as; the type of fluid, the geometry of the tank and the number of flowing fluids in the reservoir, as well as the determination of the initial conditions help in the mathematical analysis of the solution of the filtration equations [9, 11, 13].
Based on the above, I first started with the analytical treatment of the type of fluids; incompressible, slightly compressible and compressible, reflecting step by step the form of mathematical equations, which will then be combined with the physical and mathematical context of the equations of continuity, state and movement of fluids to reach the final solution, that of fluid-type filtration in steady state and isotropic porous media.
Regarding incompressible fluids whose volume does not change with the change in pressure, so their volume and density remain constant, can write:

$$\frac{\partial v}{\partial P} = 0; \frac{\partial \rho}{\partial P} = 0; (1)\, V = V_{ref} = const \; and \; \rho = \rho_{ref} = const$$

Regarding slightly compressible liquids, which show small changes in volume and density in relation to the change in pressure per unit, that is, when their volume and density do not remain the same as the reference (Initial) volume and density when the pressure changes, we can write [11]:

$$C = -\frac{1}{V} * \frac{\partial V}{\partial P} \quad (2)$$

Based on equation 2, making the relevant substitutions and then integrating it, we will get equation 3 as below:

$$V = \frac{M}{\rho} \Rightarrow \frac{\partial V}{\partial P} = \frac{\partial}{\partial P}\left(\frac{M}{\rho}\right) \Rightarrow \frac{\partial V}{\partial P} = M * \frac{\partial}{\partial P}\left(\frac{1}{\rho}\right)$$

$$\frac{\partial V}{\partial P} = -M * \frac{1}{\rho^2}$$

$$C = -\frac{1}{\frac{M}{\rho}} * \left(-M * \frac{1}{\rho^2} * \frac{\partial \rho}{\partial P}\right) \rightarrow C = \frac{1}{\rho} * \frac{\partial \rho}{\partial P}$$

$$C = -\frac{1}{\frac{M}{\rho}} * \left(-M * \frac{1}{\rho^2} * \frac{\partial \rho}{\partial P}\right) \rightarrow C = \frac{1}{\rho} * \frac{\partial \rho}{\partial P}$$

$$C * dp = \frac{d\rho}{\rho} \rightarrow \int C * dp = \int \frac{d\rho}{\rho} \rightarrow \int_{P_{ref}}^{P} C * dp = \int_{\rho_{ref}}^{\rho} \frac{d\rho}{\rho} \rightarrow C * (P - P_{ref}) = ln(\rho - \rho_{ref})$$

$$C * (P - P_{ref}) = \ln\left(\frac{\rho}{\rho_{ref}}\right) \rightarrow \frac{\rho}{\rho_{ref}} = e^{C(P - P_{ref})} \rightarrow \rho = \rho_{ref} * e^{C \times (P - P_{ref})}$$

$$e^{\times} = 1 + \frac{\times}{1!} + \frac{\times^2}{2!} + \cdots + \frac{\times^n}{n!}$$

$$e^{C \times (P - P_{ref})} = 1 + \frac{C \times (P - P_{ref})}{1!}$$

Because the exponent x [which represents the term $C * (P - P_{ref})$] is very small, the e^{X} term can be approximated by truncating equation: $e^{\times} = 1 + \times$
By this equaetion will have:

$$\rho = \rho_{ref} * e^{C \times (P - P_{ref})} \text{ or } \rho = \rho_{ref} * [1 - (P_{ref} - P)] \quad (3)$$

Where,
V → volume at pressure P
ρ→ density at pressure P
$V_{ref\rightarrow}$ Volume at initial (reference) pressure P_{ref}
ρ_{ref} → Density at initial (reference) pressure P_{ref}
It should be pointed out that crude oil and water systems fit into this category [11].

On the other hand, these are fluids whose volume changes a lot as a function of the change in pressure; this category includes all gases and according to the definition of the isothermal compressibility of gas will have equation 4 as follows:

$$C_g = -\frac{1}{V} * \frac{\partial V}{\partial P} \quad (4)$$

From the real gas equation of state:

$$P * V = Z * n * R * T \Rightarrow V = \frac{Z * n * R * T}{P}$$

Differencing the above equation with respect to pressure at constant temperature gives:

$$\frac{\partial V}{\partial P} = \frac{\partial}{\partial P}\left(\frac{Z * n * R * T}{P}\right) \Rightarrow \frac{\partial V}{\partial P} = n * R * T \times \frac{\partial}{\partial P}\left(\frac{Z}{P}\right)$$

Mathematically the formula is:

$$\frac{\partial}{\partial \times}\left(\frac{Y}{\times}\right) = \frac{Y' * \times - \times' * Y}{\times^2}$$

From this will have:

$$\frac{\partial V}{\partial P} = n * R * T * \frac{\frac{\partial Z}{\partial P} * P - \frac{\partial P}{\partial P} * Z}{P^2} \rightarrow \frac{\partial V}{\partial P} = n * R * T * \frac{\frac{\partial Z}{\partial P} * P - 1 * Z}{P^2} \leftrightarrow \frac{\partial V}{\partial P} = n * R * T * \left[\frac{1}{P} * \frac{\partial Z}{\partial P} - \frac{Z}{P^2}\right]$$

$$\frac{\partial V}{\partial P} = n * R * T * \left[\frac{1}{P} * \frac{\partial Z}{\partial P} - \frac{Z}{P^2}\right] \leftrightarrow C_g = -\frac{P}{Z * n * R * T} * n * R * T * \left[\frac{1}{P} * \frac{\partial Z}{\partial P} - \frac{Z}{P^2}\right] \leftrightarrow$$

$$\leftrightarrow C_g = \frac{1}{P} - \frac{1}{Z} * \left(\frac{\partial Z}{\partial P}\right)$$

The final equation is:

$$C_g = \frac{1}{P} - \frac{1}{Z} * \left(\frac{\partial Z}{\partial P}\right) \quad (5)$$

Figure 1 and 2 show schematic illustrations of the volume and density changes as a function of pressure for the three types of fluids [11].

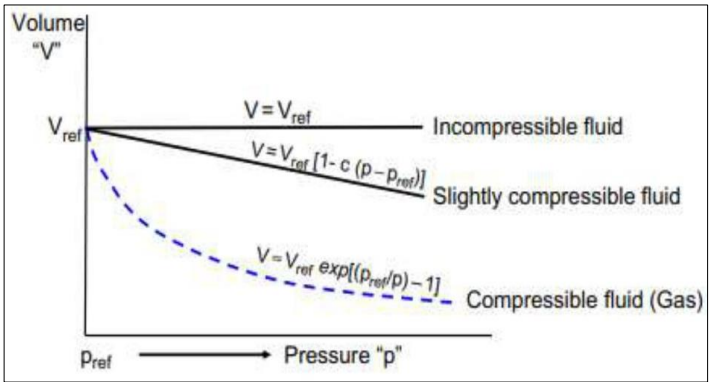

Fig 1: Pressure-volume relationships [11]

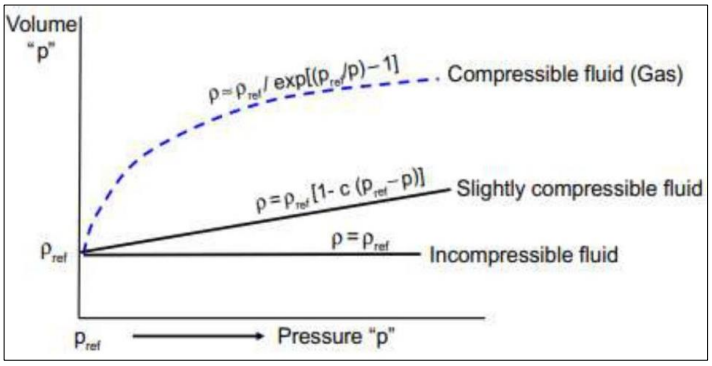

Fig 2: Fluid density versus pressure for different fluid types [11]

2. Methodology

Below, based on the fundamental physics principle of mass conservation, the mathematical treatment of the continuity equation is given. Considering an elemental volume representative of porous media Fig 3 [2], and based on the definition of the physics principle of mass conservation which describes a relationship of diference between the amount of reservoir fluids entering and leaving the representative elementary volume [8], I first wrote the corresponding equations for each direction of the fluids that will enter this volumetric element.

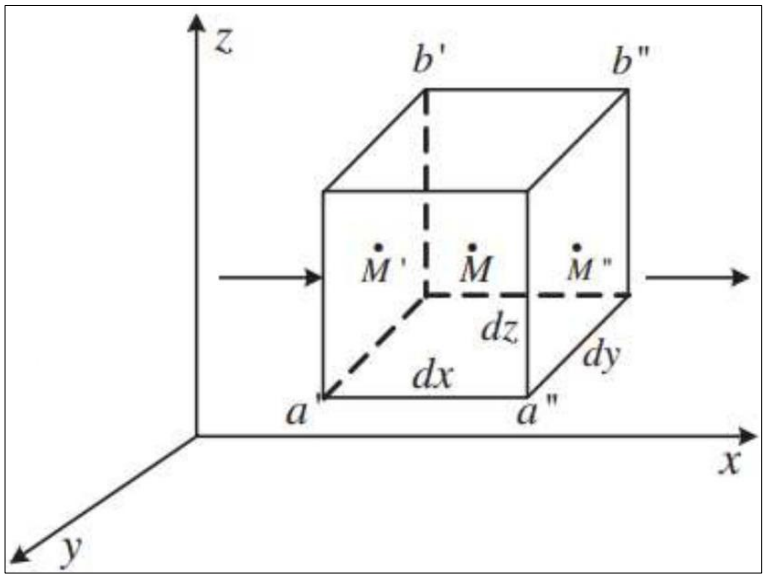

Fig 3: Representative elementary volume (REV) from the reservoir [2]

$$\begin{bmatrix} M_x = \rho * V_x * d_y * d_z * d_t \\ M_y = \rho * V_y * d_x * d_z * d_t \\ M_z = \rho * V_z * d_x * d_y * d_t \end{bmatrix} \rightarrow \text{Mass entering volume during interval dt}$$

The mass of fluid that will leave during the time interval dt is the sum of the mass of fluid that flows into this representative elementary volume of the porous medium and the change caused by this mass of fluid. Based on this below I have written the three equations related to the removal of the fluid from the three directions as follows:

$$\begin{bmatrix} M_x + dM_x = \rho * V_x * d_y * d_z * d_t + \frac{\partial}{\partial x}(\rho * V_x * d_y * d_z * d_t) * d_x \\ M_y + dM_y = \rho * V_y * d_x * d_z * d_t + \frac{\partial}{\partial y}(\rho * V_y * d_x * d_z * d_t) * d_y \\ M_z + dM_z = \rho * V_z * d_x * d_y * d_t + \frac{\partial}{\partial z}(\rho * V_y * d_x * d_z * d_t) * dz \end{bmatrix} \rightarrow \text{Mass leaving during intervaldt}$$

Bearing in mind that the change in the mass of the fluid during time dt is; $dM = M - (M + dM)$ then the equations will be as below:

$$dM_x = \rho * V_x * d_y * d_z * d_t - \rho * V_x * d_y * d_z * d_t - \frac{\partial}{\partial x}(\rho * V_x * d_y * d_z * d_t) * d_x$$
$$dM_y = \rho * V_y * d_x * d_z * d_t - \rho * V_y * d_x * d_z * d_t - \frac{\partial}{\partial y}(\rho * V_y * d_x * d_z * d_t) * d_y$$
$$dM_z = \rho * V_z * d_x * d_y * d_t - \rho * V_z * d_x * d_y * d_t - \frac{\partial}{\partial z}(\rho * V_y * d_x * d_y * d_t) * dz$$

From these equations will have:

$$dM_x = -\frac{\partial}{\partial x}(\rho * V_x * d_y * d_z * d_t * d_x)$$
$$dM_y = -\frac{\partial}{\partial y}(\rho * V_y * d_x * d_z * d_t * d_y)$$
$$dM_z = -\frac{\partial}{\partial z}(\rho * V_z * d_x * d_y * d_t * dz)$$

$$dM_{(motion)} = dM_{\times} + dM_{\gamma} + dM_z \rightarrow dM_{(motion)} - \left[\rho * V_{\times} + \rho * V_{\gamma} + \rho * V_z\right] * d_{\times} * d_{\gamma} * dz * d_t$$

$$dM_{(time)} = \frac{\partial}{\partial t}\left(\rho * \emptyset * d_{\times} * d_{\gamma} * d_z\right) * d_t$$

$$dM_{(motion)} = dM_{(time)} \rightarrow -\left[\rho * V_{\times} + \rho * V_{\gamma} + \rho * V_z\right] * d_{\times} * d_{\gamma} * dz * d_t =$$

$$= \frac{\partial}{\partial t}\left(\rho * \emptyset * d_{\times} * d_{\gamma} * d_z\right) * d_t \leftrightarrow -\left[\frac{\partial}{\partial \times}(\rho * V_{\times}) + \frac{\partial}{\partial \gamma}(\rho * V_{\gamma}) + \frac{\partial}{\partial z}(\rho * V_z)\right] - \frac{\partial}{\partial t}[\rho * \emptyset] = 0 \quad (6)$$

By multiplying both sides of the equation with the minus sign, we will have:

$$\left[\frac{\partial}{\partial \times}(\rho * V_{\times}) + \frac{\partial}{\partial \gamma}(\rho * V_{\gamma}) + \frac{\partial}{\partial z}(\rho * V_z)\right] - \frac{\partial}{\partial t}[\rho * \emptyset] = 0 \quad (7)$$

This equation is the continuity equation of single-phase fluid, which is also named as the mass conservation equation.
$\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} = \nabla \rightarrow$ Hamiltonian Operator.

From this will have:

$$\nabla(\rho * V) = -\frac{\partial}{\partial t}(\rho * \emptyset) \text{ or } div\left(\rho * \vec{V}\right) = -\frac{\partial}{\partial t}(\rho * \emptyset) \quad (8)$$

Eq 8 is called the continuity equation. Based on the same way by analyzing it step by step I proceeded with the equation of motion as well as the equation of state for the rock as follows.
In the flow mechanics of porous media, the equation of motion is generally expressed by the flow velocity in the differential form of Darcy's law [8]. The differential forms of the equation of motion in three directions are:

$$v_{\times} = -\frac{k}{\mu} * \frac{\partial P}{\partial \times};\ v_{\gamma} = -\frac{k}{\mu} * \frac{\partial P}{\partial \gamma};\ v_z = -\frac{k}{\mu} * \frac{\partial P}{\partial z}$$

It can also be written as:

$$\nu = -\frac{k}{\mu} * \text{grad}(p) \text{ or } \nu = -\frac{k}{\mu} * \nabla P \quad (9)$$

On the other hand, considering the rock elastic compressibility coefficient is defined as [1]:

$$C_f = -\frac{1}{V_f} * \frac{\partial V_P}{dP}$$

Where, V_f is the apparent volume (containing pores and grains), and V_P is the pore volume as well as definition of porosity $\emptyset = \frac{V_P}{V_f}$ I continued with substitution and integration deriving equation 10 as below:

$$C_f = -\frac{1}{V_f} * \frac{\partial V_P}{dP} = -\frac{d}{dP} * \left(\frac{V_P}{V_f}\right) = \frac{d\emptyset}{dP}$$

$$C_f = \frac{d\emptyset}{dP} \rightarrow \int_{\emptyset_0}^{\emptyset} d\emptyset = \int_{P_0}^{P} C_f * dP \rightarrow \emptyset - \emptyset_0 = C_f * (P - P_0) \rightarrow \emptyset = \emptyset_0 + C_f * (P - P_0)$$

$$\emptyset = \emptyset_0 + C_f * (P - P_0) \quad (10)$$

Based on the above equations; the equation of continuity, the equation of motion, the equation of state for the fluid and the rock, treated in the physical concept and then mathematically step by step in deriving the definitive equation, of each of them, below will help us to determine the applicable formulas for all three types of fluids in the steady state.

3. Results and Discussion

All the equations that I treated above in the physical concept as well as in the mathematical analysis for each of them starting from equation 1 to equation 10, as well as taking into account the isotropic porous medium and continuing with the combination and integration for each of them, will help me below to achieve the goal of this paper**.**

Based on equation of continuity (8), equation of motioin (9) and conditions for incompressible fluid which are $\frac{\partial v}{\partial P} = 0$ and $\frac{\partial \rho}{\partial p} = 0$ will have:

$$\nabla(\rho * V) = -\frac{\partial}{\partial t}(\rho * \emptyset) \Leftrightarrow \nabla\left(\rho * (-\frac{k}{\mu} * \nabla P)\right) = -\frac{\partial}{\partial t}(\rho * \emptyset)$$

$$\nabla(\rho * \nabla P) = -\frac{\emptyset * \mu}{k} * \frac{\partial \rho}{\partial t} = 0 \rightarrow \nabla^2 \mathrm{P} = 0 \leftrightarrow \mathbf{\nabla^2 P} = \mathbf{\frac{\partial^2 P}{\partial \times^2} + \frac{\partial^2 P}{\partial \gamma^2} + \frac{\partial^2 P}{\partial z^2}} \quad (11)$$

This is the basic differential equation of incompressible fluid for steady state flow in the isotropic porous media.
Regarding the filtration of slightly compressible fluids in isotropic porous media, based on Eq. 9 the following transformation can be made

$$\mathrm{C} * \mathrm{dP} = \frac{d\rho}{\rho} \rightarrow \int C * dp = \int \frac{d\rho}{\rho} \rightarrow \int_{P_0}^{P} C * dP = \int_{\rho_0}^{\rho} \frac{d\rho}{\rho} \rightarrow C * (P - P_0) = ln(\rho - \rho_0)$$

$$\mathrm{C} * (\mathrm{P} - \mathrm{P_0}) = \ln(\rho - \rho_0) \rightarrow \nabla[C * (P - P_0)] = \nabla\left(ln\frac{\rho}{\rho_0}\right) \rightarrow C * \nabla P = \frac{1}{\rho} * \nabla\rho \rightarrow \nabla P = \frac{1}{\rho * C} * \nabla\rho$$

$$\nabla\left(\rho * \frac{1}{\rho * C} * \nabla\rho\right) = -\frac{\emptyset * \mu}{k} * \frac{\partial \rho}{\partial t} \rightarrow \nabla^2\rho = \frac{\emptyset * \mu * C}{k} * \frac{\partial \rho}{\partial t} \rightarrow \nabla^2\rho = \frac{1}{\frac{k}{\emptyset * \mu * C}} * \frac{\partial \rho}{\partial t} \leftrightarrow \mathbf{\nabla^2 \rho} = \mathbf{\frac{1}{\aleph} * \frac{\partial \rho}{\partial t}} \quad (13)$$

Where, $\aleph = \frac{k}{\emptyset * \mu * C}$ → piezometric conductivity
From a thermodynamic point of view, gas filtration can be considered adhering to isothermal or adiabatic process conditions. Below I will present the gas equation in the case of an adiabatic process. In the case of an adiabatic process (Constant entropy), the equation of state based on the adiabatic indicator and the combination of the above equations will be as follows:

$$\nabla(\rho * V) = -\frac{\partial}{\partial t}(\rho * \emptyset)$$

$$\nu = -\frac{k}{\mu} * \nabla P$$

$$\nabla(\rho * \nabla P) = -\frac{\emptyset * \mu}{k} \times \frac{\partial \rho}{\partial t} = 0 \Rightarrow \nabla\rho \times \nabla P = \frac{\emptyset * \mu}{k} * \frac{\partial \rho}{\partial t}$$

$$\frac{P}{\rho^k} = \frac{P_0}{\rho_0^k} \rightarrow \frac{P}{P_0} = \left(\frac{\rho}{\rho_0}\right)^k \rightarrow P = \frac{P_0}{\rho_0^{\frac{1}{n}}} * \rho^{\frac{1}{n}} \rightarrow \nabla P = \frac{P_0}{\rho_0^{\frac{1}{n}}} * \frac{1}{n} * \rho^{\frac{1}{n}-1} * \nabla\rho \rightarrow \nabla\left(\frac{P_0}{\rho_0^{\frac{1}{n}}} * \frac{1}{n} * \rho^{\frac{1}{n}-1} * \nabla\rho\right) =$$

$$= \frac{\emptyset * \mu}{k} \times \frac{\partial \rho}{\partial t} \rightarrow \frac{P_0}{\rho_0^{\frac{1}{n}}} * \frac{1}{n} * \left[\nabla\rho * \rho^{\frac{1}{n}-1} * \nabla\rho\right] = \frac{\emptyset * \mu}{k} \times \frac{\partial \rho}{\partial t}$$

$$\frac{P_0}{\rho_0^{\frac{1}{n}}} * \frac{\left[\nabla\rho * \rho^{\frac{1}{n}-1} * \nabla\rho\right]}{n} = \frac{\emptyset * \mu}{k} \times \frac{\partial \rho}{\partial t} \rightarrow \nabla\rho^{\frac{1}{n}} * \nabla\rho = \frac{\rho_0^{\frac{1}{n}} * n}{P_0} * \frac{\emptyset * \mu}{k} \times \frac{\partial \rho}{\partial t}$$

$$\nabla\rho^{\frac{1}{n}} * \nabla\rho = \nabla\left[\left(\nabla\frac{\rho_0^{\frac{1}{n}} + 1}{\frac{1}{n} + 1} * \frac{1}{\frac{1}{n} + 1} * \frac{1}{n} + 1 * \rho^{\frac{1}{n}+1-1} * \rho\right)\right] = \nabla\rho^{\frac{1}{n}} * \nabla\rho$$

$$\nabla\left[\left(\nabla\frac{\rho_0^{\frac{1}{n}} + 1}{\frac{1}{n} + 1}\right)\right] = \frac{\rho_0^{\frac{1}{n}} * n}{P_0} * \frac{\emptyset * \mu}{k} * \frac{\partial \rho}{\partial t} \rightarrow \nabla\left[\left(\nabla\frac{\rho_0^{\frac{1}{n}} + 1}{\frac{n+1}{n}}\right)\right] = \frac{\rho_0^{\frac{1}{n}} * n}{P_0} * \frac{\emptyset * \mu}{k} * \frac{\partial \rho}{\partial t} \rightarrow$$

$$\rightarrow \nabla\left[\left(\nabla\frac{\rho_0^{\frac{1}{n}} + 1}{1 + n}\right)\right] = \frac{\rho_0^{\frac{1}{n}} * n}{P_0} * \frac{\emptyset * \mu}{k} \times \frac{\partial \rho}{\partial t} \leftrightarrow \nabla\left(\nabla\rho^{\frac{1}{n}+1}\right) * \frac{n}{n+1} = \frac{\rho_0^{\frac{1}{n}} * n}{P_0} * \frac{\emptyset * \mu}{k} * \frac{\partial \rho}{\partial t}$$

The final gas flow equation for steady state-flow in the case of the adiabatic process is as follows:

$$\nabla^2 \rho^{\frac{1}{n}+1} = \frac{(1+n)*\rho_0^{\frac{1}{n}}*\emptyset*\mu}{P_0*k} * \frac{\partial \rho}{\partial t} \leftrightarrow \frac{\partial^2 \rho^{\frac{1}{n}+1}}{\partial \times^2} + \frac{\partial^2 \rho^{\frac{1}{n}+1}}{\partial \gamma^2} + \frac{\partial^2 \rho^{\frac{1}{n}+1}}{\partial z^2} = \frac{(1+n)*\rho_0^{\frac{1}{n}}*\emptyset*\mu}{P_0*k} * \frac{\partial \rho}{\partial t} \quad (13)$$

For the study of the filtration of fluids in the porous media, it is necessary to know the thermodynamic conditions, in which the movement is developed, or as it is said differently, the condition of the fluids The basic filtration equations are differential equations that describe the process of fluid movements in the layer, studying it in space and time. To formulate the basic equations, we rely on the application of the principles and well-known laws of physics such as: the principle of conservation of mass, the law of state and the law of force in the process of filtering the fluid in place

4. Conclusion

1. The accuracy of the equations is related to the degree of knowledge of the physical process and its analytical expression through the above principles and laws.
2. The equation of continuity, motion and state treated above not only in the physical principle but also in detail for each equation in mathematical analysis and their combination to derive practical formulas, helps in their verification and accuracy.
3. In the basic equation of the filtration of non-homogeneous (heterogene) fluids and in anisotropic porous media, the filtration generally has an unstable character and some of the parameters are neglected in the equations due to the difficulty they present.
4. In this paper, the equations of fluid filtration in steady state in the case of uniformly isotropic porous media are treated. Based on these equations and the mathematical truth that they present, it can be said that the basic equations of homogeneous liquid filtration in uniformly isotropic porous media are the most accurate formulations achieved to date for the filtration process.

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